

Research Article

Emergent Electromagnetism from the Gauged Logarithmic Superfluid Vacuum

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The relativistic logarithmic Klein–Gordon equation for the vacuum condensate possesses a global $U(1)$ phase symmetry. We show that promoting this to a local gauge symmetry—the minimal coupling $\partial_\mu S \rightarrow \partial_\mu S - eA_\mu$ —produces Maxwell’s equations as the Euler–Lagrange equations for the gauge field A_μ , with the condensate current $J^\nu = (em_{eff}/\hbar_{eff}^2)\rho u^\nu$ as source. The Madelung decomposition of the gauged equation yields the acoustic metric equations (emergent gravity) and Maxwell’s equations (emergent electromagnetism) simultaneously from a single Lagrangian. Gravity is sourced by the soliton energy density (mass); electromagnetism is sourced by the topological winding number of vortex–Gausson defects, which quantizes electric charge as $Q = ne$ with $n \in \mathbb{Z}$. Both fields propagate at the sound speed c on the same acoustic metric, automatically ensuring $c_{grav} = c_{EM} = c$ as observed. Minimal coupling of the bulk order parameter is shown to be untenable: a charged vacuum condensate would acquire a Planck-scale photon mass through the Anderson–Higgs mechanism and a uniform vacuum charge density, both excluded by observation. Consistency forces the gauge coupling onto the topological sector alone: the neutral bulk supports free, massless electromagnetic waves, while London screening and the Meissner effect arise only where a charged condensate of defects forms—recovering superconductivity as a prediction for matter rather than a pathology of the vacuum. The Coulomb field of a charged particle is sourced by its quantized winding charge in the standard way. Gravity and electromagnetism thereby emerge from a common origin in the same condensate Lagrangian—the acoustic metric and the gauge connection—while preserving their distinct physical sources: mass versus topological charge.

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1. Introduction

The logarithmic nonlinearity was introduced in nonlinear quantum theory ^[1]; its relativistic Klein-Gordon form, on which this work is built, originates with Rosen ^[2] and was analyzed by Bartkowski and Górka ^[3]. Emergent spacetime metrics from a logarithmic condensate have also been obtained in the separate superfluid-vacuum-theory programme ^[4]; that programme and the present logarithmic-superfluid-vacuum framework share the logarithmic nonlinearity but differ in their field ontology, rest-frame structure, and relativistic formulation, and the topological-charge construction developed here has no counterpart there. The acoustic-metric formalism through which long-wavelength excitations of a condensate experience an emergent Lorentzian geometry was established in the analogue-gravity programme ^{[5][6][7]}.

In previous work, we showed that the Madelung decomposition of the relativistic logarithmic Klein-Gordon equation produces an acoustic metric equivalent to the Schwarzschild geometry in the weak-field limit, yielding the PPN parameter $\gamma = 1$ ^{[8][6][7]} and an emergent gravitational constant $G_{eff} \sim c^2/(\xi^2 \rho_0)$ ^{[9][10]}, with ξ the condensate healing length identified with the Planck length and ρ_0 at the Planck density. The vacuum condensate's density ρ and flow velocity $u_\mu = \partial_\mu S/m_{eff}$ generate the gravitational field; topological defects (vortex-Gaussons) play the role of massive particles.

This framework has a conspicuous unused structure. The logarithmic wave equation

$$\square\psi + \mu^2\psi - b\psi\ln\left(\frac{|\psi|^2}{\rho_0}\right) = 0 \quad (1)$$

is invariant under the global phase rotation $\psi \rightarrow \psi e^{i\alpha}$ for constant α . This $U(1)$ symmetry is associated, via Noether's theorem, with the conservation of the condensate particle number (the continuity equation $\partial_\mu(\rho\partial^\mu S) = 0$).

In the Standard Model, the fundamental interactions arise from *gauging* internal symmetries: promoting global invariances to local ones and introducing gauge fields to maintain covariance. Electromagnetism itself originates from gauging the $U(1)$ phase symmetry of the electron field ^[11].

