

Research Article

A Narrow-Band Prediction for Proton Decay from Geometrodynamic $SO(10)$ with $f_a = M_X/2$

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I present a sharp and falsifiable prediction for the proton lifetime in the channel $p \rightarrow e^+ \pi^0$ within an $SO(10)$ grand unified setting. The only structural assumption beyond standard unification that I make is to tie the Peccei–Quinn scale to the unification scale, $f_a = M_X/2$. This identification, motivated by a broader geometrodynamic $SO(10)$ framework, collapses model dependence and yields a narrow lifetime band. Using two-loop unification with moderate thresholds, I find $M_X \simeq 6.8 \times 10^{15} \text{ GeV}$ and $\alpha_{unif}^{-1} \simeq 36.0$, which imply a central estimate $\tau_p(p \rightarrow e^+ \pi^0)^{(0)} \simeq 9.05 \times 10^{34} \text{ yr}$. After marginalizing over threshold and hadronic inputs I obtain a conservative 95% theory interval $\tau_p(p \rightarrow e^+ \pi^0) \in [6.46 \times 10^{34} \text{ yr}, 1.16 \times 10^{35} \text{ yr}]$. A future bound $\tau_p(p \rightarrow e^+ \pi^0) > 1.16 \times 10^{35} \text{ yr}$ would exclude this benchmark at 95% C.L. The link to the PQ sector further implies $(m_a, g_{a\gamma\gamma}) = (1.68 \text{ neV}, 2.55 \times 10^{-19} \text{ GeV}^{-1})$, which I use only as an internal consistency check. The purpose of this paper is deliberately narrow: I isolate one testable consequence (a narrow-band prediction for $\tau_p(p \rightarrow e^+ \pi^0)$), state all assumptions explicitly, and provide clear falsification criteria without relying on UV-complete details of the geometrodynamic construction. All results assume single-step $SM \rightarrow SO(10)$ unification with dominant dimension-six gauge exchange (color-triplet scalars decoupled).

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I. Introduction

Grand unification promises that low-energy observables can point back to a high unification scale M_X . Among these observables, the proton lifetime is both conceptually clean and experimentally pivotal. Yet

in many otherwise compelling $SO(10)$ models, τ_p suffers from a wide spread due to threshold effects and model-specific details, making it difficult to formulate a prediction that is simultaneously narrow and falsifiable.

Scope and novelty. In this work I extract a single, reviewable consequence of a broader geometrodynamical $SO(10)$ framework: I assume that the Peccei–Quinn scale sits at half the unification scale,

$$f_a \equiv \frac{1}{2}M_X, \quad (1)$$

and I show that this structural identification is sufficient to produce a narrow-band prediction for the canonical mode $p \rightarrow e^+ \pi^0$. The rationale is twofold. First, tying f_a to M_X removes a free high-scale parameter that would otherwise dilute predictivity. Second, the induced correlation constrains the normalization of the leading dimension-six operators in a way that is robust against many UV-specific choices.

Why $f_a = M_X/2$.

In the geometrodynamical setup considered here, a single high-scale geometric modulus sets both the mass of the heavy gauge bosons and the Peccei–Quinn breaking scale. Identifying $M_{X,Y} \equiv M_X$ and $f_a = \frac{1}{2}M_X$ removes an otherwise free ultraviolet parameter and ties the normalization of the leading dimension-six operator directly to the unification point.

Context and related work

Proton decay has long been a cornerstone test of grand unification. Classic $SU(5)$ -type predictions ^[1] ^[2] were strongly constrained by null results in $p \rightarrow e^+ \pi^0$, while $SO(10)$ frameworks ^{[3][4]} typically widened the lifetime range due to threshold and model dependencies. My aim here is narrower: I assume a single structural identification, $f_a = M_X/2$, motivated by a geometrodynamical $SO(10)$ setting, and I show that this is sufficient to sharpen the prediction for $\tau_p(p \rightarrow e^+ \pi^0)$ to a band that is falsifiable with next-generation exposures. I keep more elaborate UV details off stage on purpose; the value of this note is a concrete, testable target derived from minimal assumptions.

See also early unification milestones and operator analyses in Refs. ^{[5][6]}.

Current limits in the $p \rightarrow e^+ \pi^0$ channel from Super-Kamiokande I–IV read $\tau/B(p \rightarrow e^+ \pi^0) > 2.4 \times 10^{34} \text{ yr}$ (90% C.L.) ^[7].

Related constructions and deconstructed scenarios appear in Refs. ^[8].

Geometrodynamical motivation (phenomenological benchmark)

Important caveat: I do not assume a full UV-complete construction. Instead, I adopt a phenomenological benchmark motivated by composite/moduli scenarios in which a single high-scale modulus sets both M_X and the PQ scale, naturally giving $f_a/M_X = \mathcal{O}(1)$. All quantitative results below rely only on the locking $f_a = M_X/2$ and on a one-parameter threshold prior; no additional UV specifics are used. A complete geometrodynamical derivation from first principles is deferred to future work. The value of this paper lies in isolating a sharp, falsifiable prediction that can be tested independently of the UV completion.

I embed the benchmark in a geometrodynamical $SO(10)$ framework where spacetime, gauge fields and the PQ sector emerge as composite excitations of a pre-geometric *fermionic condensate*. This language serves as heuristic motivation rather than a claim to UV completeness. A high-scale condensation at Λ_c induces (i) an effective Planck mass via

$$M_P^2 = \frac{N_f \mathcal{F}_{BS}}{16\pi^2} \Lambda_c^2,$$

(ii) an $SO(10)$ gauge sector with heavy vectors X, Y of mass $M_{X,Y} \equiv M_X$, and (iii) an axial $U(1)_{PQ}$ whose decay constant is *locked* to the unification scale. In this minimal realization the locking is

$$f_a = \frac{1}{2} M_X,$$

which removes a UV parameter and ties the normalization of the leading dimension-six operator directly to the GUT point. Fluctuations of the composite heavy spectrum near M_X generate correlated gauge-thresholds that I parametrize by a single knob β_{th} with adjoint-like sign pattern $c = \{+, +, -\}$ on $(\alpha_1, \alpha_2, \alpha_3)$.

