

[Open Peer Review on Qeios](#)

An accuracy test of Altman and Zmijewski accounting-based bankruptcy models

Antonio M. Cunha¹

¹ Instituto Superior de Administração e Gestão

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.

Abstract

The aim of this research is to test if conventional accounting-based bankruptcy prediction model parameters are still valid when applied to the most recent financial ratios of Portugal's small, medium, and large companies.

We estimate Altman's Z"-score and Zmijewski bankruptcy probability for all of Portugal's companies with more than 10 workers and 1 million euros in assets from 2016 to 2021. We excluded micro-enterprises to avoid biased results. We also excluded banks and insurance companies.

We find that both models maintain reasonable accuracy, although lower than in the initial years, and that the Zmijewski model has a lower accuracy (81%) than the Z"-score (85.5%).

This is the first study comparing the accuracy of the Z"-Score and Zmijewski models run on a sample comprising all of Portugal's companies.

Cunha, António Manuel

ISAG – European Business School, CICET – Fundação Consuelo Vieira da Costa, Portugal. Email:

antonio.cunha@isag.pt

Keywords: Z-score; Zmijewski; Accounting-based risk models; Financial Ratios; Bankruptcy Risk.

1. Introduction and Literature Review

Using accounting information and financial ratios to predict corporate bankruptcies started in the 60s with a univariate model (Beaver, 1966). Altman (1968) developed the use of accounting ratios for bankruptcy predictions by introducing the multivariate discriminant analysis methodology to predict failures, naming the model's output as the Z-score. If the Z-score of a given industrial company was below a pre-determined threshold (1.81), there was a high probability of bankruptcy in the next two years.

Altman (1983) also developed a model for non-listed companies analogous to the original, which still applied only to industrial companies. This new model was further developed (Altman et al., 1998; Altman & Hotchkiss, 2006) to be used in emerging markets, where a higher reliance on bank funding is expected and less on capital markets. The authors named the new model Z''-score.

Although the Z''-score is still widely used by bankers and other credit analysts due to Basel II and Basel III requirements (Agarwal and Taffler, 2008), several recent research papers have shown that the model is losing accuracy (Grice and Ingram, 2001; Bandyopadhyay, 2006; Bauer & Agarwal, 2014). Nevertheless, Reisz and Perlich (2007) showed that in the short run (one year), the Z''-score outperformed market-based models in predicting corporate bankruptcy. However, in longer forecast horizons, the Z''-score loses relevance.

Zmijewski (1984) innovated by applying the *probit* regression methodology to the ratio analysis, estimating any given company's bankruptcy probability (from 0 to 1) with available accounting information. This approach has been shown to perform better than the Z''-score (Wu et al., 2010; Bauer & Agarwal, 2014).

Altman et al. (2017) used a logit regression to estimate new coefficients of the Z''-score model with a sample of European Union companies, finding that it performs well in predicting bankruptcy. This study, which included Portugal, found that the improvement in predictability due to the change in estimation method is weak, suggesting that the original model coefficients estimated through multivariate discriminant analysis are still valid. It was also found that the proximity of the bankruptcy year and the company's size are statistically significant in predicting bankruptcy.

We want to verify if the original Z''-score model (parameters estimated through multivariate discriminant analysis) maintains its accuracy when applied to a sample of Portugal's companies. As a robustness test, we also want to use the Zmijewski (1984) model (parameters estimated through *probit* regression) with the same sample to check for significant differences between the two methods.

2. Methodology

2.1. Altman Z''-score model

The Z''-score model is based on the following equation (Altman & Hotchkiss, 2006, p. 267):

$$Z'' = + 3.25 + 6.56(X1) + 3.26(X2) + 6.72(X3) + 1.05(X4)$$

Where,

- X1 = working capital / total assets
- X2 = retained earnings / total assets
- X3 = operating income / total assets
- X4 = book value of equity / total liabilities

According to this model, a company with a Z''-score lower than 4.15 is distressed.

2.2. Zmijewski model

The Zmijewski model is as follows (Zmijewski, 1984, p.69, sample with 4,8% of bankrupt companies):

$$Y = - 4.336 - 4.513(ROA) + 5.679(FINL) + 0.004(LIQ)$$

Where:

- ROA = net income / total assets
- FINL = total debt / total assets
- LIQ = current assets / current liabilities

To determine the probability of bankruptcy, the following formula is used to convert the odds of bankruptcy into a value between 0 and 1:

$$P = \frac{1}{(1 + e^{-Y})}$$

Where,

- e = constant (e = 2.71828)
- Y = odds ratio output

When the obtained P (probability) is larger than 0.5, there is a high chance that a bankruptcy process will be initiated within one year.

