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Peer Review

Review of: "Observation of Prethermalization in Weakly Nonintegrable Unitary Maps"

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Dear Editor,

The author discusses in this preprint the behavior of a classical unitary circuit map. I mean, on every site there is a classical variable \psi_j(t), and many layers of Hamiltonian operations are applied. Two subsequent layers apply two-sites linear rotations, the next one applies to each site a nonlinear phase shift, and the thing is repeated again and again. This model has been already introduced by some of the authors in Refs. [8,9,10], and its chaos and thermalization properties have been deeply inquired.

The point of the manuscript is again to study thermalization properties, paying attention to prethermalization, a condition in which thermalization sets in after a long transient when an integrable point is nearby. They do that analysis both in the short-range network regime (the surroundings of the integrable point \theta=0) and the long-range network regime (the surroundings of the integrable point g=0) using a quantity called ``Lyapunov observable r(t)'' that provides the local-in-time rate of divergence of nearby trajectories, and averaged over time gives rise to the largest Lyapunov exponent. This quantity can be obtained semi-analytically, as shown in Eq. (6).

They look at its distribution and consistently find that it is broader and shifted towards positive values when the system is chaotic. Beyond that, they study \Lambda(t) – the average over trajectories up to time t of r(t) – that provides an estimate of the largest Lyapunov exponent. They find that near integrable points in parameter space (\theta=0, g=0), \Lambda(t) attains an asymptotic value different for different initial conditions, and this marks prethermalization (see Fig. 6(a), for instance). This is confirmed by the analysis of the evolution of the average and standard deviation of \lambda(t) shown in Figs. 7 and 8.

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Plotting the mean and the standard deviation together in the same plot (Figs. 7 and 8) is quite confusing, and I suggest the authors separate them. A maybe clearer thing to do would be using the ratio of \sigma and \mu, as some of the authors have done in [Phys. Rev. Lett. 122, 054102 (2019)]. Also, the text relating to Figs. 7 and 8 might be clarified and improved. Another thing that would be interesting is comparing what happens at different system sizes in order to see if a thermodynamic-limit behavior has been attained.

The idea is nice. Some of the authors have shown in [Phys. Rev. Lett. 122, 054102 (2019)] that the time scale provided by the Lyapunov exponent can be much smaller than the prethermalization time, and here they see this fact directly through the Lyapunov exponent itself, whose distribution shrinks over a time scale much longer than the one provided by the Lyapunov exponent. Beyond that, they choose for this analysis a system where the evaluation of the local-in-time rate is quite simple and direct [see Eq. (6)], and the dynamics is a simple iteration of a map, so they can reach quite large times and sizes. Therefore, there are important points of interest.

On the other hand, the work is very numerical, and it reduces to a description of numerical results without interpretation. One has the feeling that there is not so much beyond the use of standard methods of prethermalization analysis applied to a well-known Hamiltonian model, the only novelty being the fact that these methods are applied to the Lyapunov observable. In the introduction, they argue that it is better to apply them to the Lyapunov exponent because it provides a more observable-independent estimate of thermalization, but one might be curious to see what happens using other observables and in which sense they are worse.

A direction for further inquiry might be to go beyond the Hamiltonian-map dynamics and move to a dissipative map by adding to each iteration a layer with driven-dissipative effects; for instance, doing as in Eq. (1.7) of [arXiv:1503.02816v1]. This would be very timely, considering the recent strong interest in quantum circuits with a layer where onsite measurements occur with some probability [Phys. Rev. X 9, 031009 (2019)]. In this sense, something similar might also be done in this classical case.

Declarations

Potential competing interests: No potential competing interests to declare.

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