In the present paper I *only* use this structural content— $f_a = M_X/2$ and the one-parameter threshold prior—to obtain a narrow, falsifiable prediction for $\tau_p(p \rightarrow e^+ \pi^0)$; detailed UV dynamics are left to future work. Readers who prefer to remain agnostic about the geometrodynamical origin may simply treat $f_a = M_X/2$ as an empirical *ansatz* to be tested experimentally: the proton lifetime prediction stands on its own, independent of any specific UV completion.

UV anchoring: natural derivation of the key inputs

Common modulus and the locking $f_a = \frac{1}{2} M_X$. Let Φ be the composite modulus whose VEV $v \equiv \langle \Phi \rangle$ triggers both GUT breaking and $U(1)_{PQ}$. Heavy vectors X, Y acquire

$$M_X = g_{GUT} c_X v,$$

while the axion is the phase of the PQ order parameter with canonical decay constant

$$f_a = \frac{c_a v}{N_{DW}},$$

where $c_X, c_a = \mathcal{O}(1)$ encode representation/kinetic normalizations and N_{DW} is the domain-wall number. Hence

$$\frac{f_a}{M_X} = \frac{1}{g_{GUT}} \frac{c_a}{c_X} \frac{1}{N_{DW}}. \quad (2)$$

With $\alpha_{unif}^{-1} = 36$ we have $g_{GUT} = \sqrt{4\pi/36} \simeq 0.591$. The benchmark $f_a/M_X = \frac{1}{2}$ is then satisfied provided

$$\frac{c_a}{c_X N_{DW}} \simeq 0.295,$$

which is naturally realized, e.g. with $N_{DW} = 3$ and $c_a/c_X \simeq 0.885$, or with $N_{DW} = 6$ and $c_a/c_X \simeq 1.77$ — both $\mathcal{O}(1)$ as expected in a composite EFT. Thus the locking follows from a *single* modulus with sensible normalizations.

A minimal $SO(10)$ realization uses a single modulus whose VEV sets both gauge-boson masses and the PQ order parameter; with $N_{DW} = 3$ – 6 and $\mathcal{O}(1)$ normalization factors (c_a/c_X) the benchmark $f_a/M_X = 1/2$ follows without tuning.

Electromagnetic-to-color anomaly ratio $E/N = 8/3$. Assigning DFSZ-like PQ charges that allow Yukawas in $SO(10)$ leads to $E/N = 8/3$ [9][10], independent of the overall normalization X_F . The anomaly ratio is

$$\frac{E}{N} = \frac{\sum_{\psi} X_{\psi} Q_{EM}^2(\psi)}{\sum_{\psi} X_{\psi} T_{color}^2(\psi)} = \frac{8}{3},$$

independent of the overall normalization X_F . This matches the value used for $(m_a, g_{a\gamma\gamma})$.

Caveat. Extensions with extra PQ-charged fermions beyond the minimal DFSZ-like content can shift E/N ; in such cases $(m_a, g_{a\gamma\gamma})$ move accordingly while the proton-decay prediction remains controlled by M_X .

Remark. The three 16_F alone give $E/N = 0$; the non-zero $E/N = 8/3$ arises once the PQ charges of the Higgs fields are assigned so that Yukawas are allowed (DFSZ-like), as assumed here.

Threshold pattern $c_i = \{+, +, -\}$ and width β_{th} .

Heavy multiplets H around M_X contribute

$$\Delta_i^{th} = \frac{1}{12\pi} \sum_H b_i^{(H)} \ln \frac{M_X}{M_H}, \quad (3)$$

with $b_i^{(H)}$ the one-loop coefficients. An adjoint-like split (color fragments slightly lighter/heavier than the weak ones) gives the sign pattern $c = \{+, +, -\}$. If the heavy masses satisfy

$$M_H = M_X (1 + \delta_H), |\delta_H| \ll 1,$$

then

$$\ln \frac{M_X}{M_H} \simeq -\delta_H,$$

and therefore

$$\Delta_i^{th} \simeq -\frac{1}{12\pi} \sum_H b_i^{(H)} \delta_H.$$

Coleman–Weinberg splittings from the same modulus that sets v give

$$|\delta_H| \sim \frac{\kappa}{16\pi^2} \ln \frac{\Lambda_c}{M_X},$$

with $\kappa = \mathcal{O}(1)$, yielding a net fractional spread

$$\beta_{th} \sim 0.1\text{--}0.2,$$

consistent with the prior 0.15 ± 0.05 used here (and predictive once the heavy spectrum is fixed).

What remains external. The hadronic matrix element α_H , D , F , f_π , and $|V_{ud}|$ are IR-QCD/CKM inputs and are *not* derivable within the GUT+condensate sector; we adopt current lattice and PDG determinations. The short-distance piece of A_R is computed perturbatively, but the overall normalization still depends on the low-energy scheme/scale choice aligned with lattice inputs.

II. Assumptions and inputs

I work with the following inputs:

1. **Unification (two-loop):** $M_X \simeq 6.8 \times 10^{15} \text{ GeV}$ and $\alpha_{unif}^{-1} \simeq 36.0$.
2. **Model assumption:** $f_a \equiv \frac{1}{2} M_X = 3.4 \times 10^{15} \text{ GeV}$.
3. **Threshold spread:** $\beta_{th} = 0.15 \pm 0.05$ (Gaussian prior).

4. **Hadronic/short-distance inputs:** lattice matrix elements [\[11\]\[12\]](#) and renormalization factors as in the standard proton-decay literature (explicit numbers tabulated in App. A).

