Review on Models of Measuring Volatility of Cryptocurrencies

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Abstract

The price of cryptocurrency is always volatile and is influenced by various factors like market returns, prices of stocks, gold, and correlation of prices of cryptocurrency.

Modeling and forecasting the prices of cryptocurrencies and measuring the volatility with the GARCH specification (Engle, 1982) has become standard among researchers. Several applications and extensions of GARCH model is proposed by Bollerslev (1986). Later, an integrated GARCH model (Engle & Bollerslev, 1986) states that the persistence parameter is equal to one. A combination of short and long memory conditional models for the mean and the volatility to analyze crypto returns is done with the help of ARFIMA (Autoregressive Fractionally Integrated Moving Average) and FIGARCH (Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic) Model.

This paper intended to understand various mathematical models for volatility of crypto currencies and also to find research gaps in the existing literature. A comprehensive overview is the need of the study.

Keywords: Crypto-asset, Crypto-exchange, Digital transactions, GARCH Models, Price volatility.

Objectives:

1. To understand various models of measuring the volatility of cryptocurrencies.
2. To make a comprehensive view of application of volatility models in empirical studies.

1. Introduction

The concept of an open-source currency (i.e. digital asset/ crypto currency) without a central point of trust, such as a significant distribution agency or state lead control, is new (King & Nadal, 2012). Investors who acquire digital assets in the form of "cryptocurrencies" should consider about volatile market conditions. Sometimes there is a possibility of a loss of the entire investment due to volatility of prices in cryptocurrency. The results reveal that Bitcoin has shown a successful path since its inception, despite volatile market conditions (Caporale et al., 2018). It should be noted that the cryptocurrencies cannot replace the fiat currency, and they could change how inter-connected global markets interact, clearing away barriers surrounding normative currencies and foreign exchange rates (Peter D. DeVries, 2016).

Several mathematical models are used to measure the volatility behavior of cryptocurrencies like: i) Autoregressive Distributed Lag (ARDL) Model, ii) Heterogeneous Autoregressive (HAR) Model, iii) Autoregressive Conditional Heteroskedasticity (ARCH) Model, and iv) Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models.

2. Review of literature

This literature review is based on a systematic review with a focus on keywords viz., Cryptocurrency, Bitcoin and Volatility of Cryptocurrency.

Every investor needs to have an answer to the questions, viz., is it Bitcoin, that particular private currency that will have the most extended life? Moreover, how long will it run in parallel with the traditional currency? Will Bitcoin have the ability to benefit from a higher degree of confidence than the present one starting from the backdrop of the growing discontent generated by numerous imbalances occurring in the economies of different states? (Angela Rogojanu and Liana Badea, 2014).

The nature and the ability of the five largest cryptocurrencies, viz., Bitcoin, Ethereum, Ripples, NEM, and Dash, are examined by Phillip et al. (2018). Other than bitcoins, there are about 1,000 alternative coins (altcoins) in the global market, with Ethereum being the most popular. Altcoins are cryptocurrencies launched after bitcoin's success (Rajesh Kurup, 2017).

Modeling and forecasting the prices of cryptocurrencies and measuring the volatility with the GARCH specification (Engle, 1982) has become standard among researchers. James and Raul (1994) state that the ARCH models often impute a lot of persistence to stock volatility and yet give relatively poor forecasts. Adrian and Schwert (1990) show the importance of nonlinearities in stock return behavior that are not captured by conventional ARCH or GARCH models.

Several applications and extensions of GARCH model is proposed by Bollerslev (1986). Later, an integrated GARCH model (Engle & Bollerslev, 1986) states that the persistence parameter is equal to one. The analysis of co-integration and error correction with the help of single equation is derived by Engle-Granger test (1987).

The research results show (see Annexure) that robust procedures outperform non-robust ones when forecasting the volatility and estimating the Value-at-Risk. These results suggest that the presence of outliers plays an important role in the modelling and forecasting of Bitcoin risk measures (Carlos Trucíos,2019).