In this paper, we apply the same principle to the vacuum condensate. Promoting the global $U(1)$ of Eq. (1) to a local symmetry requires the introduction of a gauge field A_μ and the minimal coupling $\partial_\mu \rightarrow D_\mu = \partial_\mu - i(e/\hbar)A_\mu$. We show that the Euler-Lagrange equations for A_μ are precisely Maxwell's equations, with the condensate current as source. Together with the acoustic metric (gravity), this

exhibits two fundamental interactions emerging from a single condensate Lagrangian through distinct mechanisms. A central structural result (Sec. 7) is that the gauge coupling cannot act on the bulk condensate: a charged vacuum would acquire a Planck-scale photon mass by the Anderson-Higgs mechanism and a uniform background charge density. The coupling must instead reside in the topological sector—precisely where the charge quantization $Q = ne$ of Sec. 5 places it—leaving the bulk neutral and the photon massless.

2. The Gauged Lagrangian

The ungauged Lagrangian density for the logarithmic condensate is:

$$\mathcal{L}_0 = (\partial_\mu \psi^*)(\partial^\mu \psi) - \mu^2 |\psi|^2 + b |\psi|^2 \ln \left(\frac{|\psi|^2}{\rho_c} \right), \quad (2)$$

where ρ_c is the characteristic density of the logarithmic potential. The background density ρ_0 is defined by the stationarity condition $V'(\rho_0) = 0$, which gives $\rho_c = e\rho_0$. This is invariant under $\psi \rightarrow \psi e^{i\alpha}$ for constant α .

Promoting to local gauge invariance $\psi \rightarrow \psi e^{i\alpha(x)}$ requires replacing $\partial_\mu \rightarrow D_\mu = \partial_\mu - i(e/\hbar)A_\mu$ and adding the gauge field kinetic term. The gauged Lagrangian is:

$$\boxed{\mathcal{L} = (D_\mu \psi)^*(D^\mu \psi) - \mu^2 |\psi|^2 + b |\psi|^2 \ln \left(\frac{|\psi|^2}{\rho_c} \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}} \quad (3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor and e is the gauge coupling constant.

The term $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ is the unique Lorentz-invariant, gauge-invariant, parity-preserving kinetic term for A_μ with at most two derivatives ^[11]. Its inclusion is not optional: without it, A_μ would have no dynamics and could be gauged away.

Two remarks fix the geometric setting of this term. First, the gauge field is an excitation of the same condensate whose density and flow define the acoustic metric $g_{\mu\nu}(\rho, u)$ (Sec. 3); it therefore propagates on that metric, not on a separate flat background. The kinetic term is understood in its covariant form

$$-\frac{1}{4} \sqrt{-g} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma}, \quad (4)$$

with $g_{\mu\nu} = g_{\mu\nu}(\rho, u)$ the acoustic metric; the flat expression is its local-inertial-frame form, valid at the background where $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$. Second, the source-free Maxwell action (4) is conformally invariant on the acoustic metric: under a rescaling $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ the volume factor $\sqrt{-g}$ and the pair of inverse metrics

contribute canceling powers of Ω , leaving the action unchanged. (This cancellation is a property of the acoustic metric's Lorentzian $3 + 1$ structure, in which the emergent time direction enters with opposite signature to the three spatial directions; it is not an assumption that time is a fundamental dimension.) The acoustic metric's departure from flat is carried by an overall conformal factor ρ/c_s together with the Painlevé–Gullstrand flow term; the conformal factor drops out of the photon dynamics entirely, so the gauge sector is sensitive only to the flow. This is the geometric reason the emergent photon remains exactly achromatic (Sec. 6) while still being deflected by the gravitational flow (Sec. 8).

A scope qualification is required at the outset. As written, Eq. (3) assigns charge e to the entire field ψ , including the topologically trivial bulk condensate. Section 7 shows that this assignment is untenable—a charged bulk generates a Planck-scale photon mass and a uniform vacuum charge density—and that consistency requires the covariant derivative to act only on the charged, topological sector: the vortex–Gausson defects. Equation (3) is therefore to be read as the Lagrangian of the charged sector, with the neutral bulk governed by $\mathcal{L}_0$. The kinematic results of Secs. 3–5 concern the structure of the gauged equations and are unaffected by this restriction.