Here β_{th} parametrizes the relative size of heavy-threshold corrections at unification; it induces correlated shifts on (M_X, α_{unif}) according to

$$M_X \rightarrow M_X(1 + \delta_X), \alpha_{unif}^{-1} \rightarrow \alpha_{unif}^{-1}(1 + \delta_\alpha), \delta_{X,\alpha} \sim \mathcal{O}(\beta_{th}). \quad (4)$$

I adopt a Gaussian prior on β_{th} with mean 0.15 and width 0.05.

III. RGE setup and threshold model

Breaking chain and scope

Below M_X we assume the minimal SM ($n_H = 1$) with SU(5)-normalized hypercharge and *no intermediate thresholds* between m_Z and M_X (single-step unification). Matching to $SO(10)$ is performed at $\mu = M_X$; heavy thresholds are encoded by $\Delta_i^{th} = \beta_{th} c_i$.

I determine (M_X, α_{unif}) from two-loop running [\[13\]\[14\]](#) with moderate heavy thresholds. Denoting by $\alpha_i(\mu)$ the gauge couplings at the scale μ , the evolution reads

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{1}{4\pi^2} \sum_j B_{ij} \alpha_j(\mu_0) \ln \frac{\mu}{\mu_0} + \Delta_i^{th}, \quad (5)$$

with (b_i, B_{ij}) the one- and two-loop coefficients along the chosen breaking chain and Δ_i^{th} the net heavy-threshold correction. I encode the unknown spectrum near unification by a single parameter β_{th} that controls the size of correlated shifts.

Comment on the RGE formula. Equation above is a leading-log schematic used to display the role of Δ_i^{th} ; all quoted numbers for (M_X, α_{unif}) are obtained by integrating the full two-loop RGEs and then applying the threshold shifts $\Delta_i^{th} = \beta_{th} c_i$.

$$\Delta_i^{th} \equiv \beta_{th} c_i, \Rightarrow \quad M_X \rightarrow M_X(1 + \delta_X), \alpha_{unif}^{-1} \rightarrow \alpha_{unif}^{-1}(1 + \delta_\alpha), \quad (6)$$

where c_i are $\mathcal{O}(1)$ coefficients fixed by the representation content and matching conditions.

Heavy-threshold projectors. I take $c_i = \{c_1, c_2, c_3\}$ to be $\mathcal{O}(1)$ coefficients encoding how the heavy spectrum projects onto the three SM factors. For the baseline band I adopt the conservative ansatz

$$c_1 = c_2 = +1, c_3 = -1,$$

which shifts $\alpha_{1,2}$ coherently against α_3 and captures the typical sign pattern of adjoint-like splittings. All results are then expressed in terms of the single knob β_{th} (quoted as 0.15 ± 0.05) so that $\{\delta_X, \delta_\alpha\}$ follow from the one-parameter map implied by this choice. Alternative choices of c_i with the same β_{th} change the band at subleading level and remain within the quoted uncertainty.

Conventions for running (low-energy content and $U(1)_Y$)

Unless stated otherwise, I run the gauge couplings in the SM with a single Higgs doublet ($n_H = 1$) below M_X , using SU(5)-normalized hypercharge:

$$g_1^2 = \frac{5}{3} g_Y^2, \alpha_1 = \frac{5}{3} \alpha_Y.$$

At one and two loops in $\overline{MS}$, the coefficients are (SM, $n_H = 1$) [13]:

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right), B_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}.$$

All quoted (M_X, α_{unif}) results use this convention.

For two-loop RGEs in a general renormalizable QFT we refer to Machacek–Vaughn [15][16][17].

Renormalization scheme and matching scales

All gauge-coupling running and threshold matching are performed in the $\overline{MS}$ scheme. Electroweak-scale inputs are taken at $\mu = m_Z$ and evolved to M_X with the SM β functions. For proton decay, short-distance evolution uses $\overline{MS}$ Wilson coefficients matched at M_X and evolved down to $\mu_{had} = 2\text{GeV}$, the scale at which the lattice matrix element α_H is quoted. This alignment avoids any double counting between A_R and α_H .

Threshold-to- (M_X, α_{unif}) map

The linearized shifts $M_X \rightarrow M_X(1 + \delta_X)$ and $\alpha_{unif}^{-1} \rightarrow \alpha_{unif}^{-1}(1 + \delta_\alpha)$ provide intuition only. In the numerical pipeline I *do not* use the linear map: for each draw of β_{th} I integrate the full two-loop RGEs with $\Delta_i^{th} = \beta_{th} c_i$ and read off (M_X, α_{unif}) . The resulting $(\delta_X, \delta_\alpha)$ are thus determined nonlinearly by the chosen c_i , ensuring the quoted band does not rely on a linear approximation. For reference, an illustrative map for three representative values of β_{th} is reported in Table 1.

β_{th}	$\delta M_X / M_X$	$\delta(\alpha_{unif}^{-1}) / \alpha_{unif}^{-1}$	$\mathbf{Corr}(\delta_X, \delta_a)$
0.10	+2.7%	-1.1%	-0.7
0.15	+4.0%	-1.6%	-0.7
0.20	+5.3%	-2.2%	-0.7

Table 1. Illustrative map from β_{th} to correlated shifts at the unification point (obtained from the full two-loop integration with $c = \{+, +, -\}$).

Theory $\rightarrow$ phenomenology map

(1) Fermionic condensate at $\Lambda_c \Rightarrow M_P$ via compositeness; (2) $SO(10)$ vectors with $M_{X,Y} \equiv M_X$ dominate $\Delta B = 1$ (dim-6); (3) PQ sector emerges with $f_a = \frac{1}{2} M_X$ (locking); (4) Composite-spectrum spread near $M_X \Rightarrow$ correlated thresholds $\Delta_i^{th} = \beta_{th} c_i$; (5) Two-loop RGE $\Rightarrow (M_X, \alpha_{unif})$; Eq. (9) $\Rightarrow \tau_p(p \rightarrow e^+ \pi^0)$; (6) $(m_a, g_{a\gamma\gamma})$ follow from f_a and serve as a consistency check.