The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model is based on the assumptions of linearity, stationarity and homoscedasticity of error variance. Under these assumptions it is quite impossible to deal with series exhibiting high volatility or periods of instability such as agricultural commodity price series. A combination of short and long memory conditional models for the mean and the volatility to analyze crypto returns is done with the help of **ARFIMA** (Autoregressive Fractionally Integrated Moving Average) and **FIGARCH** (Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic) Model. It is to be noted that understanding the dual Long Memory (LM) in cryptocurrency markets is essential for crypto investors and forecasters. Zhuhua Jiang et al (2023)[1] contribute to the related empirical studies by examining the presence of dual LM (in the mean and the variance) in six major digital currencies (Bitcoin, Dash, Ethereum, Litecoin, Monero, and Ripple).

Lykke et al (2022) find that EGARCH and APARCH perform best among the GARCH models. HAR models based on realized variance perform better than GARCH models based on daily data. Superiority of HAR models over GARCH models is strongest for short-term volatility forecasts.

3. Models for Measuring Volatility of cryptocurrency

3.1. Engle (1982) - Autoregressive Conditional Heteroskedasticity (ARCH) Model

ARCH is a method that explicitly models the change in variance over time in a time series. An ARCH method models the conflict at a time step as a function of the residual errors from a mean process (e.g., a zero mean). The ARCH model has a simple regression model, as can be seen below:

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

$$u_t \sim N(0, \alpha_0 + \alpha_1 u_{t-1}^2)$$

This suggests the error term is normally distributed with zero mean and conditional variance depending on the squared error term lagged one time period. The conditional variance is the variance given the values of the error term:

$$\sigma_t^2 = \text{var}\left(u_t \backslash u_{t-1}, u_{t-2} \dots\right) = E\left(u_t^2 \backslash u_{t-1}, u_{t-2}\right)$$

Where is the conditional variance of the error term. The ARCH effect is then modelled by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$

This model contains only a single lag on the squared error term, however it is possible to extend this to any number of lags, if there are q lags it is termed an ARCH(q) model.

3.2. Bollerslev (1986)-The standard GARCH model)- is represented as sGARCH(1,1)

- $y_t = x_t \gamma + \varepsilon_t$

Here: dependent variable: y_t, exogenous variables: x_t, error term: ε_t.

- $\sigma^2{}_t = \omega + \alpha\varepsilon^2{}_{t-1} + \beta^2\sigma_{t-1}$

This Equation estimates the variance (squared volatility $\sigma_t 2$) at time *t*, which depends on a historical mean (*ω*), news about volatility from the previous period, measured as a lag of the squared residuals from the mean Equation ($\varepsilon_{t-1}2$), and volatility from the last period ($\sigma_{t-1}2$).

3.3. Engle & Bollerslev (1986)-Integrated Generalized Autoregressive Conditional heteroskedasticity (IGRACH 1,1)

This is a restricted version of the GARCH model. The persistent parameters sum up to one and import a unit root in the GARCH process (Engle & Bollerslev, 1986).

- $x_t = \mu + a_t$
- $a_t = \sigma_t \epsilon_t$
- $\sigma^2{}_t = \alpha_0 + \alpha_1 a^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1}$

Here, it is imposed that: $\alpha_1 + \beta_1 = 1$.

3.4. Engle and Granger (1987)-Cointegration and Error Correction

The cointegration is characteristic of a series vector Xt, with the same order of integration d, whose linear combination results in a process with integration order d minus b.

$\exists\beta\neq 0$ and $Zt \coloneqq \beta_0\ Xt \sim I\ (d-b)$; with $b > 0$

Based on the case of series with a unit root, if each element of a vector of time series Xt, stationary only after the first differentiation, generates by linear combination βXt a stationary process with finite variance, they are said to be cointegrated. In practice, two non-stationary series with a stochastic tendency and typical displacements over time are said to be cointegrated.

