Research Article

A Quasi-Cyclic LDPC Generator Polynomial for Navigation Signals of NavIC

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This paper proposes a novel method for constructing quasi-cyclic low-density parity-check (QC LDPC) codes with short block lengths for satellite navigation applications requiring enhanced error correction capabilities. The method constructs sparse base matrices, ensuring a parity check matrix girth greater than four, using a correlation-based method and an adaptive probability density function to avoid regular patterns in the parity check matrix. Investigations reveal that the proposed QC-LDPC codes achieve a low bit error rate of 10^{-7} at 4 dB energy per bit per unit noise density (E_b/N_0), compared to the Global Positioning Service L1C band LDPC codes, which show a bit error rate of 5×10^{-7} at the same E_b/N_0 for the shortest block length of 274 uncoded bits under consideration. This method is used for constructing LDPC parity check matrices for India's NavIC satellites by the Indian Space Research Organisation.

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1. Introduction

LDPC (Low-Density Parity-Check) codes, a type of linear error-correcting code, were first discovered by Gallager[1] and later by MacKay and Neal[2]; Sipser and Spielman[3]; MacKay[4], and have garnered attention for their outstanding performance near the Shannon limit over additive white Gaussian noise (AWGN) channels. LDPC (Low-Density Parity-Check) codes come with benefits like parallelizable decoding, self-error-detection through syndrome checks, and superior performance compared to turbo codes, as noted by Myung et al.[5]. In recent years, coding theorists have made significant advancements in constructing and decoding LDPC codes with low complexity, catering to applications such as communication systems, satellite navigation systems, and storage systems, among various others. LDPC codes have been used in the latest 5th generation (5G) new radio (NR) Li et al.[6]; Athens[7], providing a key distinction to 5G from 4th generation (4G) long-term evolution and LTE-advanced. Additionally, Wi-Fi, WiMax Gupta and Virmani[8], free space optical communication systems Sakib et al.[9]; Youssef et al.[10], underwater acoustic communication systems Zhao et al.[11] and NAND memory storage Cui et al.[12] also utilize LDPC codes. The GPS civil navigation system on L1C (CNAV-2) at 1575.42 MHz uses Bose-Chaudhuri-Hocquenghem (BCH) codes and LDPC with binary offset carrier (BOC) modulation, as defined in IS-GPS-800[13].

In LDPC codes, the structure of the associated parity check matrix plays a crucial role. LDPC codes with random parity check matrices, known as random LDPC codes, demonstrate improved convergence of iterative decoders but come with higher encoder complexity. In contrast, LDPC codes with algebraically defined structured parity check matrices offer advantages such as determinable and higher minimum Hamming distance, along with simpler encoding and reduced storage requirements. Margulis[14] introduced one of the earliest constructions of algebraic LDPC codes using Ramanujan graphs, as noted by Tomlinson et al.[15].

LDPC codes are depicted by Tanner graphs Tanner[16], where the girth represents the shortest cycle length. Short cycles in these graphs impede convergence MacKay[4]; Kim et al.[17]; Song et al.[18]; Lu et al.[19] and degrade decoder performance by compromising extrinsic information independence Lucas et al.[20]. Hence, LDPC codes with greater girth are preferred.

The LDPC codes can also be classified as quasi-cyclic (QC) and non-cyclic codes, like other linear codes. The QC LDPC codes satisfy two additional properties: a) a cyclic shift of a code makes another valid code, and b) the sum of two codes is also a valid code. A quasi-cyclic code of length Ir can also be described by a $Jr \times Ir$ parity check matrix that is formed by a $J \times I$ array of $r \times r$ circulant matrices. The main reasons for the attractiveness of QC LDPC codes are a) they have low error floors IEEE 802.11n[21] b) they need lesser storage, c) they can be encoded easily by various methods including simple shift registers Li et al.[22] and d) they can be decoded efficiently using belief-propagation-based decoding algorithms such as the sum product algorithm and the min sum algorithm Chandrasetty and Aziz[23], or LP-based decoding algorithms Feldman et al.[24]; Taghavi N. and Siegel[25] among various other algorithms. The dual diagonal structure in LDPC codes allows encoding without the need to calculate the generator matrix Richardson and Urbanke[26].