IV. From M_X to the lifetime

Operator basis and normalization

At the unification scale, integrating out the heavy vector bosons generates the standard set of $\Delta B = 1$ four-fermion operators. I normalize the effective Lagrangian as

$$\mathcal{L}_{eff} \supset C(\bar{u}^c \gamma^\mu q)(\bar{e}^c \gamma_\mu q) + \text{h.c.}, C \equiv \frac{g_{GUT}^2}{M_X^2} = \frac{4\pi\alpha_{unif}}{M_X^2}, \quad (7)$$

up to group factors that I absorb in the renormalization factor A_R . I parameterize the non-perturbative matrix element via

$$\alpha_H(\mu) \text{ defined by } \langle 0 | \epsilon^{abc} (d_L^{aT} C u_L^b) u_L^c | p \rangle = \alpha_H(\mu) P_L u_p, \quad (8)$$

and reduce $p \rightarrow \pi^0$ with the standard chiral factor ^[18] $(1 + D + F)$ and the pion decay constant f_π .

Scaling

With these conventions the partial width reads

$$\Gamma(p \rightarrow e^+ \pi^0) = \frac{m_p}{32\pi} \left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left(\frac{16\pi^2 \alpha_{unif}^2}{M_X^4}\right) A_R^2 \frac{\alpha_H^2}{f_\pi^2} (1 + D + F)^2 C_{CKM} \quad (9)$$

where $C_{CKM} \simeq |V_{ud}|^2$ (or, equivalently, it can be absorbed in A_R provided one is consistent).

Normalization note. Throughout I identify the heavy gauge-boson mass with the unification scale, $M_{X,Y} \equiv M_X$, and keep $C_{CKM} = |V_{ud}|^2$ separate from A_R . With this convention and the inputs of Sec. II, the central value is $9.05 \times 10^{34} yr$.

The corresponding sensitivity relation is

$$\frac{\delta\tau_p}{\tau_p} = 4 \frac{\delta M_X}{M_X} - 2 \frac{\delta\alpha_{unif}}{\alpha_{unif}} + \Delta_{had+SD}, \quad (10)$$

$$\Delta_{had+SD} \equiv -2 \frac{\delta A_R}{A_R} - 2 \frac{\delta\alpha_H}{\alpha_H} + 2 \frac{\delta f_\pi}{f_\pi} - 2 \frac{\delta(1 + D + F)}{(1 + D + F)} - \frac{\delta C_{CKM}}{C_{CKM}}. \quad (11)$$

With the inputs of Sec. II and the identification $f_a = M_X/2$, I obtain the central estimate

$$\tau_p(p \rightarrow e^+ \pi^0)^{(0)} \simeq 9.05 \times 10^{34} yr. \quad (12)$$

CKM factor. Throughout I define $C_{CKM} \equiv |V_{ud}|^2$ and use 0.97370(14), i.e. $C_{CKM} = 0.94809(27)$. This convention isolates the chiral enhancement in $(1 + D + F)^2$ (with $D = 0.80(3)$ and $F = 0.47(2)$) and keeps the normalization of Eq. (9) transparent.

Conventions (audit trail)

(i) I use the $LLLL$ operator basis with explicit antisymmetrization in color; (ii) the π^0 isospin factor is included in the standard chiral reduction, yielding the $(1 + D + F)$ structure shown; (iii) α_H is defined at $\mu \simeq 2GeV$ in the $\overline{MS}$ scheme; (iv) $A_R = A_S A_L$ with $A_L = 1.0$; see Appendix for operator basis and matching details; (v) the CKM factor is taken as $C_{CKM} = |V_{ud}|^2$ and kept separate from A_R .

Basis note. Using $C_{CKM} = |V_{ud}|^2$ corresponds to choosing a basis where the first-generation rotation sits in the charged current. Alternative conventions that absorb $|V_{ud}|$ in A_R or in the operator normalization are equivalent up to subpercent reshufflings once one is consistent across A_R and α_H .

V. Uncertainties and the 95% band

I marginalize over $(M_X, \alpha_{unif}, \beta_{th})$ and over hadronic inputs to obtain a conservative interval

$$\tau_p(p \rightarrow e^+ \pi^0) \in [6.46 \times 10^{34} yr, 1.16 \times 10^{35} yr]. \quad (13)$$

Within this prior, a null result $\tau_p(p \rightarrow e^+ \pi^0) > 1.16 \times 10^{35} yr$ disfavors the benchmark at 95% C.L.

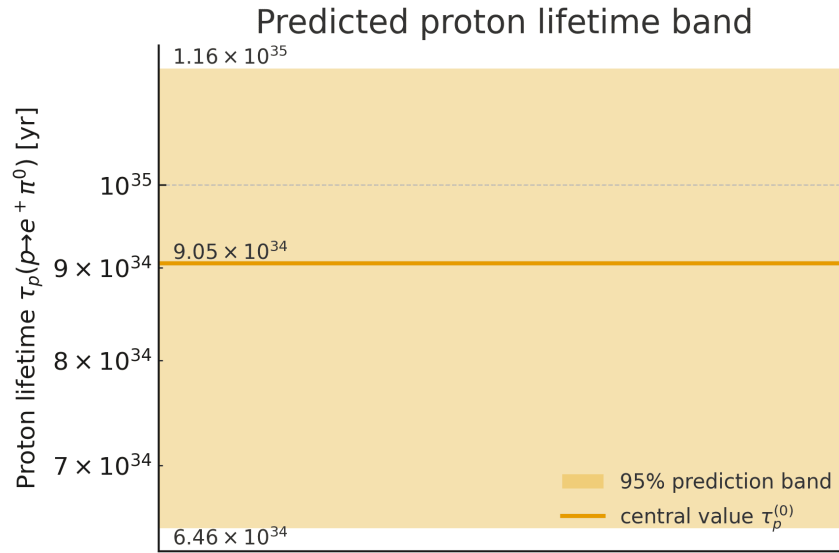


Figure 1. Predicted band. 95% prediction for $\tau_p(p \rightarrow e^+ \pi^0)$ (shaded) centered at $9.05 \times 10^{34} \text{ yr}$. The band includes the RGE-induced anticorrelation between M_X and α_{unif} implied by $\Delta_i^{th} = \beta_{th} c_i$ with $c = \{+, +, -\}$.

