Research Article

Fundamental Issues and Measurement Problem in Quantum Mechanics

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Foundations of quantum mechanics are reconsidered in relation to the measurement problem. We define two kinds of quantities. The type A quantities are observables obtained experimentally from a single measurement. A dynamical variable is a type A quantity whose measurement selects a set of eigenvalues (EVs) as c-numbers. The type B quantities represent theoretical quantities related to probabilities. State vectors, wave function, and the probability distribution (PD) are type B quantities. They are derived from an eigenvalue equation such as Schrödinger equation. A quantum jump (QJ) is the measurement of a dynamical variable to select a set of EVs. The wave function is a theoretical quantity which persists throughout an experiment and therefore it does not collapse. After a QJ, a selected set of EVs persists until it becomes macroscopic. However, an eigenstate is a theoretical notion irrelevant to a single measurement and it does not exist in reality after a QJ. One experiment is complete when an ensemble of EVs has been obtained and compared with the theoretically predicted PD. To obtain an ensemble, repeated measurements are required in general, but there are some exceptions like a Bose-Einstein condensate system for which a single measurement yields a real ensemble. We discuss the relation between the reality of the wave function and the derivation of non-relativistic quantum mechanics from quantum field theory.

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I. Introduction

The measurement problem (MP) has persisted for a century, since the birth of quantum mechanics (QM). Recently we gave an interpretation of the MP in which physical processes are described by an ordinary language instead of mathematics [1][2]. While the microscopic world governed by QM and the

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macroscopic world governed by classical physics are described by two distinct mathematical theories, there is no single mathematical theory to describe the MP, or the gap between the two worlds.

When we study QM, many new concepts are introduced along with associated mathematical quantities. They are well organized, and at a glance, there does not appear a room to raise any question. However exactly because QM is mathematically self-consistent, we need to sort out the meanings of these mathematical quantities to solve the MP which requires interpretation in words. Following questions arise. What are observables and what are theoretical quantities? What are determined by an experiment and what are theoretical predictions derived from basic equations? How are they compared with one another? In a sense, our goal of this paper is to express the concepts introduced in the first several chapters of the textbook by Dirac [3] in a more refined language.

The MP is so involved that we started from scrutinizing basic postulates adopted by unsuccessful existing theories. In those theories an observing apparatus was implicitly assumed to be classical. We call this the postulate of classicality of apparatus (PCA). We have taken a practical approach of interpreting operating principles of modern detectors and found that the apparatus is at least partly quantum mechanical. We call the interaction between the observed system and the modern detector, a microscopic quantum jump (MIJ). By adopting the postulate of MIJ and by discarding PCA, we have constructed a measurement theory consistent with standard single-particle QM in Paper I [1].

The next question we tackled was whether wave function collapses or not. We distinguished between quantum jumps (QJs) and wave function collapse (WFC). We defined a QJ as a process of selecting a set of eigenvalues (EVs) and a WFC as a process of determining the probability distribution (PD), both from a single measurement. The PD is determined from an ensemble of EVs. A single measurement in single-particle QM yields only one set of EVs and the PD is never obtained. Therefore the WFC does not happen. We extended our analysis to many-particle QM. We took a short cut of discussing the occupation number of a one-particle state. We found that the PD can be obtained in a single measurement for a many-particle system with a huge number of bosons, and in this case the WFC can occur. These are the findings of Paper II [4].

Paper I covered only single-particle QM and Paper II introduced new concepts of QJs and WFC rather abruptly. The treatment of many-particle QM was limited to its framework and detailed discussion was left out. To make up for these shortcomings, we are motivated to write this third paper. In the course of this new analysis, we inevitably encountered aforementioned fundamental issues in QM.

The paper is organized as follows. In Section II, fundamental issues are introduced and discussed. In Section III, three concrete examples of experimental setups for single-particle QM are revisited. In Section IV, the gap between single-particle QM and many-particle QM is mentioned and we proceed to the occupation number treatment of systems with similar particles. Then we discuss concrete examples of many-boson systems in some detail. In Section V, we mention the relation between the reality of the wave function and the derivation of non-relativistic QM from quantum field theory.

II. Fundamental issues

Our goal in this section is to clarify the different roles between dynamical variables and state vectors rather conceptually. In Papers I and II, we interpreted operating principles of modern detectors in three concrete experimental setups. In Section III, this interpretation is revisited and revised from the new perspective developed in this section.