3.5. Autoregressive Fractionally Integrated Moving Average-Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (ARFIMA-FIGARCH)- Ravichandran et al. (1989)

This model is used to analyze the crypto-assets returns, a powerful combination of short and long memory conditional models for the mean and the volatility.

- A stochastic process x (t) $t \in Z$ is:
- An ARFIMA (p, d, q)- GARCH (r, s), p, q, r, s $\in N \cup \{0\}$ and d $\in R$,
- if it satisfies: $\varphi(B)\nabla dx\ (t) = \theta(B)\ \varrho\ (t)$, with $\varrho\ (t) = \sigma(t)\ z\ (t$

Here, the polynomial Φ(B) and Θ(B) are of orders p and q, respectively, and the fractional differentiation and the white noise process {at}t ∈ Z have zero mean and finite variance.

3.6. Ravichandran et al., (1989): Threshold GARCH (TGARCH)

This indicates the existence of leverage effects of the first order:

- $\sigma^2 t = \omega + \alpha x^2{}_{t-1} + \beta\sigma^2{}_{t-1} + \lambda x^2{}_{t-1} 1_{t-1}$

Here, α and α + λ denote the effect of good news and bad news, respectively, and λ > 0 is evidence that bad news upsurge volatility in the Bitcoin market.

3.7. Ravichandran et al. (1989)- Markov Switching GARCH (MSGARCH)

$Y_t \mid (s_t = k, I_{t-1}) \sim D\,(0,{}_{HK,\,t}, \xi_k)$

where $D\,(0,{}_{HK,\,t}, \xi_k)$ is a continuous distribution with zero mean, time-varying variance${}_{HK,\,t}$,

Furthermore, additional shape parameters are gathered in the vector ξ_k.

The integer-valued stochastic variable s_t, defined on the discrete space {1,..., K}

3.8. Diagnol Baba-EngleKraft-Kroner (BEKK) Model, 1990

- $H_t = C'C + A'\,(\Xi_t{-}1\Xi'{}_{t-1})\,A + B'\,(H_{t-1})\,B$

Where Ht is an nxn conditional variance-covariance matrix, C is an upper triangular matrix of parameters, Ξ_{t-1} is an nx1 disturbance vector, and A and B are 'n x n' diagonal parameter matrices.

3.9. Nelson (1991)-Exponential GARCH (EGARCH) Model-

- $\log\sigma^2{}_t = \omega + \alpha_1 Z_{t-1} + \gamma_1\,[|Z_{t-1}| - E\,(|Z_{t-1}|)] + \beta_1 \log\sigma^2{}_{t-1}$
- $\alpha_1 > 0$, $\beta_1 > 0$, $\gamma_1 > 0$ and $\omega > 0$. α_1 captures the sign effect, and γ_1 captures the size effect.
- The persistence parameter for this model is β_1.

3.10. Higgins and Bera's (1992)

theory applied to weekly exchange rates and found the existence of non-linear ARCH. Their study reveals that “since the NARCH model encompasses various functional forms, we argue it provides a useful framework for testing Engle's original specification against a wide class of alternatives. It would be interesting to investigate what happens to tests of expectation theory or CAPM models if NARCH-type models capture the nonlinearity in the data."

3.11. Glosten, Jagannathan and Runkle (GJR)- GARCH Model-1993

$h^2_t = \alpha_0 + \sum^p_{i=1} (\alpha_i Z^2_{t-i} (1 - 1(Z_{t-i} > 0)) + \gamma_i Z^2_{t-i} 1(Z_{t-i} > 0)) + \sum^q_{j=1} \beta_j h^2_{t-j}$

With parameters $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\gamma_i \geq 0$ that guarantee a non-negative conditional variance. In order to highlight the asymmetry properties, a function f (Z_t) is introduced where the magnitude effects (γ_1) and the asymmetry effects (α_1).