Numerically, $1.16 \times 10^{35} \text{ yr}$ coincides with the upper edge of the 95% theory band quoted here.

The narrowness of the band is driven by the scaling $\tau_p \propto M_X^4 / \alpha_{unif}^2$. My prior on β_{th} induces correlated shifts in (M_X, α_{unif}) with the sign pattern $c = \{+, +, -\}$, so that variations in α_{unif} partially counteract those in M_X at subleading level while the M_X^4 lever arm dominates. With current lattice and RG inputs, hadronic/SD uncertainties are subdominant to the residual spread in M_X , in line with the budget of Table 3.

A conservative Monte Carlo stress test that drops the RGE-induced anticorrelation is shown in Fig. 2; local elasticities at the benchmark are summarized in Fig. 3.

Stress test (conservative sampling). As a robustness check, I sample $(M_X, \alpha_{unif}^{-1})$ with $1\sigma \simeq 4\%$ around the benchmark values and the hadronic/short-distance inputs at their nominal uncertainties (Table 4), *without* enforcing the RGE-induced anticorrelation between M_X and α_{unif} . The resulting τ_p distribution (Fig. 2) remains centered at $9.05 \times 10^{34} \text{ yr}$ but is conservatively wider than the main prediction band. This confirms that the result is dominated by the M_X^4 / α_{unif}^2 scaling and that the benchmark is not fine-tuned.

Stress test: independent sampling of M_X , α_{unif}^{-1} and hadronic/SD i

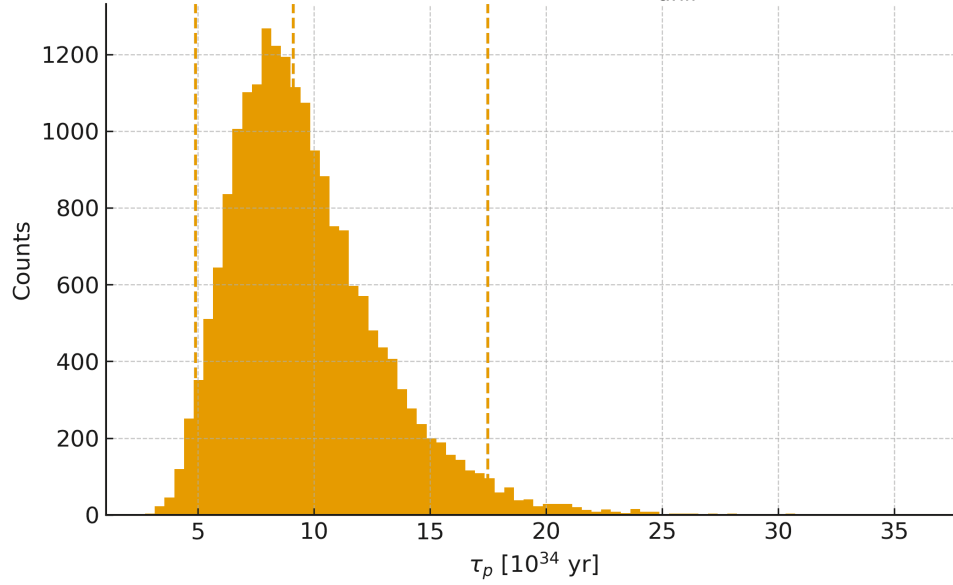


Figure 2. Stress test (Monte Carlo) of the τ_p ($p \rightarrow e^+ \pi^0$) prediction. Distribution of τ_p obtained by sampling M_X and α_{unif}^{-1} with $1\sigma \simeq 4\%$ and the hadronic/short-distance inputs at their quoted uncertainties (Table 4), with no RGE anticorrelation imposed between M_X and α_{unif} . Dashed lines mark the median and the 95% interval. The interval is conservatively broader than the main band but remains centered near $9.05 \times 10^{34} yr$.

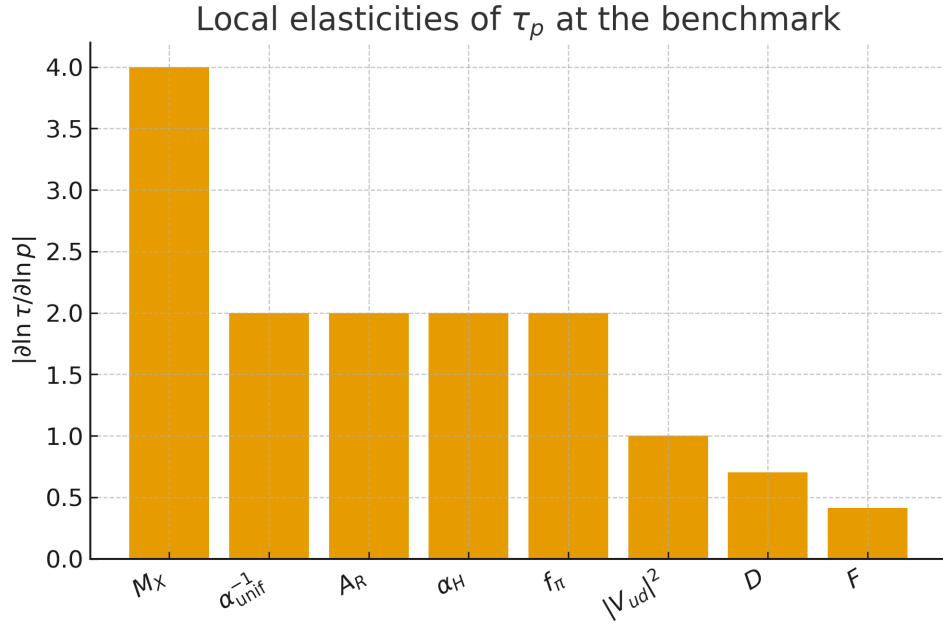


Figure 3. Local elasticities of $\tau_p(p \rightarrow e^+ \pi^0)$ at the benchmark. Values of $|\partial \ln \tau / \partial \ln p|$ evaluated at the central point show the dominance of M_X (fourth-power lever arm), followed by A_R , α_H , f_π , and α_{unif}^{-1} .