3. Madelung Decomposition of the Gauged Equation

3.1. The gauge-invariant velocity

Applying the Madelung transformation $\psi = \sqrt{\rho}e^{iS/\hbar_{eff}}$, the covariant derivative becomes:

$$D_\mu \psi = \left(\frac{\partial_\mu \sqrt{\rho}}{\sqrt{\rho}} + \frac{i}{\hbar_{eff}} (\partial_\mu S - eA_\mu) \right) \psi. \quad (5)$$

The gauge-invariant superfluid four-velocity is:

$$\boxed{u_\mu = \frac{1}{m_{eff}} (\partial_\mu S - eA_\mu)} \quad (6)$$

This is the central modification. In the ungauged theory, $u_\mu = \partial_\mu S/m_{eff}$; the gauge field shifts the velocity by $-eA_\mu/m_{eff}$. This is the relativistic generalization of the London relation in superconductors: $m\mathbf{v}_s = \hbar\nabla\phi - e\mathbf{A}$ ^[12].

3.2. The modified fluid equations

Separating the Euler–Lagrange equation for ψ into real and imaginary parts yields the gauged Madelung equations:

Continuity:

$$\partial_\mu(\rho u^\mu) = 0. \quad (7)$$

Hamilton-Jacobi:

$$\frac{m_{eff}^2}{\hbar_{eff}^2} u_\mu u^\mu = \mu^2 - b \ln\left(\frac{\rho}{\rho_c}\right) + Q[\rho], \quad (8)$$

where $Q = \square\sqrt{\rho}/\sqrt{\rho}$ is the quantum potential (Bohm potential) ^[13], unchanged from the ungauged case.

These equations are *identical in form* to the ungauged Madelung equations, with the single replacement $\partial_\mu S \rightarrow \partial_\mu S - eA_\mu$. The acoustic metric depends on ρ and u_μ , both of which are gauge-invariant. The entire gravitational sector—the PPN parameter $\gamma = 1$, the emergent G_{eff} , the Painlevé–Gullstrand flow—is preserved without modification.

4. Maxwell's Equations from the Gauge Field Variation

The Euler-Lagrange equation for A_μ is obtained by varying the Lagrangian (3) with respect to A^ν .

The condensate contribution:

$$\frac{\delta}{\delta A^\nu} [(D_\mu \psi)^* (D^\mu \psi)] = -\frac{em_{eff}}{\hbar_{eff}^2} \rho u^\nu. \quad (9)$$

The gauge field contribution, from the covariant kinetic term (4):

$$\frac{\delta}{\delta A^\nu} \left[-\frac{1}{4} \sqrt{-g} F_{\alpha\beta} F_{\rho\sigma} g^{\alpha\rho} g^{\beta\sigma} \right] = \sqrt{-g} \nabla_\mu F^{\mu\nu}. \quad (10)$$

Setting the total variation to zero gives Maxwell's equation on the acoustic metric,

$$\boxed{\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} F^{\mu\nu} \right) = J^\nu} \quad (11)$$

which at the background metric $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ reduces to the flat form $\partial_\mu F^{\mu\nu} = J^\nu$, while near a gravitating mass the metric factors encode the deflection, redshift, and time delay of electromagnetic waves by the Painlevé–Gullstrand flow (Sec. 8). The four-current is:

$$J^\nu = \frac{em_{eff}}{\hbar_{eff}^2} \rho u^\nu. \quad (12)$$

Here ρ and u^ν are the density and gauge-invariant velocity of the charged sector (Sec. 2); for the neutral bulk this current vanishes identically. In the far field of a vortex-Gausson, the circulation quantization of Sec. 5 reduces it to the current of a point charge $Q = ne$.

Equation (11) is Maxwell's inhomogeneous equation. The homogeneous equations $\partial_{[\mu} F_{\nu\lambda]} = 0$ follow identically from the definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (the Bianchi identity).

In three-vector notation, at the background metric (the flat reduction; curvature enters through the metric factors of Eq. 11):

$$\nabla \cdot \mathbf{E} = \rho_{ch}/\epsilon_0 \quad (\text{Gauss}), \quad (13)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{no monopoles}), \quad (14)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{Faraday}), \quad (15)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{Ampère-Maxwell}), \quad (16)$$

where $\rho_{ch} = J^0$ is the charge density and $\mathbf{J} = (J^1, J^2, J^3)$ is the current density. These are the full Maxwell equations, derived from the gauge field variation of the condensate Lagrangian.

