

Peer Review

Review of: "Anomalous Scaling and Magnetization Temperature Dependence in Cubic Ferromagnetic Crystals"

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The manuscript is devoted to the systematic investigation of the spontaneous magnetization in three-dimensional magnetic systems. A diagrammatic technique is developed and applied to some real ferromagnetic materials. The critical exponent β is also calculated and discussed. The manuscript is quite well-written and contains interesting results that deserve to be published. I have only one conceptual objection. In this respect, the corresponding corrections are needed in the text.

Since the concept of the "anomalous scaling" has its exact meaning in the critical phenomena, the title of the article (as well as the interpretation of the difference between the obtained index β close to $1/3$ and the $\beta=1/2$ of the mean field theory as the anomalous scaling) is misleading. Let me explain this assertion.

The scaling behavior in the studied magnetic systems is the standard scaling behavior of the form $m = (-t)^{\beta} f(x)$ (*)

where m is magnetization, $t=(T-T_c)/T_c$, T is the temperature, T_c is the critical temperature, β is the corresponding critical exponent, and $f(x)$ is the so-called scaling function of a dimensionless (in the critical sense) parameter $x=x(t, h)$, where h is the external magnetic field. We would have anomalous scaling in this system if the scaling function $f(x)$ had singular behavior that would modify the value of the critical exponent β .

However, the scaling function $f(x)$ in the ferromagnetism (as well as in the antiferromagnetism and ferrimagnetism) is a nonsingular function of x and gives no corrections to the scaling exponent β (that is why the Eq.(*) is usually written simply as $m \propto (-t)^{\beta}$, i.e., without the scaling function). Thus, since the concept of the anomalous scaling is standardly related to the singular

behavior of the scaling function that nontrivially changes the standard critical exponents, no anomalous scaling is present here.

The fact that the mean field theory gives $\beta=1/2$ and the real value of the exponent of the studied 3-dimensional systems is close to $1/3$ (and is the same for all models or systems from the same universality class) is related to the fact that the mean field approximation is effectively an infinite-dimensional system and therefore belongs to the different universality class, namely, to the so-called mean-field universality class (the class of effectively infinite-dimensional systems). In this respect, there is no sense to compare these different β indices since they characterize different universality classes. Thus, the difference between these indices is principal and has nothing to do with the standard concept of the anomalous scaling.

Note that a typical example of the anomalous scaling (in the genuine statistical sense) is the anomalous scaling in the fully developed turbulent systems, where the existence of deviations from the Kolmogorov scaling is directly related to the nontrivial (singular) behavior of the corresponding scaling functions.

Declarations

Potential competing interests: No potential competing interests to declare.