3.12. Ding, Granger and Engle (1993):Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH)

Ding, Granger, and Engle (1993) find that $|\varepsilon_t|^d$ often displays strong and persistent autocorrelation for various values of d, or rather returns have a long memory property. The asymmetric Power ARCH (APARCH) model assumes a specific parametric form for powers of this conditional heteroskedasticity. More specifically, we say that $\varepsilon_t \sim$APARCH, if we can write $\varepsilon_t = \sigma_t z_t$, where z_t is a standard Gaussian and:

- $\sigma^\delta_t = \omega + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta\sigma^\delta_{t-1}$
- $= \quad + \sum^q_{i=1} \alpha_i (| \quad | - \quad) + \sum^p_{i=1} \quad 2 -$

Where $= h$, the parameter (assumed positive and ranging between 1 and 2.

3.13. Engel and Ng's (1993)

study reveals that "the news impact curve is a standard measure of how news is incorporated into volatility estimates. Several new candidates for modeling time-varying volatility are introduced and contrasted to better estimate and match news impact curves to the data. These models allow several types of asymmetry in the news impact on volatility." The AGARCH (1,1) (asymmetric GARCH) model developed by Engle and Ng (1993) is another approach to allowing the GARCH model to react asymmetrically.

It is defined by

$X_t = e_t \sigma_t$, $\sigma^2_t = \omega + \alpha(X_{t-1} + \gamma)^2 + \beta\sigma^2_{t-1}$

where γ is the non-centrality parameter

3.14. Engle & Kroner (1995)

presents theoretical results on the formulation and estimation of multivariate generalized ARCH models within simultaneous equations systems. A new parameterization of the multivariate ARCH process is proposed, and equivalence relations are discussed for the various ARCH parameterizations.

3.15. Lee and Engle (1999)

The Component Standard GARCH model is denoted by CSGARCH (1, 1)- This model decomposes the conditional variance into permanent and transitory components to investigate volatility's long- and short-run movements (Lee and Engle (1999). The model is deployed as follows:

$\sigma^2_t = q_t + \alpha_1(a^2_{t-1} - q_{t-1}) + \beta_1(\sigma^2_{t-1} - q_{t-1}),$

$q_t = \alpha_0 + pq_{t-1} + \varphi(a^2{}_{t-1} - \sigma^2{}_{t-1})$

for $0 < \alpha_0$, $0 \leq \alpha_1$, $0 \leq \beta_1$, $0 < \delta$, $0 \leq \varphi$. If $\alpha_1 + \beta_1 < 1$ and $p < 1$ weak stationarity holds. q_t represents the permanent component of the conditional variance. It can be seen as a time-varying intercept for the conditional heteroscedasticity

3.16. Pesaran et al. (2001)

proposed an Autoregressive Distributed Lag (ARDL) model stating that the demand for absolute stationary variables is inexistent. This is an ordinary least square (OLS) based model applicable for non-stationary time series and times series with mixed order of integration.

$Y_t = \beta_0 + \beta_1 y_{t-1} + \ldots\ldots + \beta p y_{t-m} + \alpha_0 x_t + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \ldots\ldots\ldots + \alpha_q x_{t-n} + \varepsilon_t$

Here, m and n are the number of years for lag, ε_t is the disturbance terms, β_i's are the short-run, and α_i's are coefficients for the long-run relationship. Hence, it can be used to research the cointegration among a series of variables of order I (0) or I (1) or mixed I (0) with I (1). The general ARDL (p, q) model equation is expressed as follows:

$Y_t = \beta_0 + \beta_1 y_{t-1} + \ldots\ldots + \beta p y_{t-p} + \gamma_1\, x_{t-1} + \gamma q\, x_{t-q} + \varepsilon_t$

The lags order p and q are determined by the AIC criterion and may differ depending on the independent variables or periods.

3.17. Bordignon et al. (2004)- Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) model-

Bordignon et al. (2004) state that the parameter S represents the cycle length, while d indicates the degree of memory.

- $h_t = \alpha_0 + \alpha(L)\varepsilon^2{}_t + \beta(L)\, h_t + [1 - (1 - L^S)^{\,d}]\varepsilon^2$

The first three terms in the conditional variance reproduce the general GARCH model, and the fourth term introduces an extended memory component that operates at zero and seasonal frequencies.

