# Possible Description of the J/Ψ Ξ (S = -2) Structure in Terms of a First-Order Pentaquark Mass Formula

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Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.

## Abstract

A recently proposed $P^{\Xi}_{\Psi ss}$ pentaquark structure is investigated using a first-order mass formula incorporating weakly interacting J/Ψ meson and Ξ baryon clusters. The results are compared to the theoretical predictions of Marsé-Valera , Magas , and Ramos.

The first-order-mass formula predicts the J/Ψ Ξ pentaquark has a mass of 4567 MeV/$c^2$. This prediction lies between the values predicted by Marsé-Valera, Magas , and Ramos (i.e., 4493 MeV/$c^2$ (PB interaction) and 4633 MeV/$c^2$ (VB interaction)).

The first-order mass formula also predicts $J^{\pi}$ values of $1/2^-$ and $3/2^-$ for the proposed J/Ψ Ξ pentaquark. This is consistent with the predictions of the PB (VB) interaction assumption of $J^{\pi} = 1/2^-$ ($1/2^-$ and $3/2^-$).

## 1.0 Introduction

The possibility that hadrons could exist with structures beyond conventional qq or qqq quark configurations was noted by Gell-Mann[1]. Numerous data obtained by several collaborations (Belle, BABAR, LHCb, and BESII) produced exotic hadrons that appear to be inconsistent with the conventional quark model[2,3]. These structures have been suggested with the most recent example noted in Ref. 4.

Recent experimental evidence for pentaquarks with strangeness zero and one are significant hadron physics discoveries. Examples of the states are suggested in Refs. 1 – 3. As noted in Ref. 4, many of these states can be modeled as hadronic molecules and were predicted by models derived from unitarized meson-baryon amplitudes obtained from vector meson exchange interactions. This model also predicts the existence of doubly strange pentaquarks. Ref. 4 addresses the $P^{\Xi}_{\Psi ss}$ pentaquark possibly composed of a Ξ baryon and a J/Ψ vector meson.

Models based on the interaction of pseudoscalar mesons with baryons (PB interactions) predict $P^{\Xi}_{\Psi ss}$ has a mass of 4493 MeV/$c^2$ with a $J^{\pi}$ value of $1/2^-$[4]. Interactions based on vector mesons with baryons (VB interactions) predict a double strangeness state in the vicinity of about 4500 to 4600 MeV. In particular, Ref 4 suggests the VB interaction yields $P^{\Xi}_{\Psi ss}$ with a mass of 4633 MeV/$c^2$ with possible $J^{\pi}$ values of $1/2^-$ and $3/2^-$[4].

This paper utilizes the first-order mass formula model of Refs 5-22 to investigate the $\bar{P}_{\Psi ss}$. The model weakly couples a 1/2$^+$ Ξ baryon (modeled as having a uss quark content) and a 1$^-$ J/Ψ vector meson (c cbar). The proposed methodology was previously used to examine tetraquark[5-16], pentaquark[17-19], and hexaquark and other exotic quark configurations[20-22].

## 2.0 Model Formulation

Zel'dovich and Sakharov[23,24] proposed a semiempirical mass formula that provides a prediction of mesons and baryons masses in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest S state. These mass formulas are used as the basis for deriving a first-order pentaquark mass formula. In particular, the model proposed in this paper assumes the pentaquark is partitioned between two and three quark clusters with the interaction between the clusters providing a minimal contribution to the pentaquark mass.

The meson (m) mass (M) formula of Refs 23 and 24 is:

$$M_m = \delta_m + m_1 + m_2 + \frac{b_m}{3} \frac{m_0^2}{m_1 m_2} \sigma_1 \cdot \sigma_2 \quad (1)$$

where $m_1$ ($m_2$) are the mass of the first (second) quark comprising the meson, $m_0$ is the average mass of a first generation quark[25,26], and the $\sigma_i$ (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters $\delta_m$ and $b_m$ are 40 MeV and 615 MeV, respectively[24].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\boldsymbol{\sigma_1 \cdot \sigma_2}$ is the scalar product of the quark spin vectors. $\boldsymbol{\sigma_1 \cdot \sigma_2}$ has the value -3/4, +1/4 for pseudoscalar and vector mesons, respectively.

In a similar manner, the baryon (b) mass formula[23,24] is:

$$M_b = \delta_b + m_1 + m_2 + m_3 + Z \quad (2a)$$

$$Z = \frac{b_b}{3}\left[\frac{m_0^2}{m_1 m_2} \sigma_1 \cdot \sigma_2 + \frac{m_0^2}{m_1 m_3} \sigma_1 \cdot \sigma_3 + \frac{m_0^2}{m_2 m_3} \sigma_2 \cdot \sigma_3\right] \quad (2b)$$

where the $m_i$ labels the three baryon quarks (i = 1, 2, and 3) and $\delta_b$ and $b_b$ are 230 MeV and 615 MeV, respectively[24]. For a particle with a total baryon spin 1/2, the following prescription is used if the baryon (comprised of three quarks $q_1$, $q_2$, and $q_3$) contains two identical quarks[24] $q_2$, and $q_3$

$\boldsymbol{\sigma_2 \cdot \sigma_3 = 1/4}$ (3)

$\boldsymbol{\sigma_1 \cdot \sigma_2 = \sigma_1 \cdot \sigma_3 = -1/2}$ (4)

For completeness, the reader should note that $\sigma_i \cdot \sigma_j$ has the value +1/4 for a J= 3/2 baryon. In addition, these basic $\sigma_i \cdot \sigma_j$

relationships must be modified if the baryon contains three different quarks. The methodology is detailed, and described in Ref. 24.

