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Review on measuring volatility of cryptocurrencies: 1980-2020

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Abstract

The intensity of volatility persistence is sensitive to time scales, market returns and data regimes. Investors who acquire intangible digital assets in the form of "cryptocurrencies" should consider that they may or may not receive a fiat currency. Sometimes there is a possibility of a loss of the entire investment due to volatility of prices in digital currency/cryptocurrency.

Several empirical studies are conducted to measure the volatility behavior of cryptocurrencies using different mathematical models like: i) Autoregressive Distributed Lag (ARDL) Model, ii) Heterogeneous Autoregressive (HAR) Model, iii) Autoregressive Conditional Heteroskedasticity (ARCH) Model, and iv) Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models.

This paper focuses on the review of various GARCH Models studied during 1980-2020.

Keywords: Crypto-asset, Crypto-exchange, Digital transactions, GARCH Models, Price volatility.

Objectives:

1. To identify a better model for measuring the volatility of crypto currencies.
2. To appreciate GARCH models of measuring the volatility of cryptocurrencies, studied during 1980-2020.

1. Introduction

The intensity of volatility persistence is sensitive to time scales, market returns and data regimes. Investors who acquire intangible digital assets in the form of "cryptocurrencies" should consider that they may or may not receive a fiat currency. Sometimes there is a possibility of a loss of the entire investment due to volatility of prices in digital currency/cryptocurrency. Other than bitcoins, there are about 1,000 alternative coins (altcoins) in the global market, with

Ethereum being the most popular. Altcoins are cryptocurrencies launched after bitcoin's success (Rajesh Kurup, 2017).

Several empirical studies are conducted to measure the volatility behavior of cryptocurrencies using different mathematical models like: i) Autoregressive Distributed Lag (ARDL) Model, ii) Heterogeneous Autoregressive (HAR) Model, iii) Autoregressive Conditional Heteroskedasticity (ARCH) Model, and iv) Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models. This paper focuses on the review of various GARCH Models studied during 1980-2020.

2. Review of literature

This analysis is based on a systematic review and focused on keywords viz., Cryptocurrency/ Bitcoin, Volatility of Cryptocurrency, and its measures. One may acquire 'Intangible Digital Assets' created through a 'Non-Fungible Token' (NFT). The lack of interchangeability, i.e., fungibility, distinguishes NFTs from blockchain-based cryptocurrencies, such as Bitcoin. It is a non-interchangeable unit stored on a digital ledger or blockchain. NFTs can be associated with easily-reproducible items such as photos, videos, audio, and other types of digital files as unique items. Copies of the original file are not restricted to the owner of the NFT.

The concept of an open-source currency without a central point of trust, such as a significant distribution agency or state lead control, is new (King & Nadal, 2012). Every investor needs to have an answer to the questions, viz., is it Bitcoin, that particular private currency that will have the most extended life? Moreover, how long will it run in parallel with the traditional currency? Will Bitcoin have the ability to benefit from a higher degree of confidence than the present one starting from the backdrop of the growing discontent generated by numerous imbalances occurring in the economies of different states?" (Angela Rogojanu and Liana Badea, 2014).

The results reveal that Bitcoin has shown a successful path since its inception, despite volatile market conditions (Caporale et al., 2018). It is observed that the technology can potentially improve central banks' operations and can serve as a platform to launch their cryptocurrencies (Raskin & Yermack, 2016). The nature and the ability of the five largest cryptocurrencies, viz., Bitcoin, Ethereum, Ripples, NEM, and Dash, are examined by Phillip et al. (2018).

It should be noted that the cryptocurrencies cannot replace the fiat currency, and they could change how inter-connected global markets interact, clearing away barriers surrounding normative currencies and foreign exchange rates (Peter D. DeVries, 2016).

Although consumers may have digital banking credentials to access the digital financial system, consumers in many emerging markets are not active users of digital channels due to a lack of consumer trust and confidence in the new channels (Maladay, 2016).

Yates (2017) highlighted that government agencies explore the potential for cryptocurrencies to compete with government-backed money; the total value of all cryptocurrencies in circulation is over \$100 billion, arguably posing a credible threat of supplanting central-bank-issued money.

