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Peer Review

Review of: "Restoring Heisenberg-Limited Precision in Non-Markovian Open Quantum Systems via Dynamical Decoupling"

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Reviewer: Harish Parthasarathy, ECE division, Netaji Subhas University of Technology. This paper presents a novel algorithm for obtaining better (ie, lesser vari ance) estimates of a parameter or a set of parameters on which a quantum state depends. The quantum state evolves according to a Hamiltonian consist ing of the sum of a system Hamiltonian which depends upon the parameter to be estimated, an environment Hamiltonian and an interaction Hamiltonian between the system and the environment. If the initial state of the system and environment is pure and separable (ie, disentangled), ie, expressible as the ten sor product of a pure system state and a pure environment state, then after it evolves for a duration t, the total state of the system and environment will still be pure since the total evolution is unitary, however, it will be an entangled state between the system and environment because the total Hamiltonian contains an interaction component, and therefore, the state of the system alone after time t (obtained by partial tracing the total state over the environment Hilbert space) will become mixed. Both the total state and the system state will depend on the parameter because the Hamiltonian was a function of the parameter. The authors then make use of an important result, namely that the Fisher informa tion matrix of a partially traced state is smaller than that of the original state. In this context, it means that the Fisher information matrix of the total pure state is more than that of the system state, because the latter is obtained by partial tracing the former over the environment. Since the Cramer Rao lower bound on the parameter covariance is the inverse of the Fisher information, it follows that any efficient inference of the parameter based on the system state alone would be more than that based on the total pure state. This means that the presence of the interaction Hamiltonian between system and environment generates a mixed state of the system and

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therefore increases the covariance of the estimated parameter, as compared to the situation when this interaction Hamiltonian is absent. The authors also make an important remark which is well known in the literature, namely, that if the inference of the parameter on which a state depends is based on making a POV measurement, then the asso ciated classical Fisher information calculated using the probability distribution generated by this POVM on the state would be different for different POVM's and its supremum over all POVM's would yield the quantum Fisher informa tion of the state. This amounts to saying that there equals an optimal POVM for which the classical Fisher information obtained from the classical probabil ity distribution generated by making this measurement on the state equals the quantum Fisher information calculated using the symmetric logarithmic deriva tive of the state w.r.t the parameter. The authors then present what is the main result of the paper, namely, they derive some general conditions that the components of the Hamiltonian should statisfy so that there exists a control discrete time unitary system-space evolution which when applied to the total Hamiltonian and then averaged out, will result in the interaction Hamiltonian moving away to the environmental sector, ie, the resulting effective interaction Hamiltonian would act now only in the environmental Hilbert space. I think that this algorithm of causing the interaction Hamiltonian to move away to the 1 environmental sector thereby causing the controlled state of the system and environment to become disentangled and hence the controlled system state to be pure, thereby increasing its Fisher information measure is really remarkable. However, averaging unitary actions on a mixed state amounts to applying a con trol TPCP map and hence I would like to see how such a disentangled state can result by applying such control unitaries followed by averaging on the unitarily evolved state rather than on the Hamiltonian. Specifically, if U(t) denotes the total unitary evolution U(t) = exp(-it(HS ⊗IE +IS ⊗HE +HSE)) and Uc[n] are control system unitaries, then the author's give conditions for which N-1 N-1 n=0 (Uc[n]⊗IE)HSE(Uc[n]⊗IE) to be of the form IS⊗JE. Instead, one can require that N-1 N-1 n=0 (Uc[n]⊗IE).U(t)(|ψS(0)⊗ψE(0) >< ψs(0)⊗ψE(0)|)U(t)∗(Uc[n]∗⊗IE) be pure and distentangled, ie, of the form |ψS(t) ⊗ ψE(t) >< ψS(t) ⊗ ψE(t)|. A brief explanation of how this can be achieved would help me understand the situation better. The author instead considers control unitary actions on Hamiltonians for the following reason: After adding the control Hamiltonian HC(t), the total Hamiltonian becomes H(t) = HS(t)⊗IE +HC(t)⊗IE +IS ⊗HE(t)+HSE(t) and therefore using the interaction picture theory of Dirac and Dyson in which the unperturbed Hamiltonian is HC(t) ⊗ IE, the associated unitary evolution after control is given by U(t) = (UC(t)⊗IE)W(t) where UC(t) = T{exp(-i t 0 HC(s)ds)} is the evolution generated by HC(.) and W(t) satisfies the differential equation W′(t) = -iV(t)W(t) where V(t) = (UC(t)∗ ⊗IE).(HS(t) ⊗IE +IS ⊗HE(t)+HSE(t))(UC(t)⊗IE) =HSeff(t) ⊗IE +IS

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⊗HE(t)+(UC(t)∗ ⊗IE)HSE(t)(UC(t)⊗IE) with HSeff(t) = UC(t)∗HS(t)UC(t) being the effective system Hamiltonian after control as seen in the control Hamil tonian frame. The crucial idea used by the authors now is that if HSE is time 2 independent, then the contribution of the last term to the evolution is the ex ponential of -i times t 0 ((UC(t)∗ ⊗ IE)HSE(t)(UC(t) ⊗ IE)dt followed by time ordering and if the control unitaries are piecewise constant, ie they act only at discrete times nT,n = 0,1,...,N - 1, then the evolution due to the last term is the same as that corresponding to an effective Hamiltonian N-1 (1/N) n=0 (UC(nT)∗ ⊗IE)HSE(UC(nT)⊗IE) and the entire idea for decoupling thus boils down to selecting the control Hamil tonian and hence the control unitaries UC(nT) so that the above average as sumes the form IS ⊗ JE. That the authors could derive conditions for this is really very interesting and fundamental to quantum parameter estimation the ory. I would like to observe here that if G is a finite group with N elements and g →UC(g) is an irreducible unitary representation of G in the system Hilbert space, then for any system operator X, (1/N) g∈GUC(g)∗XUC(g) is a scalar c(X) times the identity in system Hilbert space (c(X) = Tr(X)/d where d is the dimension of the system Hilbert space, this is precisely the way in which Schur's lemmas are derived in representation theory of groups), and hence, such a decoupling of the interaction can also be achieved using the formula (1/N) where (UC(g)∗ ⊗IE).HSE.(U(g) ⊗IE) = IS ⊗JE g∈G JE =TrS(HSE)/d More generally, we could replace G by a compact group with UC an irreducible representation of G in HS and perform decoupling using UC(g)∗XUC(g)dg = Tr(X).IS G for any system operator X, where dg is the normalized Haar measure on G. The paper is well written and contains a fundamentally new idea that system based unitary averaging of the Hamiltonian can decouple the system and envi ronmental dynamics thereby increasing the Fisher information in it about the parameter and thereby decrease the covariance matrix of an efficient parameter estimate based on system measurements alone. System based unitary averag ing of the interaction Hamiltonian is further, naturally achieved by viewing the dynamics in the interaction picture with the unperturbed Hamiltonian taken as the control Hamiltonian.

Declarations

Potential competing interests: No potential competing interests to declare.

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