Review of: "[Commentary] Solving the Paradox of Material Implication"

Christoph Gallus[1]

1 Technische Hochschule Mittelhessen

Review of "Solving the Paradox of Material Implications"

C. Gallus, 23rd April 2024

The article is not about an obscure philosophical debate, but about an interesting problem concerning the connection between real-life natural language and formal logic. Basically, in formal logic, the implication "p -> q" is regarded as true if the proposition p is false, no matter what proposition q says. In Western philosophy, this was noticed in ancient Greece, and in the Middle Ages, it was paraphrased with the catchy formula "ex falso quodlibet." However, this principle of material implications can lead to apparent paradoxes: as an example, take p = "the sun exploded yesterday" and q = "all human beings are happy today." As of today, p is untrue (a counterfactual), so the application of the ex falso quodlibet principle forces us into a weird situation because we should regard the sentence "If the sun exploded yesterday, then all human beings are happy today" as a logical truth.

The paradox can be seen on at least two levels: as a disconnect between formal logic and the intuitive understanding of everyday language that produces weirdness, or - as outlined in Section 2 of the author's work - by working on the basis of an isomorphism between language and mental representations/real relations between beings (p.2).

Such disconnects are known in other situations where Boolean logic and natural language interpretations taken by test subjects are at odds, and they can be resolved with different formal approaches. As an example, take the conjunction fallacy detailed in [1]. In this example, subjects assign a probability to two events happening together (in conjunction) that is higher than the probability of just one of the events happening alone. While this cannot happen in standard set-based Kolmogorov probability theory and is hence seen as a paradox or a fallacy, it can be modelled by different formalisms, see [2,3]. This may be regarded as a counter-argument to a perceived necessity to be "fully in harmony with Boole's approach" (p.11). Contextuality, i.e., possible changes in meaning according to the prevalent context, is another problem when matching real-life situations to a formalism.

In the submission, the author argues that the paradox of material implications can be solved by a change in nomenclature. The author, Jan Poceij, reaches his insight about a required change in nomenclature by analyzing a football pools example, where he uses Tables 3 to 10 with different operators to connect p with q. In his summarizing Table 11, he notes that "there is some strange irregularity" and adds, "an obvious conclusion comes to mind that the implication is not what it has been recognized as […] the implication is not consequence (or entailment)." His suggestion of a required change in

nomenclature by giving up the saying that things do result from each other in a logical implication sounds reasonable. As such, I think that the submitted article contains valuable ideas and points to a direction that is worth investigating.

A couple of specific suggestions and comments for a later version of the article:

- Section 1: The introduction could benefit from some examples and at least a brief mention of non-formalist views on language in the West (e.g., along the lines of "the meaning of a word is its use in the language," Ludwig Wittgenstein, *Philosophical Investigations*) and, if interested, of Eastern thought (e.g., the use of the tetralemma in Nāgārjuna and the Madhyamaka school).

- Section 2: A quote of relevant literature regarding the stated "principle of isomorphism" is needed at the start of the section. Also, Table 2 needs a caption and headers to explain it clearly. E.g., if it is to be read as a definition of terms like "antilogy," "conjunction," etc., then why does the first row read "2,3,4,5,6"?

- Section 3: Match results stated in natural language need to be expressed more clearly, e.g., in Table 4, why is $p=1$, $q=0$ labelled as "not a goal draw"? Even here, we see problems in interpretation when parsing natural language, because the stated match result could be read as "*NOT (a draw result with goals being scored)*" which would be correct but sounds odd in everyday language, or as "*not a goal, draw",* which would be wrong. Are the tables 3 to 10 just to be read as definitions of the functions $p \wedge q$, $p \Leftrightarrow q$, etc.? Also, why can one place bets on ten different match results?

- Section 3: After the preparatory tables 4 to 10, Table 11 is introduced as a summary. However, Table 11 uses entirely different operators "x, -, =, <, >" and does not even have an explanatory caption, making it hard on the reader to understand why it is summarizing.

- Section 4: The proposed new nomenclature using "competition" is not obvious - possibly it is made clear in Moretti (2009) - but it should be made clearer in the article itself. The new nomenclature seems to allow an improvement on the classical term "implication" when an everyday natural language approach is taken, but the use of the terms "competition" and "inhibition" may also invoke everyday connotations that might lead to other problematic situations. As stated by Pociej himself, a separate study with the participation of linguists would be useful to check any newly proposed nomenclature intended to replace the term "implication. It seems desirable that any new nomenclature can be understood intuitively with a minimum of philosophical background. On the other hand, if the "principle of isomorphism" is used, then how does a change in nomenclature reflect therein?

- Section 5: This section and the link to memristor technology seems forced. It is not needed; the problem discussed in the article is interesting in its own right even without stating applications. If the author thinks that a connection to computer science and electronics is needed, more time needs to be spent on this point to make a clearer connection to the previous chapters.

[1] Tversky, A., & Kahneman, D. (1983). "Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment." *Psychological Review, 90*(4), 293-315.

[2] Busemeyer, J. R., & Bruza, P. D. (2012).*Quantum Models of Cognition and Decision*. Cambridge University Press.

[3] Trueblood, J. S., Pothos, E. M., & Busemeyer, J. R. (2014). "*Quantum probability theory as a common framework for reasoning and similarity*." Frontiers in Psychology, 5, 322.