Research Article

Nuclear Level Density in the X(1062, 312) System

Joseph Bevelacqua[1]

1. Bevelacqua Resources, United States

Single particle levels for the X(1062, 312) system as a function of energy are modeled using various functional forms. A power law form N(E) = a E^b for the total number of energy levels provides a better fit than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model.

Corresponding author: J. J. Bevelacqua, bevelresou@aol.com

1. Introduction

Level density data provide important information regarding the systematics of nuclear structure. These calculations have been performed over a range of mass values, and most are performed for nuclei with A ≥ 20[1]. These calculations have been extended to superheavy nuclei including the Z=204 A=610 system X(610, 204)[2][3], X(636, 204)[4][5], X(692, 214)[4][6] , X(730, 226) [7.8], X(766, 242)[7][8], X(888, 274)[9][10], and X(926, 282)[11][12].

This paper extends those calculations to the X(1062, 312) system[13]. This system, and previous calculations in the ^{4}He system[14], represents endpoints for nuclear level density calculations. As such, they provide additional information regarding the level density systematics in nuclear systems.

2. Calculational Methodology

Nuclear single particle energies are calculated from the Schrödinger Equation. For a spherically symmetric potential, single particle energies for a nuclear core plus nucleon, can be solved using the methodology of Lukasiak and Sobiczewski[15] and Petrovich et. al.[16].

 doi.org/10.32388/HFIEM2

For a nucleon in the field of a nuclear core, the single particle binding energy E_{NLSJ} is derived from the solution of the radial Schrödinger Equation

$$[(\hbar^2/2\mu)\,(d^2/dr^2 - L(L+1)/r^2\,) - E_{NLSJ} - V_{LSJ}(r)]\,U_{NLSJ}(r) = 0 \quad (1)$$

In Eq. 1, the radial coordinate r defines the relative motion of the nuclear core and the particle, $V_{LSJ}(r)$ is the model interaction[17][18], and $U_{NLSJ}(r)$ is the radial wave function. L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively, and N is the radial quantum number. Finally, μ is the reduced mass.

Specific details of the model and its solution are provided in Refs. 2, 4, 7, 10, 12, and 14. The numerical methods of Refs. 20 and 21 are utilized to obtain a converged solution.

3. Single Particle Level Calculations

Fig. 1 summarizes the results of the X(1062, 312) level calculations[13]. The X(1062, 312) system has an alpha decay half-life of 152 d, and is stable with respect to beta decay[13].

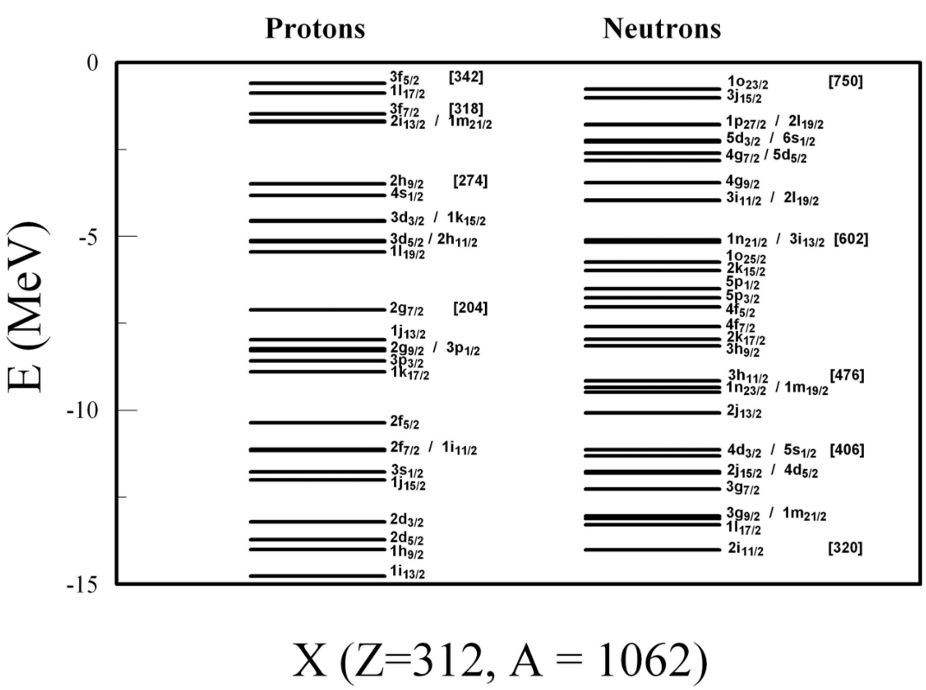

Figure 1. Calculated single particle energy levels for nucleus X(1062, 312). The notation A/B/C is used to indicate adjacent energy levels with level A more tightly bound than level B, which is more tightly bound than level C.