Structural properties of QC LDPC codes have brought their uses in various dimensions of scientific research and engineering applications. In the literature, the performance of large block-length QC LDPC codes has been well-studied[27][28][21]. The design of short block-length codes remains an open problem due to the trade-off between latency and performance[29][30][31]. Shorter block-length codes find various applications, especially but not limited to systems requiring low latency such as vehicle-to-vehicle communication, the Internet of Things, and navigation systems[32].

One of the first attempts to construct QC LDPC codes for deep space and high data rate applications was made by Andreadou et al.[33] for the Consultative Committee for Space Data Systems (CCSDS). Later, 231.1-O-1[34] provided the Experimental Specification for Short Block Length LDPC Code for TC Synchronization and Channel Coding. Medova et al.[35] propose a new construction method for short block length QC LDPC codes, providing an energy gain of 0.4 dB at a symbol error rate equal to 10^{-6} with 30% less complex decoding algorithm compared to the aforementioned CCSDC standard. Danish et al.[29] proposed short block-length QC LDPC by designing the parity-check matrix based on finite geometry properties by exploiting Euclidean geometry and circulant decomposition, resulting in a QC LDPC matrix that avoids shorter girths for iterative decoding cycles. QC Protograph-based Raptor-like LDPC (QC PBRL codes) proposed by Ranganathan et al.[36] adapt demonstrate superior performance of PBRL codes for shorter block lengths.

The motivation of our work is to address three key challenges: (a) to construct short block-length QC LDPC codes for uncoded blocks of a few hundred bits, (b) to avoid a girth of four (See Section-[2] for details on girth), and (c) to avoid regular patterns in the parity check matrix (PCM).

In this paper, we propose an algorithm to construct short block length QC LDPC codes with a dual diagonal structure, avoiding the existence of short cycles of length 4. The algorithm uses the cross-correlation for avoidance of a girth of 4 and an adaptive probability distribution function (PDF) for avoidance of regularity in the PCM. The proposed method is used for constructing LDPC PCMs for use in the Indian navigation satellites, NaVIC, of the Indian Space Research Organisation. The method is shown to have better performance than the uncoded system, Galileo convolutional codes, and GPS L1C LDPC codes for uncoded blocks of size equal to the size of GPS L1C subframe 2 and 3, each 600 and 274 bits long, respectively.

The correspondence unfolds as follows:

a. Section II defines the necessary notations and reviews QC LDPC codes with a dual diagonal structure, examining their cycle structure and discussing a simple condition for cycles,
b. Section III introduces the proposed method for constructing PCM,
c. Performance verification through simulations is presented in Section IV,
d. Concluding remarks are offered in Section V.

2. Quasi Cyclic LDPC Codes with dual diagonal structure

QC LDPC codes, a class of linear block codes, are a set of codewords $\{c_i\}$ of length n bits each, defined by the parity check matrix $H_{m\times n}$ where $m = n - k$ for encoding and decoding uncoded blocks $\{u_i\}$ of length k bits each, such that

$$Hc_i^T = 0 \tag{1}$$

where A^T represents the transpose of vector or matrix A, multiplication is the logical 'AND' operation, and addition is the logical 'XOR' operation. The parity check matrix is sparse, and the majority of it is zero. The code rate of QC LDPC is defined as k/n. Here, the quasi-cyclic nature of LDPC codes enforces that each codeword $c_i = \{c_{(i,1)}, c_{(i,2)}, \ldots, c_{(i,l)}, c_{(i,l+1)}, \ldots, c_{(i,n-1)}, c_{(i,n)}\}$ satisfies:

a. $$c_i' = \{c_{(i,n-z+1)}, c_{(i,1)}, \ldots, c_{(i.l)}, c_{(i,l+1)}, \ldots, c_{(i,n-z-1)}, c_{(i,n-z)}\} \in \{c_i\} \quad \text{for some } z \in [2, \ldots, n] \tag{2}$$

b. $$c_i' = c_p \oplus c_q \in \{c_i\} \quad \forall \quad c_p, c_q \in \{c_i\} \tag{3}$$

Equivalently, the quasi-cyclic nature forces the parity check matrix H to consist of square circulant permutation submatrices $P_{x,y}$ each of order $z \times z$ as follows:

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$$H = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n/z} \\ P_{2,1} & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ P_{m/z,1} & \cdots & \cdots & P_{m/z,n/z} \end{bmatrix} \tag{4}$$