Hacker and Thomale (2017) suggest two policy proposals to mitigate legal uncertainty concerning token sales. They are, first, tailoring disclosure requirements to the code-driven nature of token sales. An ICO-specific safe harbor would offer a clear and less burdensome path to EU law compliance for token sellers who suspect their tokens may qualify as securities. Second, overlapping and partially contradicting securities regulation regimes can undermine each other. It is noted that only a joint international regulatory authority can efficiently balance investor protection and investor access in the face of the novel generation of decentralized blockchain applications.

Blemus (2018) extensively compares the current regulatory trends in selected countries on the various applications enabled or issues raised by Blockchain technology.

Michael & Wei (2020) suggested a model for cryptocurrency as membership in a decentralized digital platform to facilitate transactions between users of certain goods or services. The problem induced by the cryptocurrency price has to clear membership demand with speculators' supply of the token.

3. Measures of Volatility of cryptocurrency-1980-2020

3.1. Engle (1982) - Autoregressive Conditional Heteroskedasticity (ARCH) Model

ARCH is a method that explicitly models the change in variance over time in a time series. An ARCH method models the conflict at a time step as a function of the residual errors from a mean process (e.g., a zero mean). The ARCH model has a simple regression model, as can be seen below:

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

$$u_t \sim N(0, \alpha_0 + \alpha_1 u_{t-1}^2)$$

This suggests the error term is normally distributed with zero mean and conditional variance depending on the squared error term lagged one time period. The conditional variance is the variance given the values of the error term lagged once, twice etc:

$$\sigma_t^2 = \text{var}(u_t | u_{t-1}, u_{t-2}, \dots) = E(u_t^2 | u_{t-1}, u_{t-2})$$

Where σ_t^2 is the conditional variance of the error term. The ARCH effect is then modelled by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$

This is an ARCH model as it contains only a single lag on the squared error term, however it is possible to extend this to any number of lags, if there are q lags it is termed an ARCH (q) model.

3.1. Bollerslev (1986)-The standard GARCH model)- is represented as sGARCH (1,1)

- $y_t = x_t\gamma + \varepsilon_t$

Here, dependent variable: y_t , exogenous variables: x_t , error term: ε_t .

- $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta^2\sigma_{t-1}^2$

This Equation estimates the variance (squared volatility σ_t^2) at time t , which depends on a historical mean (ω), news about volatility from the previous period, measured as a lag of the squared residuals from the mean Equation (ε_{t-1}^2), and volatility from the last period (σ_{t-1}^2).

3.2. Engle & Bollerslev (1986)-Integrated Generalized Autoregressive Conditional heteroskedasticity (IGRACH 1,1)

This is a restricted version of the GARCH model. The persistent parameters sum up to one and import a unit root in the GARCH process (Engle & Bollerslev, 1986).

- $x_t = \mu + a_t$
- $a_t = \sigma_t \varepsilon_t$
- $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

Here, it is imposed that: $\alpha_1 + \beta_1 = 1$.

3.4. Engle and Granger (1987)

The cointegration is characteristic of a series vector X_t , with the same order of integration d , whose linear combination results in a process with integration order d minus b .

$$\exists \beta \neq 0 \text{ and } Z_t := \beta_0 X_t \sim I(d-b); \text{ with } b > 0$$

Based on the case of series with a unit root, if each element of a vector of time series X_t , stationary only after the first differentiation, generates by linear combination βX_t a stationary process with finite variance, they are said to be cointegrated. In practice, two non-stationary series with a stochastic tendency and typical displacements over time are said to be cointegrated.

3.5. Autoregressive Fractionally Integrated Moving Average-Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (ARFIMA-FIGARCH)- Ravichandran et al. (1989)

This model is used to analyze the crypto-assets returns, a powerful combination of short and long memory conditional models for the mean and the volatility.

A stochastic process $x(t) \ t \in \mathbb{Z}$ is:

An ARFIMA (p, d, q)- GARCH (r, s), $p, q, r, s \in \mathbb{N} \cup \{0\}$ and $d \in \mathbb{R}$,

if it satisfies: $\Phi(B)\nabla dx(t) = \Theta(B)\varphi(t)$, with $\varphi(t) = \sigma(t)z(t)$

Here, the polynomial $\Phi(B)$ and $\Theta(B)$ are of orders p and q, respectively, and the fractional differentiation and the white noise process $\{z_t\}_{t \in \mathbb{Z}}$ have zero mean and finite variance.

