Review of: "Fidelity of quantum blobs"

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In the manuscript "Quantum distinguishability and symplectic topology," the author investigates the fidelity as a means to distinguish quantum states. Instead of the standard approach in terms of quantum mechanics, an approach based on "symplectic topology" is used. What it comes down to is a geometry-inspired approach that computes the overlap between spherical regions representing states in a kind of phase space. The work is unfortunately rather questionable. There are several issues that need are discussed in the following points.

a) The author provides a justification of this work based on a comparison between quantum mechanics and classical mechanics. However, such a comparison ignores the fact that there are other classical theories apart from classical mechanics. The comparison therefore gives the erroneous impression that quantum mechanics possesses certain features not present in classical theories, whereas some of the other classical theories do in fact posses the same features. For example, the superposition of complex amplitude fields is a well-known aspect of (classical) Fourier optics.

b) The author defines a phase space with a complex p by multiplying p by a phase factor that contains the Planck constant with an undefined variable/parameter α. The latter is not discussed. Since it is a non-physical quantity, there is nothing preventing it from taking on any value, in which case the Planck constant has no effect. In other words, one can simply replace h\alpha -> \phi, where \phi is an arbitrary phase value. Instead of a spherical region of uncertainty as depicted in Fig 2, the region of uncertainty would be toroidal. Here is a serious problem with the manuscript. Apart from the completely unphysical nature of this construction, the mathematical picture presented by the author is also not correct.

c) In the section "Quantum blobs" the author introduces a state consisting of N particles known with "maximum precision," which is then referred to as a "saturated state." It is not clear what is meant by the term "maximum precision," nor what it means for a state to be "saturated." When a state is represented as a Wigner function in phase space, the smallest region it can occupy is called the minimum uncertainty area. Such a state would necessarily have to be a pure state. Does the "maximum precision" imply that the state only occupies the minimum uncertainty area? Does the term "saturated" mean the same thing as "pure"? In fact, it is not clear what the actual quantum states are that are represented by the blobs. Or stated differently, how does a blob represent a particular quantum state such as a squeezed state, for example? The author needs to clarify the meaning of terms in the context of existing known concepts of phase space or, better yet, conform to the use of established terms and concepts about phase space. Although the author wishes to introduce a different phase space formalism, such a formalism would need to be presented in terms of much clearer definitions that can be understood in terms of (or by comparison with) existing formulations.

d) At some point in the section "Quantum blobs" the author refers to the N-particle state as a coherent state. This terminology is contradictory. The term "coherent state" is well established in existing usage and does not correspond to an N-particle state. Although the expectation value for the number of particles in a coherent state can take on a specific value, the number of particles in a coherent state is described by a Poisson distribution.

e) The expression in (7) describes the minimum uncertainty region as a hypersphere. Such a description flies in the face of the large diversity of quantum states. The only state with such a rotationally symmetric minimum uncertainty region (but without the hard boundary) is a coherent state, which we already addressed under the previous point. Other examples are Fock states (with fixed numbers of particles) and squeezed states. Although Fock states are pure states, their Wigner functions are described in terms of Laguerre polynomials that extend beyond the region of the minimum uncertainty area. While pure squeezed states only occupy a minimum uncertainty area, this area is not rotationally symmetric. The description provided in this manuscript is therefore very limited.

f) The term "modulus" mentioned above (8) is not properly defined. It seems to be an area. However, the area defined in (8) makes the tacit assumption that all but the two variables associated with the area are set equal to their values at the origin. Otherwise, the area would be smaller than what is given in (8). It also tacitly assumes that p_k which has been defined as a complex-valued quantity is now a real-valued quantity.

g) The author needs to provide an adequate discussion/definition of "symplectic topology" and "symplectic capacity" to make the manuscript suitably self-contained. Without such clear definitions, the arguments are nothing more than hand-waving arguments.

h) It is argued in "Overlapping quantum blobs" that the overlap between states in terms of the blobs "allow for the possibility" that the overlap is non-zero. However, if these states are coherent state, then the overlap between them is never zero. It is also not clear that the overlap between different blobs corresponds to the original definition of fidelity between the correspond density operators. The model that is used to represent states is therefore misleading or at least very limited.

i) It is not clear how (13) agrees with (7). It seems that the relationship in (13) would only be valid if all other q_k's and p_k's are set equal to their value at the origin. A clear mathematical derivation needs to be provided.

j) It is stated in "Conservation of probability" that the overlap between blobs represent the fidelity. However, the fidelity has a well-defined expression in terms of density operators of the states. It is not clear that the definition in this manuscript can be derived rigorously from the original definition. When the original definition of fidelity is applied to pure states, it simplifies to the inner product between such states, which is relatively easy to compute. The challenge is to compute the fidelity between mixed states, a situation which does not seem to be represented in the model of this manuscript. The statement therefore needs to be qualified or be supported by a rigorous mathematical derivation.

In summary, the manuscript contains a simplistic model that represents quantum states as rotationally symmetric minimum uncertainty regions with hard boundaries, which are called "blobs". The distinguishability of different states is simply represented as the overlap between different blobs. This simplistic model treats all states as if they occupy such minimum uncertainty areas. As such, the model only seems to deal with isotropic pure states.

In view of these deficiencies, and the fact that there is little hope of resolving them, the manuscript is not suitable for publication.