The submatrices $P_{x,y}$, parametrised over $p_{x,y} \in [-1, 0, 1, 2, \ldots)$, are defined as follows.

$$P_{x,y} = \begin{cases} 0_{z\times z} & \text{if } p_{x,y} = -1 \\ I_{z\times z}^{p_{x,y}} & \text{otherwise} \end{cases} \tag{5}$$

for $I^{p_{x,y}}$ is an identity matrix shifted to the right $p_{x,y}$ times column-wise. The matrix $B_{m/z\times n/z} = [p_{x,y}]$, shown in Equation-[6], is defined as the base matrix of the QC LDPC code.

$$B = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n/z} \\ p_{2,1} & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ p_{m/z,1} & \cdots & \cdots & p_{m/z,n/z} \end{bmatrix} \tag{6}$$

The graphical representation of QC LDPC codes in terms of a Tanner graph is done in terms of check nodes and bit nodes. Each row and column in the parity check matrix is represented by a check node and a bit node, respectively. The presence of a logic TRUE (1) bit in the parity check matrix represents a connection between the corresponding check node and bit node. Figure-[1] shows the Tanner graph corresponding to the parity check matrix in Equation-[7]. The girth of a Tanner graph is the minimum cycle length of the graph.

$$H = \begin{bmatrix} 1 & 1 & 1 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 1 & \cdots & 1 \end{bmatrix} \tag{7}$$

A cycle of length L in a Tanner graph is a path of L edges that closes back on itself. A cycle of length four and the corresponding ones in the parity check matrix are highlighted in orange in Figure-[1] and Equation-[7], respectively.

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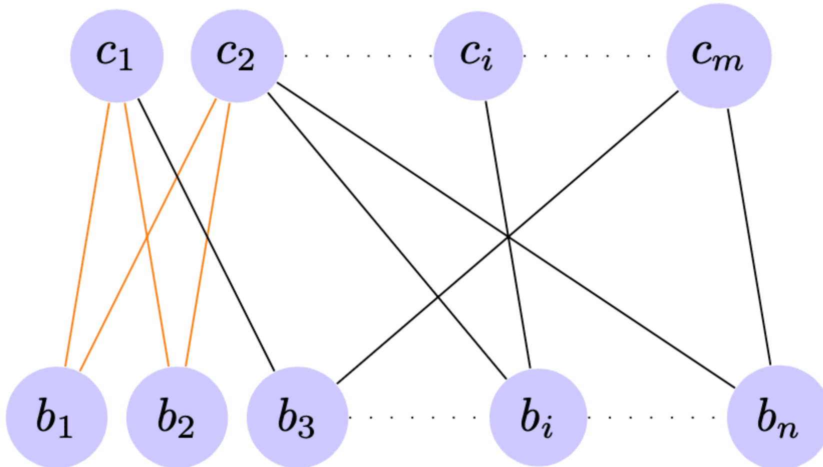

Figure 1. Exemplar Tanner graph corresponding to the parity check matrix in Equation-[7]. Highlighted connections represent the cycle of length four.

The LDPC codes, however, in general, suffer from high encoding complexity. Encoding with a generator matrix is quadratic in the LDPC code's block length[37]. QC LDPC codes with a special dual diagonal structured base matrix offer tremendous advantages in simplifying the encoder architecture, and the same has been adopted in multiple standards IEEE 802.11n[21]; Bhuvaneshwari and Tharini[28]; Li et al.[6]. There are two popular dual diagonal structures defined in the literature, also shown in Equation-[8], where all blank entries are -1; however, various permutations of the two by changing the direction of the diagonal, flipping the diagonal, truncating the diagonal, fragmenting the diagonal are possible and not shown explicitly here.

