# Qeios

## Peer Review

# Review of: "Variational Conditional Normalizing Flows for Computing Second-Order Mean Field Control Problems"

#### Michal Kloc<sup>1</sup>

1. University of Basel, Switzerland

The manuscript introduces an extension of conditional normalizing flows for solving mean-field control problems (MFCs) in high-dimensional spaces. The approach leverages a general formulation of an MFC in Lagrangian coordinates (eq. 2.4). In the model implementation (Section 3), the push-forward map f is represented by autoregressive layers, and the optimal velocity field can be computed using automatic differentiation or finite differences of f minimizing the loss function. The model uses Monte Carlo sampling to approximate the spatial discretization of the loss functions during training epochs. Benchmarking studies for several tasks are presented, and the scalability of the algorithm is discussed. The code is available on GitHub.

In my opinion, the manuscript presents an interesting advancement in the field of optimal control. Its usefulness can only be proven by time, but I can imagine practical tasks in fields such as engineering or computational biology where such approaches might bring new efficient solutions. The structure of the manuscript is well organized; I appreciate Section 2, which allows the reader to revise the necessary mathematical background. I have some (generally minor) comments.

- 1. A figure picturing the NN architecture as described in 3.1 and App. 1 would be helpful.
- 2. I would try to make a more explicit link to the conditional normalizing flows (CNFs). It is implicitly on page 8, but a more direct link would be nice.
- 3. On page 4, first paragraph of 2.1, when referring to Eqs. 2.1 and 2.2, the references in the text show only as (1) and (2).

- 4. From page 6 onwards: There is inconsistency in annotations in  $\grad_X \log p(x,t)$  vs  $\grad X \log p(x,t)$  (cf. Eq. 2.3 and the one above).
- 5. Page 10, first equation: I would add a direct reference to the equation describing the OT problem on page 7 (Eq. 2.6) for clarity.

### Declarations

Potential competing interests: No potential competing interests to declare.