2.3. Data

We collected the data from the SABI (Iberian Balance Sheet Analysis System) database of Bureau Van Dijk, a Moody's Analytics company, where companies are classified according to the NACE codes and can be selected according to their status: active or bankrupt. We excluded banks and insurance companies, selecting all companies in the remaining sectors. We also excluded subsidiaries from the sample.

We have seen in the literature review that the company's size affects the reading of the financial ratios, so we excluded micro-enterprises. According to the definition of the European Union, micro-enterprises are companies with less than ten workers and less than 2 million euros in assets. We included non-bankrupt companies that fulfilled all these criteria in any sample year. Regarding the bankrupt companies, we kept the ten workers criterion, but we relaxed the size criterion to at least one year of total assets above 2 million euros in the sample years, as we expect that failing companies see their total assets decrease with time. Also, we know that the Z'' -score is more potent in predicting bankruptcy in one-year horizons, so, following Altman et al. (2017), we limited the analysis of the financial ratios of bankrupt companies to their last year's accounting report before declaring bankruptcy.

The sample includes financial data with accounting information on bankrupt and nonbankrupt companies. As the recent years' mean number of yearly bankruptcies was 60 for a total of around 9001 nonbankrupt companies, we included 6 years of data in the sample to reach a bankrupt-to-nonbankrupt ratio between the 1.5% ratio used by Altman et al. (2017) and the 4.8% ratio used by Zmijewski (1984). The total sample has 9,371 companies, of which 370 (3.9%) went bankrupt from 2017 to 2022 (one year after the last account filing). We filled with a "1" any missing values of ratio denominators that would produce errors in the computation of the ratios.

3. Results and Discussion

The 9001 nonbankrupt companies' ratios were computed for each of the six years of the sample, as they kept operating from 2017 to 2022. The 370 bankrupt companies' ratios were computed only for the year preceding bankruptcy to estimate the accuracy of the models with a one-year horizon. There are ratios of both nonbankrupt and bankrupt companies in each of the sample years (from 2016 to 2021). The descriptive statistics for each ratio were computed by pooling all companies in all years.

3.1. *Altman Z'' -score model pooled descriptive statistics*

We can find below the univariate statistics of the Z'' -score model.

Table 1. Altman's descriptive statistics

	X1		X2		X3		X4	
Company	NonB.	Bankr.	NonB.	Bankr.	NonB.	Bankr.	NonB.	Bankr.
Median	0.234	-0.058	0.193	-0.111	0.0452	-0.046	0.758	-1.007
Mean	0.232	-11.042	0.197	-3.708	0.0597	-7.817	2.511	-0.924
Std.Dv.	0.342	207.391	0.415	62.581	0.0992	147.009	49.789	0.535
Sample	9001	370	9001	370	9001	370	9001	370

NonB. means nonbankrupt companies pooled sample (all years). Bankr. means bankrupt companies pooled sample (all years). X1 = working capital / total assets; X2 = retained earnings / total assets; X3 = operating income / total assets; X4 = book value of equity / total liabilities.

3.2. Zmijewski model pooled descriptive statistics

We can find below the univariate statistics of the Zmijewski model.

Table 2. Zmijewski descriptive statistics

	ROA		FINL		LIQ	
Company	NonB.	Bankr.	NonB.	Bankr.	NonB.	Bankr.
Median	0.030	-0.061	0.569	0.655	1.639	0.186
Mean	0.042	-11.011	0.559	-2.869	4.303	-7.211
Std.Dv.	0.088	207.392	0.331	62.632	318.229	147.051
Sample	9001	370	9001	370	9001	370

NonB. means nonbankrupt companies pooled sample (all years). Bankr. means bankrupt companies pooled sample (all years). ROA = net income / total assets; FINL = total debt / total assets; LIQ = current assets / current liabilities.

3.3. Models' results and accuracy comparison

We computed the Z''-scores and Probabilities for all company-year observations of nonbankrupt and bankrupt companies. According to the previous section, we saw that some ratios are not normally distributed, suggesting skewed distributions and outliers. We used the median values of the pooled Z''-scores and Probabilities of nonbankrupt and bankrupt companies to avoid biased results.

Table 3 shows the models' accuracy comparison, including the cut-off points and the type I and type II errors for each model.