$$
\begin{aligned}
B &= \begin{bmatrix}
p_{1,1} & p_{1,2} & \cdots & p_{1,k/z} & 0 & & & \\
p_{2,1} & \ddots & \cdots & p_{2,k/z} & 0 & 0 & & \\
p_{3,1} & \ddots & \cdots & p_{3,k/z} & & 0 & 0 & \\
\vdots & \vdots & \ddots & \vdots & & & \ddots & \ddots \\
p_{m/z,1} & \cdots & \cdots & p_{m/z,k/z} & & & 0 & 0
\end{bmatrix} \\
\mathit{or} \\
&= [B_S \quad B_P] \text{ where } B_S = \begin{bmatrix}
p_{1,1} & p_{1,2} & \cdots & p_{1,k/z} \\
p_{2,1} & \ddots & \cdots & p_{2,k/z} \\
p_{3,1} & \ddots & \cdots & p_{3,k/z} \\
\vdots & \vdots & \ddots & \vdots \\
p_{m/z-1,1} & \cdots & \cdots & p_{m/z-1,k/z} \\
p_{m/z,1} & \cdots & \cdots & p_{m/z,k/z}
\end{bmatrix} \text{ and } B_P = \begin{bmatrix}
\alpha & 0 & & & & \\
& 0 & 0 & & & \\
& & 0 & 0 & & \\
0 & & & \ddots & \ddots & \\
& & & & 0 & 0 \\
\alpha & & & & & 0
\end{bmatrix}
\end{aligned} \tag{8}
$$

The two dual diagonal structures are similar to one another; the second structure, however, has been adopted in the IEEE 802.11n/e standards due to more flexibility for parallelization. The second structure is also adapted in this work.

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3. Proposed method

3.1. Construction of PCM

Decoding algorithms for LDPC codes are usually iterative. When cycles exist in the Tanner graph, they degrade decoding performance due to dependent information. However, cycles can improve the minimum code distance. In LDPC code design, we must consider both aspects. Specifically, 4-cycles negatively impact decoding convergence and performance, so removing them significantly enhances performance. This sub-section presents a method that completely avoids 4-cycles, and the method is inspired by work by Yang et al.[38]. The method uses correlation to determine the presence of cycles of 4. The cross-correlation of two sequences χ and ψ with the length of N is defined as

$$R_{\chi,\psi}(y) = \sum_{i=1}^{N} \chi(i) \cdot \psi((i+y) \bmod N) \tag{9}$$

No 4-cycle exists in the PCM means the cross-correlation value $R_{\chi,\psi}(0)$ is less than 2 between any two arbitrary rows if we view each row of PCM as a sequence χ and ψ. The PCM has a structure as described by the base matrix B discussed in Section-[2]. The B_P sub-matrix of B is pre-defined for a dual-diagonal structure, and the B_S sub-matrix needs to be determined. The values of B_S are determined ensuring the aforementioned correlation constraint is satisfied.

PCM construction is an iterative process of determining m rows $r_{iz+j} \quad \forall \quad i = 0 : m/z - 1, j = 0 : z - 1$ of PCM. The method used to construct PCM for LDPC in NavIC satellites is shown in Figure-[2]. It includes mainly two steps:

a. The rows are determined in groups of z rows at a time as per the proposed Algorithm-[3.1]. The proposed algorithm provides a balance between decoding performance (by allowing randomness) and low encoding-decoding complexity (by preserving structure). The algorithm takes a PDF (ζ) which ensures no two consecutive values in a column of the base matrix B_S are the same. This results in the avoidance of regular patterns in the contribution of uncoded bits to parity bits. The algorithm can result in multiple solutions for a given k, z, B_P and.

b. The determined rows are checked for forming cycles of 4 with previously determined rows.

The described method results in the non-existence of -1 in the basis matrix B ensuring the row and column weights are k/z and m/z respectively, resulting in a density not low enough. We set some of the sub-matrices of H corresponding to B_S as zero matrices to reduce the density, which has been verified to be an efficient method.

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Algorithm 1 Algorithm for sparse sequence determination

Require: n, k, z, B_P, ζ

$w_k \leftarrow 0:1:k-1$

$v_k \leftarrow \lfloor w_k/z \rfloor$

$w_n \leftarrow 0:1:n-1$

$v_n \leftarrow \lfloor w_n/z \rfloor$

$s_{iz} \leftarrow \left[0, 0, \dots, 0\right]_{1\times k}$

while $\sum s_{iz} < {}^{k}/_{z}$ **do**

  $\lambda \leftarrow$ Random draw from w as per PDF ζ

  $v_\lambda \leftarrow \text{where}(v_k == v_k[\lambda])$

  if $\sum_{t\in v_\lambda} s_{iz}[t] = 0$ **then**

    $s_{iz}[\lambda] \leftarrow 1$

  end if

end while

$p_{iz} \leftarrow iz^{\text{th}}$ Row of H corresponding to B_P $r_{iz} \leftarrow \begin{bmatrix} s_{iz} & p_{iz} \end{bmatrix}$

