Review of: "Visualizing the Contraction Mapping Theorem"

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Potential competing interests: No potential competing interests to declare.

I enjoy reading this paper and watching the video lecture. The concept of visualizing the operator iteration process is cool. I am confident that this paper has the potential to enhance readers' comprehension of DP theory, making it a valuable contribution. I just have a few minor suggestions for improvement:

1. As indicated by the title, this paper contributes to visualizing the convergence process of operator iterations that are contraction mappings, with a scope broader than just value function iteration (VFI). Therefore, there is no necessity to assert in the abstract and introduction that the paper exclusively focuses on VFI. It may be beneficial to discuss VFI and policy function iteration (PFI) from a unified perspective.
2. Given that the primary audiences are first-year graduate students who may lack a solid foundation in real analysis, I recommend making the paper more self-contained:
    1. Providing a more detailed introduction to normed (resp., Banach) spaces in Section 2.2, similar to the treatment of metric (resp., complete metric) spaces, and demonstrating the relationship between them.
    2. Clarifying the mathematical definitions of Euclidean distance (Example 1) and $O(\beta^n)$ (Theorem 2.1).
3. Consider replacing the maximum in (3.3), (3.4), and (3.6) by supremum, or add a footnote to clarify that the maximum may not exist without further assumptions.
4. To avoid confusion, I recommend refining the language in the proof of Proposition 3. Beginners may find it unclear why compactifying the state space is necessary and why it is adequate to verify the well-definedness of $V(f(a-c))$ after the compactification. It might be worth emphasizing that the objective is to demonstrate that T maps the space of bounded continuous functions on $[0, \bar{a}]$ into itself.
5. Including pseudocode to explain the VFI and PFI algorithms in Sections 3.2 and 4.2 could be beneficial. This addition would help beginners better understand the practical implementation of these algorithms.
6. The equation between (4.1) and (4.2) should be labeled, and the surrounding paragraphs should correctly reference these equations.
7. In Section 5.2, it might be valuable to add a simple example illustrating how the transformation of the Bellman equation can facilitate solving unbounded DP problems that are hard to address using the standard Bellman equation.