5. Charge Quantization from Vortex Topology

The vortex-Gausson solitons of the logarithmic condensate have quantized phase winding. For a vortex of winding number n , the phase increases by $2\pi n \hbar_{eff}$ around any closed loop encircling the core:

$$\oint \partial_\mu S dx^\mu = 2\pi n \hbar_{eff}, n \in \mathbb{Z}. \quad (17)$$

The gauge-invariant circulation is:

$$\oint u_\mu dx^\mu = \frac{1}{m_{eff}} \oint (\partial_\mu S - e A_\mu) dx^\mu = \frac{2\pi n \hbar_{eff}}{m_{eff}} - \frac{e}{m_{eff}} \oint A_\mu dx^\mu. \quad (18)$$

In the absence of an external gauge field ($A_\mu = 0$ at infinity), the total charge enclosed by the loop is determined by the winding number:

$$\boxed{Q = ne, n \in \mathbb{Z}} \quad (19)$$

Electric charge is quantized because vortex winding numbers are integers. This is the same mechanism by which magnetic flux is quantized in superconductors ^[14]: the single-valuedness of the order parameter forces the enclosed quantity to be an integer multiple of the fundamental quantum.

The physical identification is:

- Vortex-Gaussons with $n = +1$: positively charged particles (charge $+e$).
- Anti-vortices with $n = -1$: negatively charged particles (charge $-e$).
- Pure Gaussons with $n = 0$: electrically neutral particles (charge 0).

The vortex-Gausson thus carries both mass (from its soliton energy E_v , generating gravity) and charge (from its winding number n , generating electromagnetism).

6. Electromagnetic Waves in the Condensate

In the vacuum background the neutral bulk does not couple to A_μ (Sec. 2); with no charged defects present, $J^\nu = 0$ and the Maxwell equations (11) reduce to:

$$\square A_\mu = 0 \quad (\text{in Lorenz gauge}), \quad (20)$$

where $\square = \partial_t^2/c^2 - \nabla^2$ and $c = c_s(\rho_0)$ is the sound speed at the background density. Electromagnetic waves propagate at the speed of sound in the condensate.

This provides a unified explanation: the speed of light is the speed of sound because the photon is a gauge excitation propagating on the same condensate whose acoustic properties define c . The identity $c_{EM} = c_{grav} = c_{sound}$ is not a coincidence but a structural consequence of both fields emerging from the same substrate.

Near a gravitating mass, the acoustic metric modifies the effective spacetime seen by all excitations equally. Electromagnetic waves are deflected, redshifted, and time-delayed by the Painlevé-Gullstrand flow, reproducing the standard general relativistic predictions. The Shapiro time delay, the gravitational redshift, and the bending of light are automatically consistent with $\gamma = 1$ because the photon (gauge excitation) and the phonon (density excitation) share the same acoustic metric. The gapless excitation branch of the relativistic logarithmic condensate is exactly linear, $\omega = ck$ to all orders [8]: the second-order relativistic kinetic operator admits no k^4 correction, so electromagnetic propagation in the bulk is exactly achromatic, satisfying gravitational-wave and Lorentz-invariance constraints identically rather than by numerical suppression.

7. The London Equation and Electromagnetic Screening

Consider first the consequence of letting the *bulk* condensate carry the gauge coupling, as a literal reading of Eq. (3) would imply. In a region of uniform density ($\rho = \rho_0$) and smooth phase, the gauge-invariant velocity (6) contains the piece

$$u_\mu = -\frac{e}{m_{eff}} A_\mu, \quad (21)$$

which, substituted into the current (12), yields a response proportional to the gauge field itself (m_{eff} cancels):

$$J^\nu = -\frac{e^2 \rho_0}{\hbar_{eff}^2} A^\nu. \quad (22)$$

Combined with Maxwell's equation (11), this gives the Proca equation

$$\square A^\nu + \frac{1}{\lambda_L^2} A^\nu = 0, \quad (23)$$

with London penetration depth

$$\lambda_L = \frac{\hbar_{eff}}{e} \frac{1}{\sqrt{\rho_0}}. \quad (24)$$

This is the Anderson-Higgs mechanism: the gauge field absorbs the condensate phase and propagates with mass $m_\gamma = \hbar_{eff}/(\lambda_L c)$. With ρ_0 at the Planck density, λ_L is microscopic and m_γ is of order the Planck mass up to the dimensionless coupling—at least forty orders of magnitude above the observational bound $m_\gamma < 10^{-18} eV$. A minimally gauged bulk would be a Planck-scale superconductor, opaque to electromagnetic radiation. Independently, the background carrier phase $\partial_t S = -m_{eff} c^2$, structural throughout the framework [8], makes $u^0 \neq 0$: the gauged bulk would also carry a uniform charge density $J^0 \propto e\rho_0$, for which Gauss's law admits no consistent solution in an infinite homogeneous medium.