$j \leftarrow 1$

while $j < z-1$ **do**

  $r_{iz+j}[w_n] \leftarrow r_{iz}[zv_n + (w_n - j) \mod z]$

  $j \leftarrow j+1$

end while

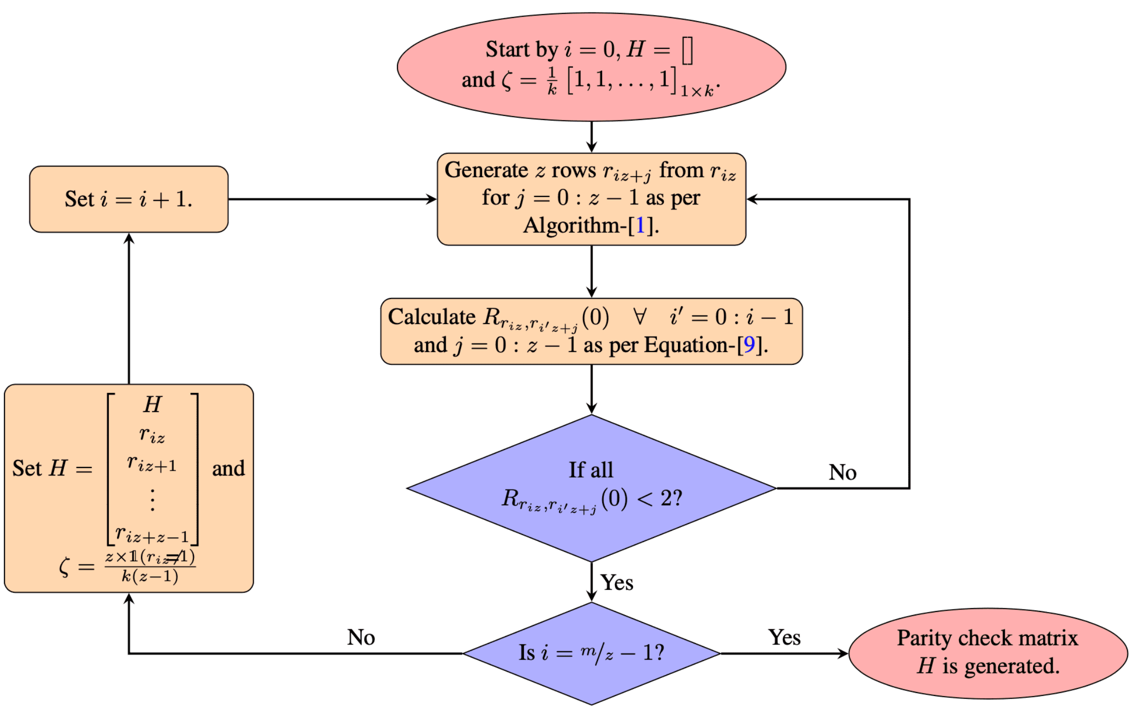
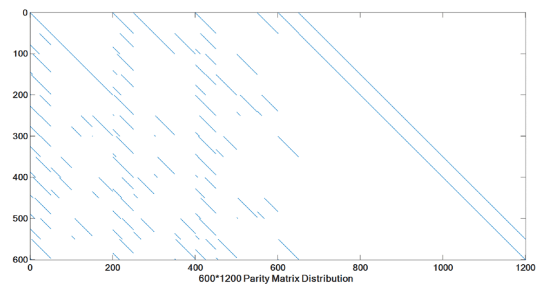


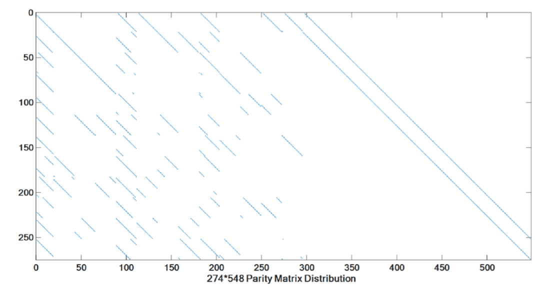
Figure 2. Algorithm for Parity Check Matrix generation. Here the function 1 is a standard indicator function which is 1 when the argument is True, else 0.