A. Dynamical variables as type A quantities

We first consider a typical eigenvalue equation in QM.

$$\alpha|Q\rangle = a|Q\rangle, \tag{1}$$

where α is a dynamical variable, a is an eigenvalue (EV), and $|Q\rangle$ is an eigenket. We now ask what are observed quantities upon measurement and what are not. This measurement is also called a microscopic quantum jump (MIJ) described in detail in Section III. Here we comment on a technical term. In Papers I and II, we used a term, "system eigenvalue (SEV)" to emphasize an experimental setup consisting of an observed system and an observing apparatus. However, the SEV is a non-standard technical term used only by us and here we use a standard term, "eigenvalue (EV)" throughout this paper. The answer to the above question is as follows.

The dynamical variable α is an "observable" and the measurement yields a as an EV of α. The concept of an observable originated from W. Heisenberg [5] and was made popular by P.A.M. Dirac [3]. Typical examples of α are q and p, which are also classical dynamical variables. If we adopt the Heisenberg picture (dynamical variables are time-dependent), it is apparent that an observable usually has a classical counterpart [3].

However, this is true only for single-particle QM. For a system with multiple similar particles, "permutations" are considered as dynamical variables. There is no permutation operator in classical

physics, because particles are distinguishable. It is also noted that a permutation is not a Hermitian operator and does not have a real EV.

Real dynamical variables which are Hermitian operators mathematically, are observables. This is a standard definition. These observables are reality.

Eigenvalue a is selected upon measurement of the observable α. In Section III, we will show that a is selected by a MIJ and the MIJ emits a microscopic particle (MIP) carrying the information of a. After amplification or accumulation, the carrier of the information of a becomes macroscopic and a is obtained as a c-number.

After the MIJ, the EV a, persists throughout the amplification/accumulation process until it becomes a c-number.

Eigenket $|Q\rangle$ is not observed by the measurement. The measurement itself does not have any effect on the eigenket. $|\langle Q|Q\rangle|^2$ theoretically predicts the probability for a to be selected upon measurement.

It is wrong to imagine that the eigenstate persists after a quantum jump.

Type A quantity

We define a type A quantity as follows. The type A quantity is an observable that can be obtained from a single measurement. A real dynamical variable is a type A quantity whose measurement selects a set of EVs. Real dynamical variables are the only type A quantities.

B. Probability distribution and wave function as type B quantities

Let us consider another eigenvalue equation, which is more like an equation of motion.

$$H\psi = E\psi, \tag{2}$$

where H is a Hamiltonian, E is an energy eigenvalue, and ψ is a wave function. Since the nature of H (real dynamical variable) and E (EV) is known, the question is what the nature of the wave function ψ is.

Now we define a type B quantity as follows.

Type B quantity

The type B quantity is not an observable. It is theoretically predicted. The probability distribution (PD), P, and associated wave function, ψ, are type B quantities. They are derived from the eigenvalue equation. To calculate a theoretical PD, we also need an initial condition and/or boundary conditions and we will come

back to this point in Section II.D. The solution of Equation (2) gives ψ and P$= \psi^* \psi$ gives the PD, which should be compared with the ensemble of EVs obtained in an experiment.

One observation is complete when the PD is compared with the observed ensemble of EVs.

To complete an observation, an ensemble of EVs is needed. In case of single-particle QM, one measurement yields only one EV (or one set of EVs) and repeated measurements are required to obtain a virtual ensemble.

In case of many-particle QM, a real ensemble of bosons may be found in a quantum state. We will discuss such systems in Section IV.

C. Wave function never collapses

We consider that eigenkets are used to calculate the PD theoretically, and they are not affected by measurements. Therefore we do not think that a wave function collapses upon measurement. Equivalently we do not consider that von Neumann projection postulate is valid. In Paper II, we thought that the wave function collapses along with a macroscopic quantum jump. Now we think that our interpretation in Paper II was wrong. Our conclusion is that the wave function never collapses.

We consider that Dirac introduced the principle of superposition of states and the bra-ket notation to calculate "theoretical PD" and associated expectation values. A state vector has nothing to do with a single measurement. Only real dynamical variables are observed and their EVs are selected. They alone are relevant to the measurement outcomes and they are real.