3.6. Ravichandran et al., (1989): Threshold GARCH (TGARCH)

This indicates the existence of leverage effects of the first order:

$$\sigma_t^2 = \omega + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2 + \lambda x_{t-1}^2 1_{t-1}$$

Here, α and $\alpha + \lambda$ denote the effect of good news and bad news, respectively, and $\lambda > 0$ is evidence that bad news upsurge volatility in the Bitcoin market.

3.7. Ravichandran et al. (1989)- Markov Switching GARCH (MSGARCH)

$$Y_t | (s_t = k, I_{t-1}) \sim D(0, H_{k,t}, \xi_k)$$

where $D(0, H_{k,t}, \xi_k)$ is a continuous distribution with zero mean, time-varying variance $H_{k,t}$,

Furthermore, additional shape parameters are gathered in the vector ξ_k .

The integer-valued stochastic variable s_t , defined on the discrete space $\{1, \dots, K\}$

3.8. Diagonal Baba-EngleKraft-Kroner (BEKK) Model, 1990

$$H_t = C'C + A'(\bar{\Xi}_{t-1} - \Xi_{t-1})A + B'(H_{t-1})B$$

Where H_t is an $n \times n$ conditional variance-covariance matrix, C is an upper triangular matrix of parameters, $\bar{\Xi}_{t-1}$ is an $n \times 1$ disturbance vector, and A and B are ' $n \times n$ ' diagonal parameter matrices.

3.9. Nelson (1991)-Exponential GARCH (EGARCH) Model-

- $\log \sigma_t^2 = \omega + \alpha_1 Z_{t-1} + \gamma_1 [|Z_{t-1}| - E(|Z_{t-1}|)] + \beta_1 \log \sigma_{t-1}^2$
- $\alpha_1 > 0, \beta_1 > 0, \gamma_1 > 0$ and $\omega > 0$. α_1 captures the sign effect, and γ_1 captures the size effect.
- The persistence parameter for this model is β_1 .

3.10. Higgins and Bera's (1992)

theory applied to weekly exchange rates and found the existence of non-linear ARCH. Their study reveals that "since the NARCH model encompasses various functional forms, we argue it provides a useful framework for testing Engle's original

specification against a wide class of alternatives. It would be interesting to investigate what happens to tests of expectation theory or CAPM models if NARCH-type models capture the nonlinearity in the data."

3.11. Glosten, Jagannathan and Runkle (GJR)- GARCH Model-1993

$$h_t^2 = \alpha_0 + \sum_{i=1}^p (\alpha_i Z_{t-i}^2 (1 - 1(Z_{t-i} > 0)) + \gamma_i Z_{t-i}^2 1(Z_{t-i} > 0)) + \sum_{j=1}^q \beta_j h_{t-j}^2$$

With parameters $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\gamma_i \geq 0$ that guarantee a non-negative conditional variance. In order to highlight the asymmetry properties, a function $f(Z_t)$ is introduced where the magnitude effects (γ_i) and the asymmetry effects (α_i).

3.12. Ding, Granger and Engle (1993):Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH)

Ding, Granger, and Engle (1993) find that ε_t^d often displays strong and persistent autocorrelation for various values of d , or rather returns have a long memory property. The asymmetric Power ARCH (APARCH) model assumes a specific parametric form for powers of this conditional heteroskedasticity. More specifically, we say that $\varepsilon_t \sim \text{APARCH}$, if we can write $\varepsilon_t = \sigma_t z_t$, where z_t is a standard Gaussian and:

$$\begin{aligned} \sigma_t^\delta &= \omega + \alpha(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta \sigma_{t-1}^\delta \\ &= \omega + \sum_{i=1}^q \alpha_i (|\varepsilon_{t-i}| - \gamma \varepsilon_{t-i})^\delta + \sum_{i=1}^p \beta_i \sigma_{t-i}^\delta \end{aligned}$$

Where $\delta = h$, the parameter δ (assumed positive and ranging between 1 and 2).

3.13. Engel and Ng's (1993)

study reveals that "the news impact curve is a standard measure of how news is incorporated into volatility estimates. Several new candidates for modeling time-varying volatility are introduced and contrasted to better estimate and match news impact curves to the data. These models allow several types of asymmetry in the news impact on volatility." The AGARCH (1,1) (asymmetric GARCH) model developed by Engle and Ng (1993) is another approach to allowing the GARCH model to react asymmetrically.