doi.org/10.32388/HFIEM2

The level systematics summarized in Sections 4.0, 5.0, and 6.0 are based on Fig. 1. These results are compared to the constant temperature model[19] in Section 4.0, power law in Section 5.0, and equidistant model[20][21][22][23] in Section 6.0. In addition to illustrating nuclear level systematics, level density calculations have also been applied to superheavy nuclei[24], nuclear astrophysical reaction rates[25], and machine learning[26].

4. Constant Temperature Model

The constant temperature model[19] is commonly utilized to evaluate nuclear level data and theoretical models, and it has the form

$\rho(E) = a \exp(E/T)$ (2)

In Eq. 2, $\rho(E)$ is the number of energy levels per MeV. The terms **a** and T are constants derived from a numerical fit to the X(1062, 312) single particle levels summarized in Fig. 1. Simple level density approximations may not reproduce the nuclear density that exhibits a unique single particle level structure. Accordingly, the systematics of level density parameters illuminate the unique differences between nuclei.

The level density $\rho(E)$ functional form is expected to be a simple exponential based on data from A = 36-66 even-even nuclei[19]. When the level scheme of Fig. 1 is fit to Eq. (2), the values **a** = 1.76999688 and T = 1/0.104675408 = 9.553 are obtained. These values are similar to the fits to the X(610, 204)[3], X(636, 204)[5], X(692, 214)[6], X(730, 226)[27], X(730, 242)[8], X(888, 274)[10], and X(929, 282)[12]. Fig. 2 illustrates the fit of the constant temperature model to the model results for the X(1062 312) system.

 doi.org/10.32388/HFIEM2

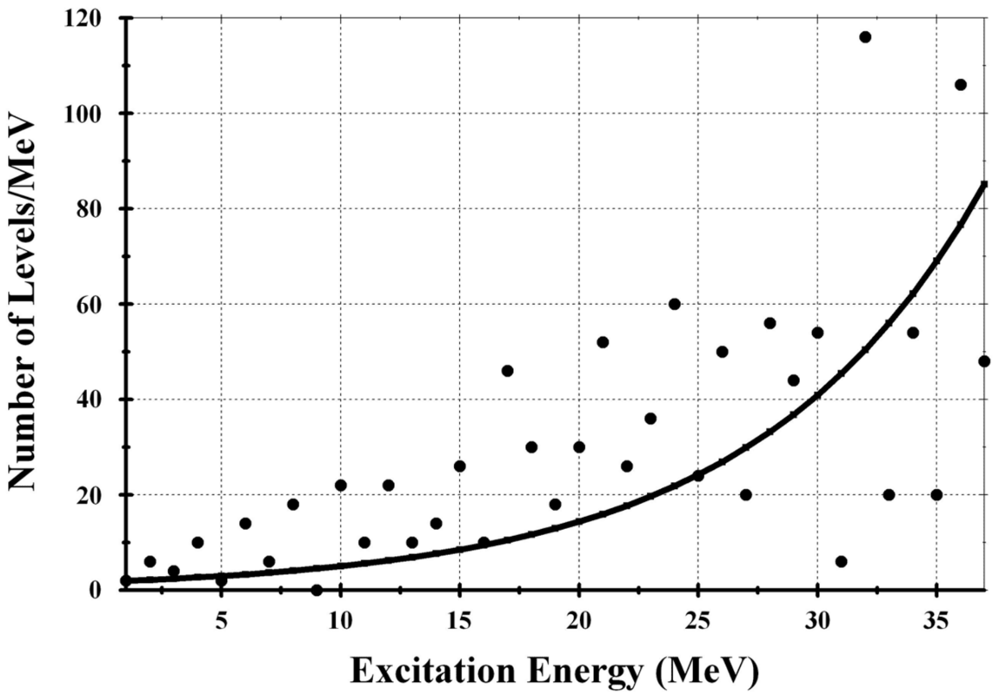

Figure 2. Energy level density for X(1062, 312) using the constant temperature model. The "•" symbol represents the state density for a 1 MeV energy bin. The solid curve is a fit to the constant temperature functional form ρ(E) = a exp(E/T).