3.2. Parity Check Matrices generated by the proposed method

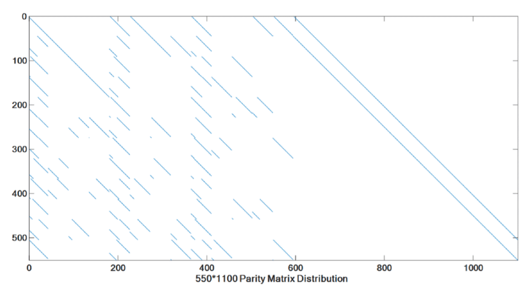
As per the navigation system requirements of NavIC, PCMs are generated for $(m,n,z) = (600, 1200, 12), (274, 548, 12), (550, 1100, 50)$ and $(324, 648, 12)$. It is to be noted that the PCM for $(m,n,z) = (274, 548, 12)$ was generated with an expansion factor of 12, but the last two rows were truncated. The truncation had minimal effect on performance. The generated matrices are shown in Figure-[3].



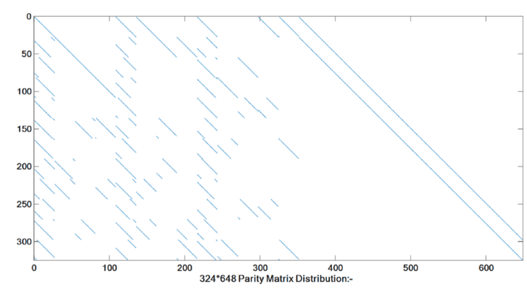
(a) $(m,n,z) = (600, 1200, 12)$



(b) $(m,n,z) = (274, 548, 12)$



(c) $(m,n,z) = (550, 1100, 50)$



(d) $(m,n,z) = (324, 648, 12)$

Figure 3. Generated Parity Check Matrices by the proposed method. Blue regions in the plot indicate the indices of PCM with value 1.

4. Performance Evaluation

We have simulated the bit error ratio (BER) performance of the proposed LDPC code in AWGN and Land Mobile Satellite (LMS) channels. Quadrature Phase Shift Keying (QPSK) modulation is used as the modulator and demodulator, where the QPSK demodulator provides a 3-bit soft log-likelihood ratio output. The maximum number of iterations is set to 50 for each decoding process. The LDPC Decoder block uses the belief propagation algorithm to decode a binary LDPC code, which is input to the block as the soft-decision output (log-likelihood ratio of received bits) from demodulation. The block decodes generic binary LDPC codes where no patterns in the parity-check matrix are assumed. In order to evaluate the performance and power of LDPC codes, a ½ convolutional code used in the

GALILEO L1 band and the uncoded performance of QPSK modulation are also simulated. A comparative simulation is performed with respect to the GPS LDPC polynomial in the identical simulation environment. Figures [4] and [5] show the comparative performance results of the new proposed Quasi-cyclic LDPC code with respect to the GPS L1C LDPC codec for $m = 600$ and 274 respectively. Figures [6] and [7] show the performance of the proposed LDPC codec for $m = 550$ and 324 against the GPS L1C codec for $m = 600$ and 274 respectively.