It is defined by

$$X_t = e_t \sigma_t, \sigma_t^2 = \omega + \alpha(X_{t-1} + \gamma)^2 + \beta \sigma_{t-1}^2$$

where γ is the non-centrality parameter

3.14. Engle & Kroner (1995)

presents theoretical results on the formulation and estimation of multivariate generalized ARCH models within simultaneous equations systems. A new parameterization of the multivariate ARCH process is proposed, and equivalence

relations are discussed for the various ARCH parameterizations.

3.15. Lee and Engle (1999)

The Component Standard GARCH model is denoted by CSGARCH (1, 1)- This model decomposes the conditional variance into permanent and transitory components to investigate volatility's long- and short-run movements (Lee and Engle (1999). The model is deployed as follows:

$$\sigma_t^2 = q_t + \alpha_1(a_{t-1}^2 - q_{t-1}) + \beta_1(\sigma_{t-1}^2 - q_{t-1}),$$

$$q_t = \alpha_0 + \rho q_{t-1} + \phi(a_{t-1}^2 - \sigma_{t-1}^2)$$

for $0 < \alpha_0$, $0 \leq \alpha_1$, $0 \leq \beta_1$, $0 < \delta$, $0 \leq \phi$. If $\alpha_1 + \beta_1 < 1$ and $\rho < 1$ weak stationarity holds. q represents the permanent component of the conditional variance. It can be seen as a time-varying intercept for the conditional heteroscedasticity

3.16. Pesaran et al. (2001)

proposed an Autoregressive Distributed Lag (ARDL) model stating that the demand for absolute stationary variables is inexistent. This is an ordinary least square (OLS) based model applicable for non-stationary time series and times series with mixed order of integration.

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \dots + \beta_p Y_{t-p} + \alpha_0 X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha_n X_{t-n} + \varepsilon_t$$

Here, m and n are the number of years for lag, ε_t is the disturbance terms, β_t 's are the short-run, and α_t 's are coefficients for the long-run relationship. Hence, it can be used to research the cointegration among a series of variables of order $I(0)$ or $I(1)$ or mixed $I(0)$ with $I(1)$. The general ARDL (p, q) model equation is expressed as follows:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \dots + \beta_p Y_{t-p} + \gamma_1 X_{t-1} + \gamma_q X_{t-q} + \varepsilon_t$$

The lags order p and q are determined by the AIC criterion and may differ depending on the independent variables or periods.

3.17. Bordinon et al. (2004)- Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) model-

Bordinon et al. (2004) state that the parameter S represents the cycle length, while d indicates the degree of memory.

$$h_t = \alpha_0 + \alpha(L)\varepsilon_t^2 + \beta(L)h_t + [1 - (1 - L^S)^d]\varepsilon_t^2$$

The first three terms in the conditional variance reproduce the general GARCH model, and the fourth term introduces an extended memory component that operates at zero and seasonal frequencies.

3.18. Corsi (2009)-Heterogeneous Autoregressive (HAR) MODEL

The basic specification in Corsi (2009) estimates by ordinary least squares under the assumption that at time t , the conditional mean of $\text{var } \epsilon_{t+1}$ is equal to zero. Where the conditional volatility is made dependent on past volatilities aggregated at different frequencies (HMEM; D (Daily), 5 is for W (Weekly), 22 is for M (Monthly)):

- $x_t = \mu_t \epsilon_t$, $\epsilon_t \sim \text{Gamma}(a, 1/a)$ for each t
- $\mu_t = \omega + \alpha D x_{t-1} + \alpha W \bar{x}_{t-1}^{(5)} + \alpha M \bar{x}_{t-1}^{(22)}$,

with the possible introduction of regimes (MS (n)- HMEM)

- $x_t = \mu_t s_t \epsilon_t$,
- $\epsilon_t | s_t \sim \text{Gamma}(a s_t, 1/a s_t)$ for each t
- $\mu_t, s_t = \omega + \sum_{i=1}^n k_i |s_{t-i}| + \alpha D, s_t x_{t-1} + \alpha W, s_t \bar{x}_{t-1}^{(5)} + \alpha M, s_t \bar{x}_{t-1}^{(22)}$.

3.19. Mohammadi and Rezakhah. (2017)- Hyperbolic GARCH- (HYGARCH)

HYGARCH model is basically used to model long-range dependence in volatility. This model provides a flexible structure to capture different levels of volatilities and also short and long memory effects. The equation is as given below:

$$r_t = h_t^{1/2} z_t$$

$$h_t = u(h_{t-1}, \dots, h_{t-p}, x_{t-1}, \dots, x_{t-q})$$

here r_t is the return, x_t a realized measure of volatility, $(z_t)_t$ are identically independently distributed (i.i.d) with mean zero and variance one, $(u_t)_t$ are also i.i.d with mean zero and variance $\sigma^2 u$. Here $(z_t)_t$ and $(u_t)_t$ are mutually independent.

market using trend-following and mean-reverting techniques.