In view of the limited number of energy levels, there is considerable variation between the model and constant temperature values of ρ(E). This variation is minimized by considering the total number of levels N(E). N(E) is also assumed to have the constant temperature model form

N(E) = c exp(E/d) (3)

The curve shown in Fig. 3 results when the X(1062, 312) levels N(E) of Fig. 1 are fit to the constant temperature model of Eq. (3). Fitting parameters c = 14.79799784 and d = 1/0.133141478 = 7.511 are utilized in Fig. 3. The values of these parameters are similar to the X(610, 204)[3], X(636, 204)[5], X(692, 214)[6], X(730, 226)[27], X(730, 242)[8], X(888, 274)[10], and X(926, 282)[12] results.

The fit to the functional form of Eq. 3 to the X(1062, 312) system underestimates the number of energy levels below about 28 MeV, and overestimates N(E) above about 29 MeV. Table 1 provides a comparison of the X(1062, 312) system d value to lighter systems using the constant temperature model.

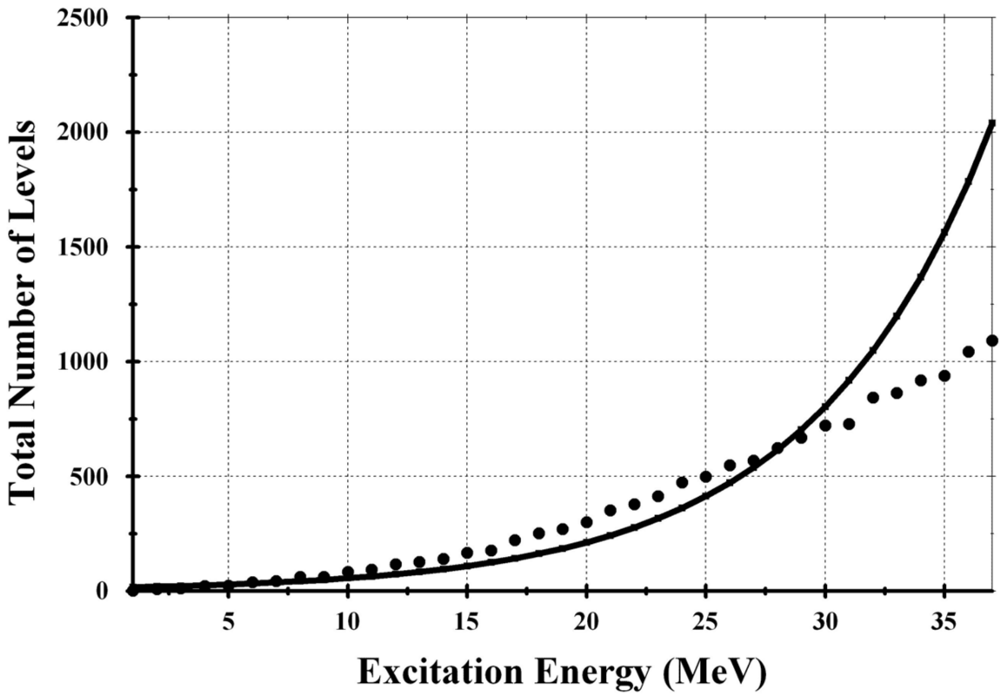

Figure 3. Total number of energy levels N(E) for X(1062, 312) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the constant temperature functional form N(E) = c exp(E/d).

doi.org/10.32388/HFIEM2

Nucleus	d (MeV)
^{4}He	2.79[a]
^{36}Ar	1.87
^{38}Ar	1.47
^{40}Ca	1.73
^{50}Cr	1.29
^{52}Cr	1.43
^{54}Cr	1.22
^{54}Fe	1.40
^{56}Fe	1.40
^{58}Fe	1.31
^{68}Zn	0.90
X(610, 204)	7.31[b]
X(636, 204)	7.15[c]
X(692, 214)	6.90[d]
X(730, 226)	7.26[e]
X(766, 242)	8.25[f]
X(888, 274)	7.65[g]
X(926, 282)	7.41[h]
X(1062, 312)	7.51[i]

