

Peer Review

Review of: "Picture Fuzzy, Hesitant Fuzzy, and Spherical Fuzzy SuperHyperGraph"

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Dear Editor,

Thank you for the opportunity to review the manuscript titled "Picture Fuzzy, Hesitant Fuzzy, and Spherical Fuzzy SuperHyperGraph" submitted by Takaaki Fujita. It is a pleasure to evaluate this contribution, which addresses advanced theoretical constructs in fuzzy graph theory.

Below I provide my detailed review, including the strengths, areas for improvement, and final recommendation. I hope my comments are helpful to both the editorial board and the author.

Sincerely,

Majid Khalili

Manuscript Title: Picture Fuzzy, Hesitant Fuzzy, and Spherical Fuzzy SuperHyperGraph

Author: Takaaki Fujita

Manuscript Type: Theoretical Research Article

General Evaluation

The manuscript presents a novel and mathematically rich extension of SuperHyperGraphs by incorporating Picture Fuzzy Sets, Hesitant Fuzzy Sets, and Spherical Fuzzy Sets. The work is highly theoretical and offers unified mathematical formulations and reduction principles among the proposed models.

Strengths

The author defines new classes of fuzzy SuperHyperGraphs which have not been previously unified in the literature. Definitions such as 2.1, 2.4, and 2.7 are systematically constructed and exhibit internal consistency. Membership constraints like $\alpha^2 + \beta^2 + \gamma^2 \leq 1$ in Spherical Fuzzy sets are appropriately formulated. In section 2.3, the reduction from spherical to intuitionistic fuzzy structures is rigorously proved.

Weaknesses and Suggestions

1. The article contains only theoretical examples such as “Let $V = \{\text{Alice, Bob, Carol, Dave}\}$ ” on page 5, without real-world data application.

Suggestion: Include a basic use case from social networks or multi-agent systems using real datasets.

2. Despite complex graph structures, the paper lacks visual aids or schematic diagrams.

Suggestion: Provide at least one illustrative graph per structure (e.g., Picture Fuzzy SuperHyperGraph) to enhance clarity.

3. There is no section detailing how to derive or normalize values like α, β, γ from real systems.

Suggestion: Add a section titled “Parameter Assignment Strategy” or include guidelines based on fuzzy decision-making methods.

Final Recommendation

- Major Revision Required.

The manuscript is theoretically solid and offers meaningful innovation. However, it requires improvements in empirical grounding, visualization, and reference credibility before it can be accepted for publication. Inclusion of a basic real-world scenario would significantly increase its impact and applicability.

Additional Comments

As a further contribution to enhance the manuscript's clarity and applicability, I would like to offer the following suggestions:

Real-world datasets:

To demonstrate the practical value of the proposed SuperHyperGraph models, the author may consider applying them to publicly available datasets such as:

- The Facebook Social Circles dataset from Stanford SNAP for social network modeling,
- The DBLP co-authorship network for academic collaboration analysis,
- Fuzzy MCDM case studies such as supplier selection or site allocation problems from operations research literature,
- Or biomedical datasets such as the Breast Cancer Wisconsin dataset for uncertainty modeling in healthcare.
- The section that would benefit most from a real-world example is **Section 2.1 (Picture Fuzzy SuperHyperGraph)**. Given its complexity and multi-layered membership degrees, applying the model to a dataset such as a real-world social network (e.g., Facebook circles or DBLP co-authorship network) could clarify its potential for practical modeling.

Suggested implementation steps for a real-world use case:

To illustrate the practical utility of Picture Fuzzy or Spherical Fuzzy SuperHyperGraphs, I suggest the authors consider the following steps using the Facebook Social Circles dataset:

- Step 1: Extract a subset of nodes (e.g., 10–20 users) and identify overlapping social circles (e.g., friends, family, colleagues).
- Step 2: For each SuperEdge, assign fuzzy membership values based on interaction strength (α), uncertainty (β), and opposition (γ) — possibly derived from message frequency, post likes, or mutual friends.
- Step 3: Represent these relationships in a Picture Fuzzy SuperHyperGraph with visual encoding (see below).
- Step 4: Analyze centrality, clustering, or path-related metrics under the fuzzy membership assumptions.

- Step 5: Compare insights with a classic fuzzy or intuitionistic fuzzy graph to demonstrate the benefits of higher-order modeling.

Visual illustrations:

Given the complexity of the fuzzy SuperHyperGraph structures, the addition of visual aids would significantly benefit the reader. I recommend including:

- Schematic diagrams of sample Picture Fuzzy and Spherical Fuzzy SuperHyperGraphs,
- Comparative diagrams that highlight differences between classical fuzzy, intuitionistic fuzzy, and spherical fuzzy approaches,
- And a hierarchy chart that maps how each new model reduces to its predecessors under special conditions.
- As for visual representation, I recommend including:
 - A layered node-edge diagram that shows how SuperHyperGraphs differ from standard HyperGraphs.
 - Color-coded illustrations for fuzzy degrees (truth, neutrality, falsity in Picture Fuzzy; or α , γ , β in Spherical Fuzzy).
 - A hierarchical reduction chart demonstrating how the proposed models reduce to classical fuzzy or intuitionistic fuzzy graphs under specific constraints.

. Recommended visual aids:

- A multi-layered graph diagram showing SuperEdges with nodes grouped in nested clusters.
- Color-coded edge thickness and node sizes to reflect degrees of α (positive), β (neutral), and γ (negative) in Picture Fuzzy models.
- A matrix-based visualization (e.g., fuzzy incidence matrix) for high-dimensional edge membership.
- A model hierarchy tree that visually maps the relationships between Spherical, Picture, Intuitionistic, and Classical fuzzy graphs.
- Example output of a fuzzy clustering or ranking algorithm applied to the fuzzy graph structure.

Sincerely,

Majid Khalili

Declarations

Potential competing interests: No potential competing interests to declare.