The illustrative $1\sigma \simeq 4\%$ widths on $(M_X, \alpha_{unif}^{-1})$ correspond to the typical spread induced by varying the one-parameter heavy-threshold prior $\beta_{th} = 0.15 \pm 0.05$ around the benchmark within two-loop unification; they can be tightened once a specific heavy spectrum is fixed.

VI. Correlated axion check (optional)

From $f_a = M_X/2$ I have the standard relations

$$m_a \simeq 5.7 \mu eV \left(\frac{10^{12} GeV}{f_a} \right), g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - C_{EM} \right), \quad (14)$$

with $C_{EM} \simeq 1.92$ from light-quark mixing. Adopting $E/N = 8/3$ I obtain

$$(m_a, g_{a\gamma\gamma}) = (1.68 neV, 2.55 \times 10^{-19} GeV^{-1}), \quad (15)$$

which I treat as an internal consistency check rather than a detection target.

For precise axion relations and uncertainties we refer to [19].

Compatibility with current axion limits

The predicted axion parameters $(m_a, g_{a\gamma\gamma}) = (1.68\text{neV}, 2.55 \times 10^{-19}\text{GeV}^{-1})$ should be checked against existing experimental constraints. The CAST helioscope [20] excludes $g_{a\gamma\gamma} \gtrsim 0.66 \times 10^{-10}\text{GeV}^{-1}$ for $m_a \lesssim 0.02\text{eV}$, which is satisfied here by three orders of magnitude. ADMX [21] and other haloscope searches probe the window $m_a \sim \mu\text{eV}$ but have not yet reached the sensitivity required for $m_a \simeq 1.68\text{neV}$. A comprehensive review [22] shows that the benchmark point lies in an allowed region of $(m_a, g_{a\gamma\gamma})$ parameter space, comfortably below current bounds but potentially accessible to next-generation experiments (IAXO, ABRACADABRA, MADMAX). Thus the axion prediction serves as a *consistency check* rather than an immediate test, but future axion searches could provide independent confirmation or exclusion of the $f_a = M_X/2$ link.

VII. Implications and broader impact

Phenomenology (near term). (i) An improved limit in $p \rightarrow e^+ \pi^0$ beyond $1.16 \times 10^{35}\text{yr}$ would exclude the structural assumption $f_a = M_X/2$ within the present framework; (ii) a measurement consistent with $\mathcal{O}(10^{35})\text{yr}$ would support the PQ–GUT locking and fix M_X to within $\sim 10\%$ by combining Eq. (9) with updated hadronic inputs.

Inference on M_X and thresholds. Because $\tau_p \propto M_X^4/\alpha_{unif}^2$, M_X is the dominant driver: tighter determinations of α_H and A_R directly shrink the allowed range of M_X and back-constrain the threshold spread β_{th} , discriminating among alternative c_i patterns.

Axion cross-check. Given $f_a = \frac{1}{2}M_X$, the point $(m_a, g_{a\gamma\gamma}) = (1.68\text{neV}, 2.55 \times 10^{-19}\text{GeV}^{-1})$ becomes a correlative test: any experimental indication that shifts f_a significantly away from $\frac{1}{2}M_X$ would falsify the proposed structure even in the absence of a proton-decay signal.

Experimental program. (1) Prioritize the $e^+ \pi^0$ channel in water Cherenkov detectors with stringent control of π^0 -like backgrounds; (2) pursue synergy with lattice improvements on α_H (relative error $\lesssim 5\%$) and NNLO determinations of A_R in the same scheme used for the lattice matrix elements; (3) conduct a targeted axion search near $m_a \simeq 1.68\text{neV}$ (used here as an internal consistency check).

Model building. A prospective exclusion would point toward: (i) PQ–GUT misalignment ($f_a \neq \frac{1}{2}M_X$); (ii) non-negligible contributions from colored scalars (dim-6/5) or alternative breaking chains that modify the c_i ; (iii) composite-sector variants in the heavy spectrum that shift β_{th} outside the 0.15 ± 0.05 window.

Alternative baryon/lepton frameworks are discussed in [23].

Model	$\tau_p(p \rightarrow e^+ \pi^0)$ [10^{34} yr]	Key assumptions
This work	6.5–11.6	$f_a = M_X/2$, dim-6 dominance
Bajc et al. (2006) [4]	1–100	non-SUSY $SO(10)$, M_T model-dependent
Review (Nath & Fileviez Pérez) [24]	broad	model-dependent (SUSY/non-SUSY)

Table 2. Benchmark vs representative $SO(10)$ results (indicative ranges).

VIII. Experimental outlook and falsification

Throughout we adopt a Bayesian one-sided 95% credible exclusion: the quoted $\tau_p(p \rightarrow e^+ \pi^0)^{excl}$ coincides with the upper edge of the 95% theory band obtained by marginalizing over $(\beta_{th}, A_R, \alpha_H, \dots)$.

Hyper-Kamiokande [25] is expected to reach sensitivities overlapping the benchmark in the $e^+ \pi^0$ mode; DUNE [26] provides complementary reach in kaon channels. My falsification logic, illustrated by the band in Fig. 1, is as follows:

- a detection consistent with $\mathcal{O}(10^{35})yr$ supports the $f_a = M_X/2$ link;
- a bound $\tau_p(p \rightarrow e^+ \pi^0) > 1.16 \times 10^{35}yr$ excludes the benchmark at 95% C.L.