Table 3. Models' accuracy				
	Altman		Zmijewski	
Company	NonB.	Bankr.	NonB.	Bankr.
Z''-score / P	7.221	1.018	0.214	0.806
Bankruptcy if statistic	< 4.15		> 0.5	
Type I error		13.2%		16.2%
Type II error	14.5%		19%	
Accuracy	85.5%	86.8%	81%	83.8%
Total accuracy	85.5%		81%	

NonB. means nonbankrupt companies' pooled sample (all years). Bankr. means bankrupt companies' pooled sample (all years). Type I error is the percentage of bankrupt companies classified by the model as nonbankrupt. Type II error is the percentage of nonbankrupt companies classified by the model as bankrupt. Total accuracy is the percentage of correctly predicted types in the total sample.

4. Conclusion and Future Research

Both models still show accuracy levels above 80%, although initially, they started with accuracy above 95%. Altman's Z''-score model performs better than the Zmijewski probability model. This is surprising because, although both models are accounting-based, Zmijewski's model was developed with a more robust technique (probit regression) than Altman's (multivariate discriminant analysis). The latter is more prone to produce biased results due to the methodology based on equal-size samples of bankrupt and nonbankrupt companies and assuming a normal distribution and equal covariances of the ratios, a scenario that is not real.

In this study, we ran both models with their original coefficients. Although the *probit* regression methodology produces better estimates than multivariate discriminant analysis, the Zmijewski model coefficients were computed in 1984, whereas Altman's Z''-score coefficients were computed more than ten years later, which suggests that the coefficients change with time. Moreover, the cut-off points in the Z''-score were defined according to bond rating benchmarks, thus increasing the model's validity.

As a suggestion for future research, it would be interesting to re-estimate both models' coefficients with this sample and test the estimates against a future sample of bankrupt and nonbankrupt companies in the same market (Portugal), for comparison with the present study.

References

- Agarwal, V. & Taffler, R. (2008). Comparing the Performance of Market-Based and Accounting-Based Bankruptcy Prediction Models, *Journal of Banking and Finance*, 32, 1541–1551.

- Altman, E.I. (1968). Financial Ratios, Discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance*, 23(4), 589-609. doi:10.2307/2978933.
- Altman, E.I. (1983). *Corporate Financial Distress: A Complete Guide to Predicting, Avoiding, and Dealing with Bankruptcy (Frontiers in Finance Series)*. New York: John Wiley & Sons.
- Altman, E.I., Hartzell, J. & Peck, M. (1998). Emerging market corporate bonds — a scoring system. In: Levich, R.M. (eds) *Emerging Market Capital Flows. The New York University Salomon Center Series on Financial Markets and Institutions*, 2. Springer, Boston, MA. https://doi.org/10.1007/978-1-4615-6197-2_25.
- Altman, E.I. & Hotchkiss, E. (2006). *Corporate Financial Distress & Bankruptcy*, 3rd ed. Hoboken, NJ. John Wiley.
- Altman, E.I., Iwanicz-Drozdowska, M., Laitinen, E.I., & Suvas, A. (2017). Financial distress prediction in an international context: A review and empirical analysis of Altman's Z-score model. *Journal of International Financial Management & Accounting*, 28, 131–71.
- Bandyopadhyay, A. (2006). Predicting probability of default of Indian corporate bonds: logistic and Z-score model approaches. *Journal of Risk Finance*, 7(3), 255-272. doi:10.1108/15265940610664942.
- Bauer, J. & Agarwal, V. (2014). Are Hazard Models Superior to Traditional Bankruptcy Prediction Approaches? A Comprehensive Test, *Journal of Banking and Finance*, 40, 432–442.
- Beaver, W.H. (1966). Financial Ratios As Predictors of Failure. *Journal of Accounting Research*, 4, 71-111. doi:10.2307/2490171.
- Grice, J.S., & Ingram, R.W. (2001). Tests of the generalizability of Altman's bankruptcy prediction model. *Journal of Business Research*, 54(1), 53–61. doi:10.1016/S0148-2963(00)00126-0.
- Reisz, A.S. & Perlich C. (2007). A Market-Based Framework for Bankruptcy Prediction. *Journal of Financial Stability*, 3, 85–131.
- Wu, Y., Gaunt, C., & Gray, S. (2010). A comparison of alternative bankruptcy prediction models. *Journal of Contemporary Accounting & Economics*, 6(1), 34-45. doi:10.1016/j.jcae.2010.04.002.
- Zmijewski, M.E. (1984). Methodological Issues Related to the Estimation of Financial Distress Prediction Models. *Journal of Accounting Research*, 22, 59-82. doi:10.2307/2490859.