Table 1. Constant Temperature Model Parameters for Nuclear Densities[a–j]

[a] *Ref. 6.* [b] *Ref. 3.* [c] *Ref. 5.* [d] *Ref. 6.* [e] *Ref. 8.* [f] *Ref. 9.* [g] *Ref. 11*, and [h] *Ref, 13.* [i]*This work.* [j]*All others Ref. 22.*

doi.org/10.32388/HFIEM2

Table 1 summarizes the systematics of the constant temperature model parameter d. This parameter behaves differently in the A < 70 region compared to A > 600 systems. The A > 600 d values are about a factor of four larger than those in the A < 70 region.

5. Power Law Model

The total number of levels N(E) is also fit to the power law functional form

$N(E) = a\,E^{b}$ (4)

where **a** = 1.65910055 and b = 1.75738835. Fig. 4 summarizes the use of these parameters in Eq. 4 to fit the X(1062, 312) energy levels of Fig, 1. The power law (Eq. 4) yields an improved fit compared to the constant temperature model (Eq. 3) for the total number of energy levels. The X(1062, 312) parameters are similar to the values derived from X(610, 204)[3], X(636, 204)[5], X(692, 214)[6], X(730, 226)[27], X(730, 242)[8], X(788, 274)[10] and X(926, 282)[12].

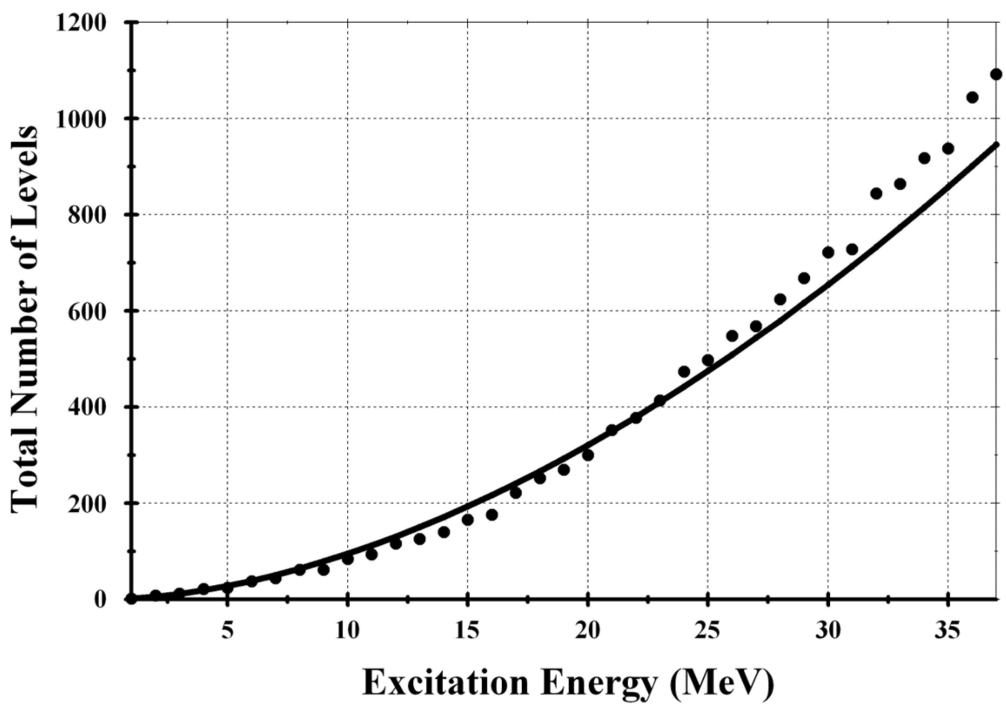

Figure 4. Total number of energy levels N(E) for X(1062, 312) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the power law functional form $N(E) = aE^{b}$.

doi.org/10.32388/HFIEM2

6. Equidistant Model

Single particle energy levels are assumed to be equidistant and nondegenerate in the equidistant model[20][21][22][23]. The total number of energy levels for a system of neutrons and protons is given by

$N(E) = (\pi)^{1/2} \exp(2 [a E]^{1/2}) / (12 E^{5/4} a^{1/4})$ (5)

where **a** is a level density parameter. For known nuclei, the parameter **a** has the value of A/8[23].