In formulating the pentaquark mass formula, effective quark masses provided by Griffiths[25] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/$c^2$, respectively. These masses are utilized in Eqs. 1 and 2.

These six quarks are arranged in three generations: [d(-1/3 e), u(+2/3 e)], [s(-1/3 e), c(+2/3 e)], and [b(-1/3e), t(+2/3 e)] [26]. The three generations are specified by the square brackets and the quark charges are given within parenthesis in terms of the proton charge e.

The first-order mass formula used in this paper partitions the pentaquark into two clusters. The first cluster is a meson (m) and the remaining cluster is a baryon (b). Zero angular momentum is assumed between the two clusters. These simplifications are incorporated to minimize model complexity that is consistent with an initial first-order formulation. In addition, the pentaquark mass formula (M) is assumed to have the form:

$$\mathbf{M = M_m + M_b + \Phi} \quad (5)$$

where Φ defines the interaction between the meson and baryon clusters. Within the scope of this mass formula, the meson-baryon cluster interaction is assumed to be weak and sufficiently small to be ignored. Accordingly, Eq. 5 represents a quasimolecular five quark system whose basic character is a weakly bound meson-baryon system. In the case of the J/Ψ Ξ pentaquark system, the first cluster is a J/Ψ meson and the remaining cluster is a Ξ baryon.

The first-order pentaquark model only permits a primitive coupling structure between the clusters

$$J^{\pi}(J/\Psi\Xi) = J^{\pi}(J/\Psi) \bigotimes L \bigotimes J^{\pi}(\Xi) (6)$$

where L is the angular momentum between the clusters. The values utilized in Eq. 6 are $J^{\pi}(J/\Psi) = 1^-$, $J^{\pi}(\Xi) = 1/2^+$, and L = 0. Eq. 6 does not uniquely predict the total angular momentum of the pentaquark state, but does permit spin coupling to be specified for the individual meson and baryon clusters.

## 3.0 Results and Discussion

Eq. 5 is used to calculate the mass of the J/Ψ Ξ pentaquark. The first-order-mass formula predicts the J/Ψ Ξ pentaquark has a mass of 4567 MeV/$c^2$. This prediction lies between the values predicted by Ref. 4 (i.e., 4493MeV/$c^2$ (PB interaction) and 4633 MeV/$c^2$ (VB interaction)).

Using Eq. 6, the first-order mass formula predicts $J^{\pi}$ values of $1/2^-$ and $3/2^-$ for the proposed J/Ψ Ξ pentaquark. This is consistent with the predictions Ref 4 summarized in Table 1.

Table 1 provides a summary of the mass and $J^{\pi}$ values derived from the first-order pentaquark model and Ref. 4. The vector meson (J/Ψ) plus baryon (Ξ) pentaquark mass and $J^{\pi}$ values are consistent with the predictions of Ref. 4.

However, the first-order mass is about 1.4% smaller than the VB interaction mass of Ref. 4.

| Table 1 Predicted Characteristics of the $P^{\Xi}_{\Psi ss}$ | | | |
|---|---|---|---|
| | | | |
| Model | Mass (MeV/$c^2$) | $J^{\pi}$ | Assumptions |
| $P^{\Xi}_{\Psi ss}$ | 4493 | $1/2^-$ | PB Interaction (Ref. 4) |
| $P^{\Xi}_{\Psi ss}$ | 4633 | $1/2^-$ and $3/2^-$ | VB Interaction (Ref. 4) |
| J/Ψ Ξ Pentaquark | 4567 | $1/2^-$ and $3/2^-$ | First-order mass formula |

## 4.0 Conclusions

A recently proposed **$P^{\Xi}_{\Psi ss}$** pentaquark structure is investigated using a first-order mass formula incorporating weakly interacting meson (J/Ψ) plus baryon (Ξ) clusters. The results are compared to the theoretical predictions of Marsé-Valera , Magas , and Ramos[4].

The first-order-mass formula predicts the J/Ψ Ξ pentaquark has a mass of 4567 MeV/$c^2$. This prediction lies between the values predicted by Ref. 4 (i.e., 4493 MeV/$c^2$ (PB interaction) and 4633 MeV/$c^2$ (VB interaction)).

The first-order mass formula also predicts $J^{\pi}$ values of $1/2^-$ and $3/2^-$ for the proposed J/Ψ Ξ pentaquark. This is consistent with the predictions of Ref 4.

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