3.20. Alina and Dieyo (2019) & Corbet et al. (2018)-Conditional Mean Equation (CME)

The CME is studied by Alina and Dieyo (2019) and Corbet et al. (2018) are explained here: Conditional Mean Equation (CME): $r_t = \mu + \epsilon_t$

- r_t is the vector of the price returns,
- μ is a vector of parameters that estimates the mean of the return series, and
- ϵ_t is the vector of residuals with a conditional covariance matrix
- H_t given the available information set I_{t-1} .

The daily price returns:

$$R_{it} = \ln(P_{i,t}) - \ln(P_{i,t-1})$$

- ### Another Equation:

- Where:

- #### 4. Pros and Cons of GARCH Models

- There is an interrelation between the non-normality and heteroskedasticity of the returns on cryptocurrencies. Investment managers should select asymmetric GARCH-type models with a long memory to forecast the VaR of cryptocurrencies.
- The cross-correlation matrix of cryptocurrency price changes will reflect the 'non-trivial hierarchical structures' and 'groupings of cryptocurrency pairs.'
- The transaction volume, the stock, the EUR/USD exchange rate, and the macroeconomic and financial development do not determine the crypto-currency price in the short and long term.
- ARDL models play a vital role in analyzing an economic scenario. In an economy, changing any economic variable may bring change in another economic variable beyond time. This change in a variable is not reflected immediately but distributed over future periods.
- Some results show that it is possible to predict cryptocurrency markets using machine learning/ artificial intelligence and sentiment analysis.
- The "Efficient Market Hypothesis" is not valid and that speculation is feasible via trading. Nevertheless, significant steps toward cryptocurrency efficiency have been traced in recent years. It can lead to less profitable trading strategies for

speculators.

5. Conclusion

This paper analyzed about the intensity of volatility persistence is sensitive to time scales, market returns and data regimes of the entire investment due to volatility of prices in digital currency/cryptocurrency.

In this paper several empirical studies conducted by various eminent research scholars are analyzed and presented an overview of different mathematical models like: i) Autoregressive Distributed Lag (ARDL) Model, ii) Heterogeneous Autoregressive (HAR) Model, iii) Autoregressive Conditional Heteroskedasticity (ARCH) Model, and iv) Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models. Investors who acquire intangible digital assets in the form of "cryptocurrencies" should aware that they may or may not receive a fiat currency. Sometimes there is a possibility of a loss of the entire investment due to volatility of prices in digital currency/cryptocurrency.

References

1. Angela Rogojanu and Liana Badea ((2014). The Issue of Competing Currencies-Case study-Bitcoin.*Theoretical Applied Economics*, XXI. (1), 103-114.
2. Adrian R. Pagan, G.William Schwert (1990). Alternative models for conditional stock volatility, *Journal of Econometrics*, 45(1–2), 267-290. ISSN 0304-4076, [https://doi.org/10.1016/0304-4076\(90\)90101-X](https://doi.org/10.1016/0304-4076(90)90101-X).
3. Baba, Y., Engle, R. F., Kraft, D. F., & Kroner, K. F. (1990). Multivariate simultaneous generalized ARCH.*Manuscript, University of California, San Diego, Department of Economics*.
4. Bollerslev T (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31,307–327.
5. Bordignon, S., Caporin, M., and Lisi, F. (2004). A seasonal fractionally integrated GARCH model. Working paper. University of Padova, Italy.
6. Blemus, S. (2018). Law and Blockchain: A Legal Perspective on Current Regulatory Trends Worldwide.*Revue Trimestrielle de Droit Financier (Corporate Finance and Capital Markets Law Review)* RTDF N°4-2017 - December 2017, Available at SSRN: <https://ssrn.com/abstract=3080639> or <http://dx.doi.org/10.2139/ssrn.3080639>
7. Caporale, G.M., Gil-Alana, L., Plastun, A. (2018). Persistence in the cryptocurrency market.*Research in International Business and Finance*, 46, 141–148.
8. Chaim, P., Laurini, M.P. (2018). Volatility and return jump in bitcoin. *Economics Letters* 173, 158–163.
9. Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7, 174–196.
10. Ding, Z., Granger C.W.J. and Engle, R.F., (1993). A long memory property of stock market returns and a new model, *Journal of Empirical Finance*, 1 (1).
11. Engle, R.F. and Bollerslev, T. (1986). Modelling the Persistence of Conditional Variance. *Econometric Reviews*, 5, 1-