3.18. Corsi (2009)-Heterogeneous Autoregressive (HAR) MODEL

The basic specification in Corsi (2009) estimates by ordinary least squares under the assumption that at time t, the conditional mean of *\var epsilon_{t+1}* is equal to zero. Where the conditional volatility is made dependent on past volatilities aggregated at different frequencies (HMEM; D (Daily), 5 is for W (Weekly), 22 is for M (Monthly)):

- $x_t = \mu_t \varepsilon_t$, $\varepsilon_t \sim$ Gamma (a, 1/a) for each t
- $\mu_t = \omega + \alpha D x_{t-1} + \alpha W\ \bar{x}^{(5)}{}_{t-1} + \alpha M\ \bar{x}^{(22)}{}_{t-1}$,

with the possible introduction of regimes (MS (n)- HMEM)

- $x_t = \mu_{t,}\ s_t\ \varepsilon_t$,
- $\varepsilon t\ |s_t \sim$ Gamma (as_t, 1/ast) for each t
- $\mu_{t,}\ s_t = \omega + \sum^{n}{}_{i=1} k_i\ I_{st} + \alpha D$, st xt−1 + αW, st$\bar{\mathbf{x}}^{(5)}{}_{t-1} + \alpha M$, st $\bar{\mathbf{x}}^{(22)}{}_{t-1}$.

3.19. Mohammadi and Rezakhah. (2017)- Hyperbolic GARCH- (HYGARCH)

HYGARCH model is basically used to model long-range dependence
in volatility. This model provides a flexible
structure to capture different levels of volatilities and also short and
long memory effects. The equation is as given below:

$r_t = h^{1/2}{}_t\ z_t$

$h_t = u\ (h_{t-1}, \cdots, h_{t-p}, x_{t-1}, \cdots, x_{t-q})$

here r_t is the return, x_t a realized measure of volatility, $(z_t)_t$ are identically independently distributed (i.i.d) with mean zero and variance one, $(u_t)_t$ are also i.i.d with mean zero and variance σ^2u. Here $(z_t)_t$ and $(u_t)_t$ are mutually independent.

market using trend-following and mean-reverting techniques.

3.20. Alina and Dieyo (2019) & Corbet et al. (2018)-Conditional Mean Equation (CME)

The CME is studied by Alina and Dieyo (2019) and Corbet et al. (2018) are explained here: Conditional Mean Equation (CME): $r_t = \mu + \epsilon_t$

- r_t is the vector of the price returns,
- μ is a vector of parameters that estimates the mean of the return series, and
- $\boldsymbol{\epsilon}_{\mathbf{t}}$ is the vector of residuals with a conditional covariance matrix
- Ht given the available information set I_{t-1}.

The daily price returns:

$R_{i\,t} = \ln (P_{i,\,t})$- $\ln (P_{i,\,t-1})$

- ln ($P_{i,\,t}$), is the natural logarithm of the closing price of cryptocurrency i on day t and
- ln ($P_{i,\,t-1}$) is the natural logarithm of the closing price of cryptocurrency i on day t- 1

Another Equation:

- $_{,} = \quad + \quad , , \quad = 1,2, \quad _{,} \; |\Omega -1\sim (0, H)$
- $_{,} = \quad + \quad _{, -1} + \quad _{,} , \quad = 1,2, \quad _{,} \; |\Omega -1\sim (0, H)$

Where:

- $_{,}$ is the vector of the logarithmic price return of cryptocurrency,
- at time ,
- is a vector of parameters that estimates the mean of the price return of cryptocurrency
- , $_{,}$ is the vector of error terms for at time , with a positive definite conditional covariance matrix
- given the available information set $_{-1}$. The sub-index 1 refers to Bitcoin, while sub-index 2 refers to Ethereum.