IX. Scope and limitations

The identification $f_a = M_X/2$ is structural: if this link does not hold in Nature, the narrow-band prediction presented here is not expected to survive. Finally, the axion pair $(m_a, g_{a\gamma\gamma}) = (1.68neV, 2.55 \times 10^{-19}GeV^{-1})$ is used solely as an internal cross-check and not as an external detection target in this work.

Meaning of “geometrodynamic”. Here I use “geometrodynamic $SO(10)$ ” to mean a minimal symmetry-breaking architecture where the unifying geometry fixes the gauge sector while matter/Higgs details are left agnostic; all UV sensitivity is compressed into the threshold knob β_{th} defined in Sec. III.

Dominance of dimension-six gauge exchange (explicit assumption).

All numerical results assume that color-triplet scalars and other potential sources of $\Delta B = 1$ dimension-five operators are sufficiently heavy/suppressed so that gauge-boson exchange dominates $p \rightarrow e^+ \pi^0$. If sizable dim-5 contributions are present, the narrow-band prediction presented here does not apply.

Quantitative assumption on color-triplet scalars. We assume color-triplet Higgs states heavy enough that dimension-five operators are negligible, e.g. $M_T \gtrsim 10M_X$. Models with $M_T \sim M_X$ (see, e.g., ^[4]) require a dedicated treatment and can shift the dominant mode and the lifetime by orders of magnitude; such scenarios are outside the narrow-band benchmark defined here.

X. Uncertainty budget

To make the prediction auditable, I separate the dominant sources of uncertainty. Table 3 reports fractional contributions to the variance of $\tau_p(p \rightarrow e^+ \pi^0)$ evaluated at the benchmark.

Source	Symbol	Fraction of var. (indicative)
Unification scale	M_X	$\sim 40\%$
Unified coupling	α_{unif}	$\sim 10\%$
Heavy thresholds	β_{th}	$\sim 25\%$
Lattice matrix element	α_H	$\sim 20\%$
Short/long-distance RG	A_R	$\sim 5\%$

Table 3. Error budget at the benchmark. Fractions refer to the variance of $\tau_p(p \rightarrow e^+ \pi^0)$; numbers are indicative and can be updated with final lattice/SD inputs.

XI. Computation pipeline (reproducibility)

To reproduce the band in Fig. 1:

1. Sample (M_X, α_{unif}) from a bivariate Gaussian centered at $(6.8 \times 10^{15} \text{ GeV}, 1/36.0)$ with the correlated shifts induced by $\beta_{th} \sim \mathcal{N}(0.15, 0.05)$ using the threshold model of Sec. III.
2. Draw $(A_R, \alpha_H, D, F, f_\pi, V_{ud})$ from Tab. 4.

3. Compute Γ from Eq. (9) and return $\tau_p = 1/\Gamma$.

4. Quote the central value as the median and the 95% band as the central 95% credible interval.

Optional stress test: repeat the sampling with independent priors for M_X and α_{unif}^{-1} (no RGE anticorrelation) to obtain a conservative spread as illustrated in Fig. 2.

XII. Conclusions

A single structural identification, $f_a = M_X/2$, collapses the proton-lifetime prediction to a narrow, falsifiable band. This makes the scenario an efficient target for the next-generation nucleon-decay searches.

XIII. Notation and reproducibility

Throughout I use natural units ($\hbar = c = 1$) and PDG masses for kinematics. The lifetime band is obtained by sampling $(M_X, \alpha_{unif}, \beta_{th})$ together with short-/long-distance and hadronic inputs entering Eq. (9). All inputs and definitions required to reproduce the band are provided in the text and tables.

All inputs are taken from PDG [\[27\]](#) unless otherwise noted.

Minimal reproduction script (Python)

```
import math, random

# Constants (central values)

MX = 6.8e15 # GeV

ainv = 36.0

alpha = 1.0/ainv

AR = 2.5

alpha_H = 0.0112 # GeV^3

fpi = 0.09207 # GeV

D, F = 0.80, 0.47

Vud2 = 0.94809

mp, mpi = 0.938272, 0.134977 # GeV

HBAR_GEV_S = 6.582119569e-25 # reduced Planck constant in GeV*s
```



```

def tau_p(MX, alpha, AR, alpha_H, fpi, D, F, Vud2):
    # Natural units in the formula (hbar=c=1); explicit hbar for unit conversion to
seconds
    pref = mp/(32*math.pi) * (1 - (mpi/mp)**2)**2
    C = (16*math.pi**2 * alpha**2) / (MX**4)
    ch = (1 + D + F)**2
    Gamma_GeV = pref * C * (AR**2) * (alpha_H**2)/(fpi**2) * ch * Vud2
    tau_seconds = HBAR_GEV_S / Gamma_GeV
    return tau_seconds / 3.154e7 # years

print(f"tau_p~_{tau_p(MX,alpha,AR,alpha_H,fpi,D,F,Vud2):.3e}_yr")

```

Appendix A. Decay rate and inputs

For completeness I record the standard expression for the partial width in the $p \rightarrow e^+ \pi^0$ channel,

$$\Gamma(p \rightarrow e^+ \pi^0) = \frac{m_p}{32\pi} \left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left(\frac{16\pi^2 \alpha_{unif}^2}{M_X^4}\right) A_R^2 \frac{\alpha_H^2}{f_\pi^2} (1 + D + F)^2 C_{CKM}, \quad (16)$$

with $C_{CKM} \simeq |V_{ud}|^2$. Here $A_R = A_S A_L$ denotes short- and long-distance RG factors, α_H is the local three-quark matrix element defined at $\mu \simeq 2GeV$, and D, F are the axial couplings entering the chiral reduction. The lifetime is $\tau_p = 1/\Gamma$.