Two seemingly natural escapes fail. Electrical *neutrality* of the bulk does not help: the screening kernel in Eq. (22) is proportional to $e^2 \rho_0$, not to the net charge, and laboratory superconductors—globally neutral systems—expel magnetic fields for precisely this reason. Nor does spatial variation of the macroscopic phase: far from gravitating masses $\nabla S \rightarrow 0$ and $\rho \rightarrow \rho_0$, so the screening would be *strongest* exactly where electromagnetic waves are observed to propagate freely. (Note also that $\lambda_L \propto 1/\sqrt{\rho}$: screening weakens, not strengthens, where the density drops toward a vortex core.)

The resolution is supplied by the charge quantization of Sec. 5. Charge is winding, $Q = ne$; the bulk condensate is topologically trivial, $n = 0$, and therefore carries no charge. Consistency thus forces the reading of Eq. (3) announced in Sec. 2: the covariant derivative acts only on the charged, topological sector—the vortex-Gausson defects—while the neutral bulk is governed by the ungauged $\mathcal{L}_0$. With the bulk decoupled, the photon mass term vanishes identically, electromagnetic waves propagate freely at $\omega = ck$ (Sec. 6), and the Coulomb field of a charged particle is sourced by its localized winding charge through Gauss's law in the standard way.

The London response then reappears where it belongs. Wherever charged defects condense into a coherent macroscopic state—a laboratory superconductor—the charged-sector density ρ_{ch} is nonzero over a macroscopic region, and Eqs. (21)–(24) apply with $\rho_0 \rightarrow \rho_{ch}$: a finite penetration depth and the Meissner effect. The screening that consistency forbids in the vacuum is recovered as the correct electrodynamics of superconducting matter: the vacuum is a transparent dielectric; a charged condensate of defects is a superconductor.

8. Common Origin of Gravity and Electromagnetism in the Condensate

The gauged logarithmic condensate produces two emergent interactions from a single Lagrangian:

Gravity is the emergent geometry itself: the acoustic metric $g_{\mu\nu}(\rho, u)$ on which all condensate excitations propagate. The soliton energy density (mass) sets the scale of the metric perturbation, and self-consistency of the metric selects the Painlevé–Gullstrand form equivalent to Schwarzschild, with coupling $G_{eff} \sim c^2/(\xi^2 \rho_0)$, $\xi \sim \ell_P$, ρ_0 at the Planck density [\[9\]\[10\]](#). The microscopic origin of the collective flow that realizes this metric remains an open problem [\[9\]](#).

Electromagnetism arises from the gauge field A_μ required by local $U(1)$ invariance. Its source is the topological winding number (charge). Its field equation is Maxwell's $\partial_\mu F^{\mu\nu} = J^\nu$. Its coupling constant is e .

The two interactions are coupled through the condensate:

- Charged particles (vortex-Gaussons) create both gravitational (acoustic metric) and electromagnetic (gauge) fields simultaneously.
- Within charged matter (condensed defects), the electromagnetic field modifies the charged-sector flow through the London relation (21); the neutral bulk vacuum does not couple.
- The gravitational flow (PG inflow) modifies the effective spacetime for electromagnetic wave propagation—this is the gravitational deflection of light.

The two interactions are not symmetric in status. Electromagnetism is a genuine gauge field propagating on the condensate: it is sourced by the topological winding charge, satisfies Maxwell's equations on the acoustic metric, and is mechanically well-defined. Gravity is not a field on the same footing but the emergent *geometry* on which the photon and all other excitations propagate; the acoustic metric

reproduces the weak-field observables ($\gamma = 1$, the Shapiro delay, deflection) but its collective flow is not derived from the microscopic dynamics in the present single-field treatment. What the framework does establish cleanly is the difference in coupling: electromagnetism is topological (only $n \neq 0$ defects carry charge, which cancels for neutral matter), whereas the geometric coupling is universal (every soliton carries energy). This is the condensate origin of the fact, postulated in general relativity, that electromagnetism couples to charge while gravity couples to all matter.