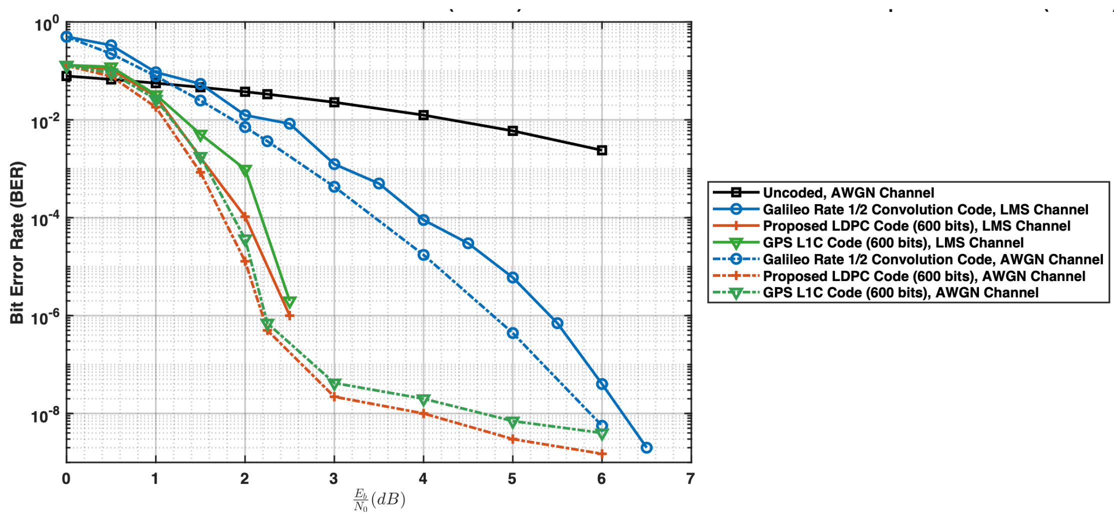


Figure 4. BER performance comparison of proposed QC-LDPC codes ($m = 600$), GPS L1C LDPC codes ($m = 600$), and Galileo $1/2$ convolutional codes for AWGN and LMS channels.

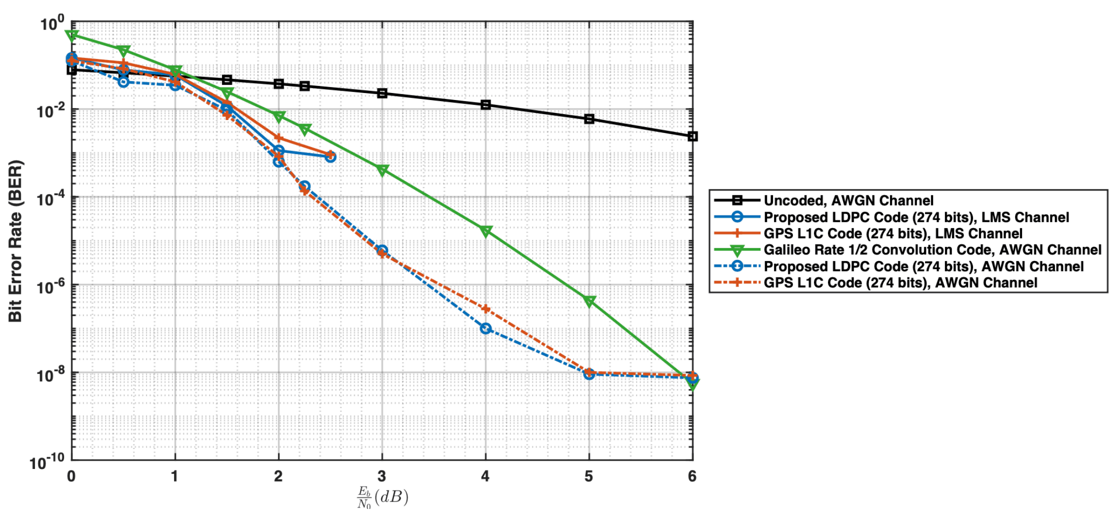


Figure 5. BER performance comparison of proposed QC-LDPC codes ($m = 274$), GPS L1C LDPC codes ($m = 274$), and Galileo $1/2$ convolutional codes for AWGN and LMS channels.