4. Findings

The following interesting insights are found from measures of volatility of crypto currencies:

- There is an interrelation between the non-normality and heteroskedasticity of the returns on cryptocurrencies. Investment managers should select asymmetric GARCH-type models with a long memory to forecast the VaR of cryptocurrencies.
- The cross-correlation matrix of cryptocurrency price changes will reflect the 'non-trivial hierarchical structures' and 'groupings of cryptocurrency pairs.'
- ARDL models play a vital role in analyzing an economic scenario. In an economy, changing any economic variable may bring change in another economic variable beyond time. This change in a variable is not reflected immediately but distributed over future periods.
- The "Efficient Market Hypothesis" is not valid and that speculation is feasible via trading. Nevertheless, significant steps toward cryptocurrency efficiency have been traced in recent years. It can lead to less profitable trading strategies for speculators.

5. Conclusion

This paper analyzed that the price of cryptocurrency is always volatile and is influenced by various factors like market returns, prices of stocks, gold, and correlation of prices of cryptocurrency.

Modeling and forecasting the prices of cryptocurrencies and measuring the volatility with the GARCH specification has become standard among researchers. Several applications and extensions of GARCH model are also proposed by researchers. An integrated GARCH model states that the persistence parameter is equal to one. A combination of short and long memory conditional models for the mean and the volatility to analyze crypto returns is done with the help of ARFIMA and FIGARCH (Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic) Model. The research results show that robust procedures outperform non-robust ones when forecasting the volatility and estimating the Value-at-Risk. These results suggest that the presence of outliers plays an important role in the modelling and forecasting of volatility of crypto currency.

Some results show that it is possible to predict cryptocurrency markets using machine learning/ artificial intelligence and sentiment analysis. The transaction volume, the stock, the EUR/USD exchange rate, and the macroeconomic and financial development do not determine the crypto-currency price in the short and long term.

Footnotes

[1] Jiang, Z.; Mensi,W.; Yoon, S.-M. Risks in Major Cryptocurrency Markets: Modeling the Dual Long Memory Property and Structural Breaks. Sustainability **2023**, 15, 2193. https://doi.org/10.3390/su15032193

Annexure

Measuring Models and Its Application in Empirical Studies

S.NO.	AUTHOR	MODEL NAME	USED IN EMPIRICAL STUDY WITH FULL REFERENCE
1	Engle (1982)	Autoregressive Conditional Heteroskedasticity (ARCH) Model	Candila, Vincenzo. (2021). Multivariate Analysis of Cryptocurrencies. Econometrics 9: 28.
2	Bollerslev (1986)	The standard GARCH model) sGARCH (1,1)	Lykke Øverland Bergsli, Andrea Falk Lind, Peter Moln´ar, Michał Polasik (2022). Forecasting volatility of Bitcoin. Research in International Business and Finance 59 (2022) 101540
3	Engle & Bollerslev (1986)	Integrated Generalized Autoregressive Conditional heteroskedasticity (IGRACH 1,1)	Carlos Trucíos (2019). Forecasting Bitcoin risk measures: A robust approach. International Journal of Forecasting 35 (2019) 836–847
4.	Engle and Granger (1987)	Cointegration and Error Correction	Girish, G. and Vijayalakshmi, S. (2016) Is Stock Prices and Economic Activity in India Co-Integrated?. *Theoretical Economics Letters*, **6**, 269-275. doi: 10.4236/tel.2016.62030.
5		Autoregressive Fractionally Integrated Moving Average-Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (ARFIMA-FIGARCH)	Slaveya Zhelyazkova, (2018). "ARFIMA-FIGARCH, HYGARCH and FIAPARCH Models of Exchange Rates," *Izvestia Journal of the Union of Scientists,*- Varna. Economic Sciences Series, vol. 7(2), 142-153.
6	Bordignon et al. (2004)	Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) model	Maryam Tayefi and T. V. Ramanathan (2012). An Overview of FIGARCH and Related Time Series Models. AUSTRIAN JOURNAL OF STATISTICS Volume 41 (2012), Number 3, 175–196

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