The X(1062, 312) energy levels are fit to the functional form of Eq. (5), and are illustrated in Fig. 5. An **a** = 1.298 value is utilized in Eq. 5. This **a** value is similar to the X(610, 204)[3], X(636, 204)[5], X(692, 214)[6], X(730, 226)[27], X(730, 242)[8], X(788, 274)[10] and X(926, 282)[12] parameters.

The reader should note that Eq. 5 is an asymptotic expression that should become more accurate as the atomic mass tends to infinity. This suggests that Eq. 5 should provide an improved description of the level density as the nucleus mass increases. However, the equidistant model of Eq. 5 does not reproduce the systematics of the level spectrum of Fig. 1 as well as the power law model.

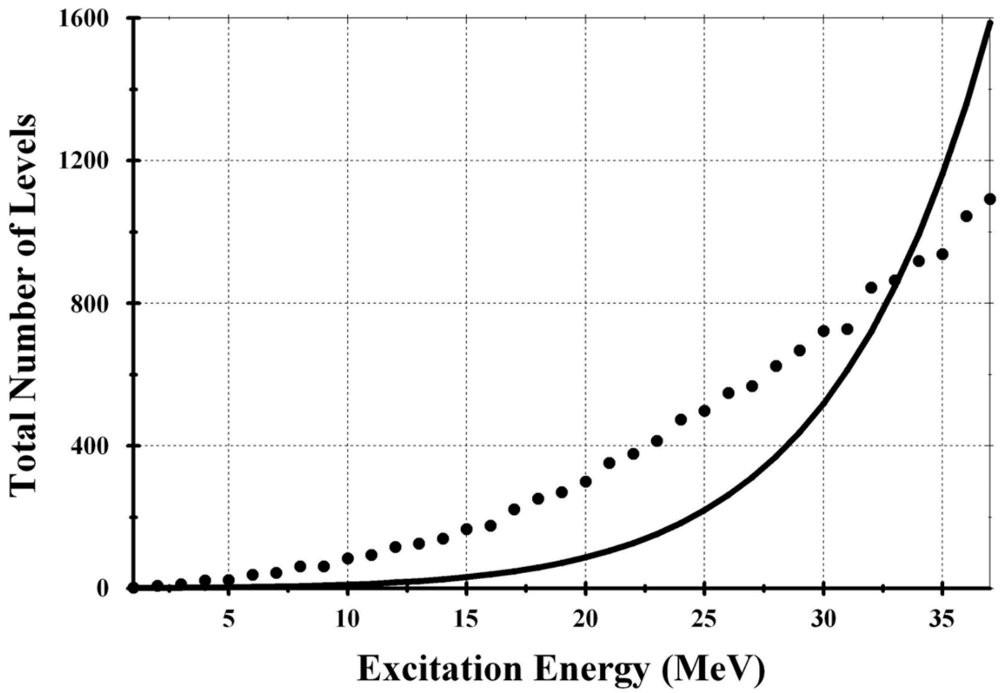

Figure 5. Total number of energy levels N(E) for X(1062, 312) as a function of energy using the equidistant model. The "•" symbol represents the total number of energy levels up to energy E.

7. Conclusions

The single particle levels for the X(1062, 312) system as a function of energy can be modeled using various functional forms. A power law form N(E) = $\mathbf{a}\ E^{b}$ for the total number of energy levels provides a better fit than the than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model.

References

1. ^Smith BC, Choudhury FN, Das Gupta S. "Nuclear level density with schematic forces". Phys. Rev. C. 17: 318 (1978).
2. ^Bevelacqua JJ. "Physics Essays, 570 ≤ A ≤ 620 Superheavy Nuclei". Physics Essays. 25: 475 (2012).
3. a, b, c, d, eBevelacqua JJ. "Nuclear Level Density in the X (610, 204) System". Qeios. OVWPON: 1 (2022). doi:10.32388/OVWPON.