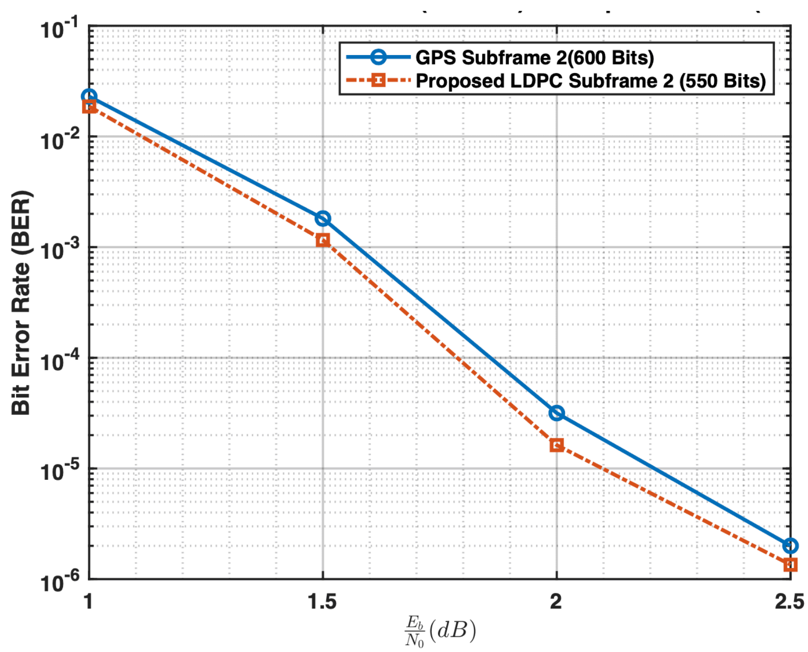


Figure 6. BER performance comparison of proposed QC-LDPC codes ($m = 550$) and GPS L1C LDPC codes ($m = 600$).

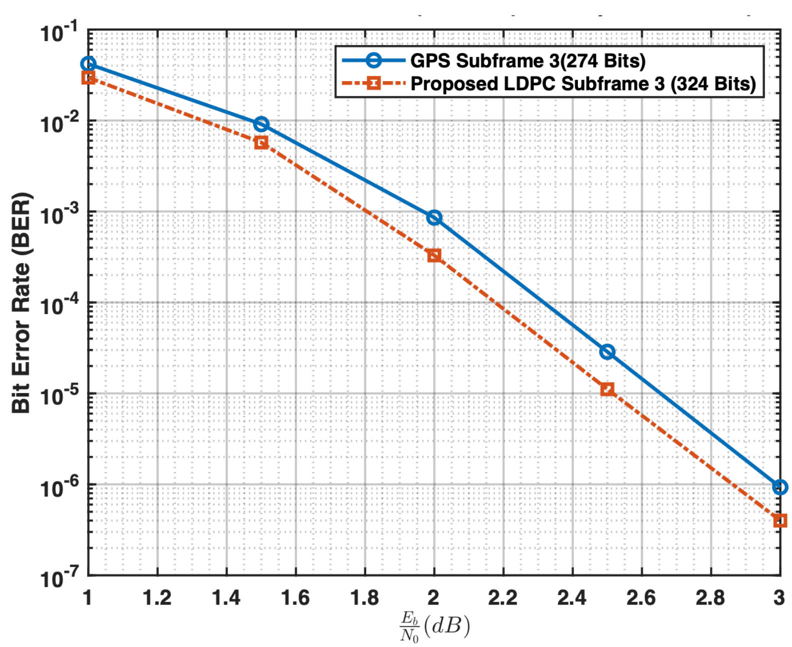


Figure 7. BER performance comparison of proposed QC-LDPC codes ($m = 324$), and GPS L1C LDPC codes ($m = 274$).

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Using the 802.11n protocol as a reference, the parity-check part is double diagonal, which makes the encoding process possible directly with the parity-check matrix. The results highlight that as the frame size increases, the forward error correction capability of the proposed codes increases. The proposed codes offer a coding gain of nearly 7 dB, 7.2 dB, 7.5 dB, and 7.8 dB in the case of subframe lengths of 274, 324, 550, and 600 bits, respectively. They offer a coding gain of 2.5 dB in reference to the Galileo Rate 1/2 convolutional code for a subframe length of 600 bits. Simulation results show that the proposed Quasi-cyclic (QC) LDPC code performs comparably better than the GPS L1C LDPC codec, while providing lower encoding and decoding complexity, making it more suitable for Navigation Band signals. Moreover, the performance of the proposed codes is comparable in both AWGN and LMS channels.

5. Conclusion

This paper presents an algorithm for constructing Quasi-Cyclic LDPC codes for Navigation Band signals, designed to avoid short cycles of length 4 in the Tanner graph. The QC-LDPC code's parity-check matrix, composed of circulant permutation or zero matrices, requires significantly less memory than random LDPC codes. The paper examines the bit error ratio (BER) vs. energy per bit per unit noise density (E_b/N_0) performance, showing comparable results to GPS L1C LDPC and Galileo Convolution codes. These QC-LDPC codes are currently operational in NVS-01 NavIC Satellites of the Indian Space Research Organization.

Statements and Declarations

Availability of data and materials

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Conflicts of interest

The authors declare no conflict of interest.

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Author contribution

All the authors have contributed equally to the article.

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