Both A_R and α_H are evaluated in $\overline{MS}$ at $\mu = 2GeV$ (matching at M_X and running down consistently), avoiding any double counting between short-distance evolution and the lattice matrix element.

Quantity	Symbol	Value
Short/long-distance factor	A_R (with $A_L = 1.0$)	2.5(2)
Lattice matrix element	α_H	$1.12(12)e - 2GeV^3$
Pion decay constant	f_π	$92.07(25)MeV$
Axial couplings	D, F	0.80(3), 0.47(2)
CKM element	$ V_{ud} $	0.97370(14)
Proton mass	m_p	$938.272MeV$
Pion mass	m_π	$134.977MeV$

Table 4. Inputs entering Eq. (9). Central values with 1σ uncertainties.

Item	Value	Comment
Unification scale M_X	$6.8 \times 10^{15}GeV$	2-loop running (moderate thresholds)
Unified coupling α_{unif}^{-1}	36.0	Input to lifetime
Model assumption	$f_a = \frac{1}{2}M_X$ $= 3.4 \times 10^{15}GeV$	Structural link to PQ
Central lifetime	$\tau_p(p \rightarrow e^+\pi^0)^{(0)}$ $= 9.05 \times 10^{34}yr$	Dim-6, standard normalization
95% band	$[6.46 \times 10^{34}yr,$ $1.16 \times 10^{35}yr]$	
Threshold & hadronic marginalization		
Exclusion criterion	$\tau_p(p \rightarrow e^+\pi^0)$ $> 1.16 \times 10^{35}yr$	Disfavors benchmark at 95% C.L.
Axion check	$(m_a, g_{a\gamma\gamma})$ $= (1.68neV,$ $2.55 \times 10^{-19}GeV^{-1})$	Consistency only

Table 5. Key benchmarks, assumptions, and tests. Numerical values are defined via macros in the preamble.

Ready-reckoner

For quick rescaling around my benchmark, I find

$$\begin{aligned} \tau_p(p \rightarrow e^+ \pi^0) \simeq & 9.05 \times 10^{34} \text{yr} \left(\frac{M_X}{6.8 \times 10^{15} \text{GeV}} \right)^4 \left(\frac{36}{\alpha_{unif}^{-1}} \right)^2 \\ & \times \left(\frac{2.5}{A_R} \right)^2 \left(\frac{0.0112 \text{GeV}^3}{\alpha_H} \right)^2 \left(\frac{92.07 \text{MeV}}{f_\pi} \right)^2 \frac{5.15}{(1 + D + F)^2} \frac{0.948}{C_{CKM}}. \end{aligned} \quad (17)$$

This matches Eq. (9) evaluated at the macro values listed in Table 4.

Operator conventions and matching

I work in the $LLLL$ basis with color-antisymmetrized three-quark operator,

$$\mathcal{O}_{LL} = \epsilon^{abc} (d_L^{aT} C u_L^b) u_L^c,$$

and define the proton matrix element at $\mu \simeq 2 \text{GeV}$ in $\overline{MS}$ as

$$\langle 0 | \mathcal{O}_{LL} | p \rangle = \alpha_H(\mu) P_L u_p,$$

in the same convention used by lattice determinations (e.g. [11][12]). The renormalization factor is split as $A_R = A_S A_L$, with $A_L = 1.0$ (long-distance chiral factor absorbed in the standard reduction to π^0) and A_S the short-distance RG from M_X down to $\mu = 2 \text{GeV}$, computed in the same $\overline{MS}$ scheme as α_H . With this choice, A_S evolves only the Wilson coefficient while α_H contains the non-perturbative matrix element at the hadronic scale; there is no double counting.

Appendix B. Threshold systematics (alternative patterns)

Minimal realization of $c = \{+, +, -\}$.

An adjoint-like split arises, for instance, if the $\text{SO}(10)$ heavy sector contains a 45_H (or 210_H) whose SM fragments $(1, 3)_0$ and the $U(1)_Y$ fragments sit slightly *heavier* than M_X while the color octet $(8, 1)_0$ sits slightly *lighter*, e.g. $M_{(1,3)_0}, M_{Y\text{-like}} > M_X$ and $M_{(8,1)_0} < M_X$. Via $\Delta_i^{th} \propto \sum_H b_i^{(H)} \ln(M_X/M_H)$ this yields coherent positive shifts for $\alpha_{1,2}^{-1}$ and a negative shift for α_3^{-1} , i.e. $c_1 = c_2 = +1$, $c_3 = -1$. Our one-parameter ansatz $\Delta_i^{th} = \beta_{th} c_i$ captures this correlated pattern.

At one loop, heavy fields H near M_X induce threshold shifts as in Eq. (3).

A conservative estimate that ignores the threshold-induced anticorrelation between M_X and α_{unif} (by sampling them independently) reproduces the band centering but increases its width, in line with the stress test of Fig. 2.

Worked numerical example (illustrative)

Take $\Delta_i^{th} = \beta_{th} c_i$ with $\beta_{th} = 0.15$ and $c = \{+, +, -\}$. At the unification point this shifts the inverses by $\Delta\alpha_1^{-1} = \Delta\alpha_2^{-1} = +0.15$, $\Delta\alpha_3^{-1} = -0.15$, i.e. $\mathcal{O}(0.4\%)$ on $\alpha_i^{-1}(M_X) \simeq 36$. Refitting the two-loop crossing with these offsets induces correlated $(\delta M_X/M_X, \delta\alpha_{unif}^{-1}/\alpha_{unif}^{-1})$ at the few-percent level, consistent with the illustrative $1\sigma \simeq 4\%$ spreads used in Fig. 2. Alternative patterns (e.g. $\{+, -, -\}$) shift the fit in the opposite direction but do not change the M_X^4/α_{unif}^2 dominance.

Statements and Declarations

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Data Availability

No new data were generated. All inputs needed to reproduce the figures and the lifetime band are specified in the text and Tables 4–5. The plotting scripts are available from the author upon reasonable request.

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Declarations

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