4. [a, b]Bevelacqua JJ. "Physics Essays, Superheavy Nuclei II: $620 < A < 700$ Systems". *Physics Essays*. 26: 516 (2013).
5. [a, b, c, d, e]Bevelacqua JJ. "Nuclear Level Density in the X (636, 204) System". *Qeios*. CBI922: 1 (2022). doi:10.32388/CBI922.
6. [a, b, c, d, e]Bevelacqua JJ. "Nuclear Level Density in the X (692, 214) System". *Qeios*. 8SBVPK: 1 (2022). doi:10.32388/8SBVPK.
7. [^]Bevelacqua JJ. "Superheavy Nuclei III: $700 \leq A < 800$ Systems". *Physics Essays*. 27: 655 (2014).
8. [a, b, c, d, e]Bevelacqua JJ. "Nuclear Level Density in the X (766, 242) System". *Qeios*. C2V8T7: 1 (2023). doi:10.32388/C2V8T7.
9. [^]Bevelacqua JJ. "Superheavy Nuclei IV: $800 \leq A < 900$ Systems". *Physics Essays*. 29(4): 490 (2016).
10. [a, b, c, d, e]Bevelacqua JJ. "Nuclear Level Density in the X (888, 274) System". *Qeios*. QEOPGL: 1 (2024). doi:10.32388/QEOPGL.
11. [^]Bevelacqua JJ. "Superheavy Nuclei V: $900 \leq A < 1000$ Systems". *Physics Essays*. 30(1): 1 (2017).
12. [a, b, c, d, e]Bevelacqua JJ. "Nuclear Level Density in the X (926, 282) System". *Qeios*. 05USEO: 1 (2024). doi:10.32388/05USEO.
13. [a, b, c]Bevelacqua JJ. "Superheavy Nuclei VI: $1000 \leq A < 1100$ Systems". *Physics Essays*. 30(4): 392 (2017).
14. [^]Bevelacqua JJ. "Nuclear Level Density in the 4He System". *Phys. Rev. C*. 19: 2050 (1979).
15. [^]Lukasiak A, Sobiczewski A. "Estimations of Half-Lives of Far-Superheavy Nuclei with $Z \approx 154$-164". *Acta Phys. Pol. B*. 6: 147 (1975).
16. [^]Petrovich F, Philpott RJ, Robson D, Bevelacqua JJ, Golin M, Stanley D. "Comments on Primordial Superheavy Elements". *Phys. Rev. Lett.*. 37: 558 (1976).
17. [^]Rost E. "Proton shell-model potentials for lead and the stability of superheavy nuclei". *Phys. Lett.*. 26B: 184 (1968).
18. [^]Blomqvist J, Wahlborn S. "Shell Model Calculations in the Lead Region with a Diffuse Nuclear Potential". *Ark. Fys.*. 16: 545 (1959).
19. [a, b, c]Huizenga JR, Katsa AA. "Distribution of spacings of nuclear energy levels of mixed spin and parity". *Nucl. Phys. A*. 98: 614 (1967).
20. [a, b]Bethe H. "An Attempt to Calculate the Number of Energy Levels of a Heavy Nucleus". *Phys. Rev.*. 50: 332 (1936).
21. [a, b]Bethe HA, Bacher RF. "Stationary States of Nuclei". *Rev. Mod. Phys.*. 8: 82 (1936).

22. a, b*Bethe H. "Nuclear Dynamics, Theoretical". Rev. Mod. Phys.. 9: 69 (1937).*

23. a, b, c*Huizenga JR. "in Statistical Properties of Nuclei, edited by J. B. Garg (Plenum, New York, 1972), p. 425."*

24. ^*Rahmatinejad A, et al. "Energy dependent ratios of level-density parameters in superheavy nuclei". Phys. Rev.. 105: 044328 (2022).*

25. ^*Sangeeta T, et al. "Astrophysical reaction rates with realistic nuclear level densities". Phys. Rev. C. 105: 044320 (2022).*

26. ^*Wu XH, Ren ZX, Zhao PW. "Nuclear energy density functionals from machine learning". Phys. Rev. C. 105: L031303 (2022).*

27. a, b, c, d*Bevelacqua JJ. "Nuclear Level Density in the X (730, 226) System". Qeios. 9LZL9I: 1 (2023). doi:10.32388/9LZL9I.*

Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.