50. Source: <http://dx.doi.org/10.1080/07474938608800095>.
12. Engle, R.F. and Ng, VK (1993) Measuring and Testing the Impact of News on Volatility. *The Journal of Finance*, 48, 1749-1778. <https://doi.org/10.1111/j.1540-6261.1993.tb05127>.
13. Engle, R., & Kroner, K. (1995). Multivariate Simultaneous Generalized ARCH. *Econometric Theory*, 11(1), 122-150. doi:10.1017/S0266466600009063
14. Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3), 339–350.
15. James D Hamilton, Raul Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64(1–2), 307-333, ISSN 0304-4076, [https://doi.org/10.1016/0304-4076\(94\)90067-1](https://doi.org/10.1016/0304-4076(94)90067-1).
16. Hacker, P., & Thomale Chris. (2018). Crypto-Securities Regulation: ICOs, Token Sales and Cryptocurrencies under EU Financial Law. *European Company and Financial Law Review* 645-696, Available at SSRN: <https://ssrn.com/abstract=3075820> or <http://dx.doi.org/10.2139/ssrn.3075820>.
17. Higgins, M.L. and Bera, A.K. (1992) A Class of Nonlinear Arch Models. *International Economic Review*, 33, 137–158. <https://doi.org/10.2307/2526988>.
18. Kotz S., Kozubowski T., Podgorski K. (2012). *The Laplace Distribution and Generalizations: A Revisit with Applications to Communications, Economics, Engineering, and Finance*. SpringerLink, Birkhäuser Boston, Bücher.
19. King, S., and S. Nadal. (2012). "PPCoin: Peer-to-Peer Crypto-Currency with Proof-of-Stake." Source: <https://decred.org/research/king2012.pdf>.
20. Nara Kim and Jang Mook Kang. (2018). A Case study of Public Blockchain and CryptoCurrency Technology Focus on Authentication System. *Journal of Engineering and Applied Sciences*, 13(3), 689-690. ISSN: 1816-949x.
21. Malady L. (2016). Consumer Protection Issues for Digital Financial Services in Emerging Markets. *Banking & Finance Law Review*, 31 (2), 389 – 401.
22. Michael Sorkin & Wei Xiong. (2020). "A Model of Cryptocurrencies," NBER Working Papers 26816, National Bureau of Economic Research, Inc.
23. Mohammadi, F., and S. Rezakhah. (2017). Smooth transition HYGARCH model: Stability and testing. <http://arxiv.org/pdf/1701.05358.pdf>.
24. Peter D. DeVries. (2016). An Analysis of Cryptocurrency, Bitcoin, and the Future. *International Journal of Business Management and Commerce*, 1(2).
25. Pesaran, M. H., Shin, Y. & Smith, R. J. (2001). 'Bounds testing approaches to the analysis of level relationships, *Journal of Applied Econometrics*, 16(3), 289– 326.
26. Petro, Hrytsiuk & Babych, Tetyana. (2022). The cryptocurrencies risk measure based on the Laplace distribution.
27. Phillip, A., Chan, J. and Peiris, S. (2018). A new look at Cryptocurrencies. *Economic Letters*. 163, 6-9.
28. Raskin, M., & D. Yermack. (2016). "Digital Currencies, Decentralized Ledgers, and the Future of Central Banking." National Bureau of Economic Research. NBER Working Paper No. 22238.
29. Ravichandran K, Bose S, Akgiray V, et al. (1989) Threshold generalized autoregressive conditional heteroskedasticity models. *Research Journal of Business Management*, 6, 55–80.
30. Robert F. Engle & Tim Bollerslev. (1986) Modelling the persistence of conditional variances, *Econometric Reviews*,

5(1), 1–50, DOI: 10.1080/07474938608800095.

31. Yates, T. (2017). The consequences of allowing a cryptocurrency takeover or trying to head one off. Source; <https://ftalphaville.ft.com/2017/06/07/2189849/guestpost-the-consequences-of-allowing-a-cryptocurrency-takeover-or-trying-to-head-oneoff/>