9. Discussion

9.1. What is derived and what is assumed

The derivation rests on four inputs:

- i. The logarithmic wave equation (1), which defines the condensate dynamics.
- ii. The requirement of local $U(1)$ gauge invariance, which forces the introduction of A_μ .
- iii. The gauge field kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, which is the unique allowed term.
- iv. The restriction of the gauge coupling to the topological sector, leaving the bulk neutral. This is forced by consistency—the photon-mass and vacuum-charge obstruction of Sec. 7—and is consonant with the charge quantization $Q = ne$, but it is an input at the level of the effective Lagrangian: a microscopic derivation of why the bulk phase is not gauged is left open.

Of these, (i) is the foundation of the entire framework. Input (ii) is a symmetry principle—the same one used in the Standard Model to derive electromagnetism from the electron field's $U(1)$ symmetry [11]. Input (iii) is uniquely determined by Lorentz invariance, gauge invariance, and the requirement of at most two derivatives. Input (iv) is forced by observational consistency rather than assumed for convenience: a bulk-gauged condensate is excluded by the photon mass and vacuum charge it would generate (Sec. 7), and the topological sector is the only place the coupling can consistently reside. Its status as an input—rather than a derived feature—marks the principal open problem of this work: a microscopic account of why the bulk phase decouples from A_μ .

The coupling constant e is *not* determined by the condensate parameters. Deriving its value (equivalently, the fine structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$) would require computing the self-energy of a vortex-Gausson in the gauged condensate and matching it to the observed electron mass. This remains an open problem.

9.2. Comparison with the Standard Model

In the Standard Model, electromagnetism arises from gauging the $U(1)_Y$ hypercharge symmetry of the fermion and Higgs fields. The photon is the massless gauge boson associated with the unbroken $U(1)_{EM}$ after electroweak symmetry breaking. The electron's charge is a coupling constant.

In the present framework, electromagnetism arises from gauging the $U(1)$ phase symmetry of the vacuum condensate. The photon is the massless gauge excitation propagating on the condensate. The electron's charge is the same coupling constant e , and charge quantization follows from vortex topology rather than from anomaly cancellation.

The key structural difference is that here, the condensate provides both the gravitational field (acoustic metric) and the electromagnetic field (gauge connection), whereas the Standard Model treats gravity as external.

9.3. Limitations

The framework produces only a $U(1)$ gauge field. The Standard Model requires $SU(3) \times SU(2) \times U(1)$. Extending to non-Abelian gauge groups would require a multi-component order parameter (e.g., a spinor or matrix-valued condensate), which is beyond the scope of the present single-component model.

The identification of electric charge with vortex winding number predicts that all charged particles have integer multiples of e . This is consistent with observation (quarks carry fractional charges, but they are confined, and all observable states carry integer charges). However, deriving quark confinement from the condensate would require the $SU(3)$ extension mentioned above.

10. Conclusion

We have shown that gauging the $U(1)$ phase symmetry of the logarithmic superfluid vacuum—with the coupling necessarily restricted to the topological sector, the bulk remaining neutral—produces Maxwell's equations alongside the acoustic metric equations from a single Lagrangian, with a massless photon and the Meissner effect recovered as a property of charged matter rather than of the vacuum. The Madelung decomposition of the gauged equation separates cleanly into:

1. The gravitational sector: the acoustic metric $g_{\mu\nu}(\rho, u)$, governed by the Bernoulli relation and the Painlevé–Gullstrand flow, sourced by mass (soliton energy).

2. The electromagnetic sector: the gauge field A_μ , governed by Maxwell's equations $\partial_\mu F^{\mu\nu} = J^\nu$, sourced by charge (vortex winding number).

Electric charge is quantized ($Q = ne$, $n \in \mathbb{Z}$) because vortex winding numbers are integers. Both fields propagate at the sound speed c , explaining the identity $c_{EM} = c_{grav}$ as a structural consequence rather than a coincidence.

The coupling constant e (the fine structure constant) is not determined by the condensate parameters and remains an open problem. Extension to non-Abelian gauge groups requires a multi-component condensate and is deferred to future work.

Statements and Declarations

Acknowledgments

The methodological stance of this paper—treating the order parameter as a real classical field whose hydrodynamic behavior underlies emergent phenomena—is indebted to the Madelung-de Broglie-Bohm tradition, particularly as developed in P. R. Holland's exposition of the quantum theory of motion. Claude (Anthropic) was used as an editorial and computational tool during manuscript preparation. All scientific content, equations, and interpretations are the author's own.

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