Quantum mechanical Kepler problem

There is a famous work by W. Pauli who obtained the energy levels of a hydrogen atom without using Schrödinger equation. He adopted an algebraic approach utilizing a conserved quantity called Runge-Lenz-Pauli vector to solve the quantum mechanical Kepler problem [6]. He only used dynamical variables with commutation relations, but did not use a wave function. Pauli's approach, later known as algebraic spectroscopy, obtains energy levels or energy EVs without using state vectors. This implies that state vectors are necessary only to calculate the PD theoretically. Again we can see the difference in roles between a dynamical variable and a state vector. The former is an observable to be obtained by a measurement, while the latter is used to calculate the theoretical PD.

Eigenvalue equation

The type A quantity, a real dynamical variable, exists from microscopic to macroscopic at the level of its EV. In a sense, the real dynamical variable covers from quantum to classical. On the other hand, the type B quantity, a wave function describes theoretical probability in the quantum world. The type A and type B quantities confront at the eigenvalue equation like Schrödinger equation.

However, there is a caveat that the wave function may not fully belong to the quantum world. According to the Copenhagen interpretation, the probabilistic nature of QM originates from the interface between the microscopic and macroscopic worlds. If that is the case, the wave function exists at the boundary between the two worlds. This interface is exactly where the MP is located. In that sense, the MP is a central issue and not a peripheral issue of QM.

D. Boundary conditions

To solve a real physical problem we need an initial condition and/or boundary conditions. To make it clear, let us consider a steady state scattering problem in nuclear physics in which only the boundary conditions matter.

Let us consider the scattering of a proton by a heavy nucleus. We do not consider Rutherford scattering and the interaction is due to nuclear force of a short range. The boundary conditions are given at $r = 0$ and $r = \infty$. $r = 0$ corresponds to the origin of polar coordinates and the location of the scatterer (heavy nucleus). The boundary condition at $r = 0$ constrains the functional form of the wave function, but it does not introduce extra complexity. The size of the interaction region is order of 10^{-13} cm. Since typical particle detectors are placed far away from the interaction region ($r \gg 10$ cm), the theoretical boundary condition at $r = \infty$ is a good approximation of reality. Therefore the physical problem is well defined.

An interesting case is an inverse problem. If the microscopic physics is not well understood, the model Hamiltonian is only tentatively assigned. The difference between the theoretical PD and the observed PD obtained from an ensemble of EVs will tell us about the true Hamiltonian including the interaction energy. An actual procedure will be to adjust the model Hamiltonian by fitting so that the two PDs coincide.

After all, QM is a theory and its only connection to the real world is the relation between the theoretical PD and the observed PD derived from an ensemble of EVs. The ambiguity of the previously accepted

interpretation was in that theoretical notions were treated as reality by misunderstanding.

In the rest of this paper, when we mention a PD, it means a theoretical PD.

III. Single-particle quantum mechanics

We examine three concrete experimental setups, which have been analyzed in Papers I and II, but here we revisit them from a different perspective. We focus on “the persistence of eigenvalues” and “the absence of an eigenstate after a quantum jump”. We also comment on the fact that a wave function never collapses.

A. Two-dimensional photon-counting detection

Let us consider a double-slit experiment using a two-dimensional photon-counting detector. The details of this experiment are described in Paper I. At low light levels, individual photons arrive at the detector surface sequentially.

Microscopic quantum jump (MIJ)

The first step of the measurement is a MIJ. In a MIJ, an incident photon interacts with a single atom on the detector surface (photocathode), gets absorbed and a photoelectron is emitted as a microscopic particle (MIP) by the external photoelectric effect. The MIJ selects a two-dimensional position of the photon’s arrival, (x, y), as a set of EVs. The time of arrival t is also recorded. The information of (x, y) and t is carried by the MIP. The selection of (x, y) follows the PD or the interference pattern, while t is irrelevant to the PD. Since each photon event obeys the same PD, the PD or equivalently the wave function never collapses.

Amplification of secondary electrons

A micro-channel plate (MCP) is placed behind the photocathode. The MCP is a two-dimensional array of capillary-shaped electron multipliers. A photoelectron as a MIP enters one of the multiplier tubes, triggers secondary electron multiplication in the tube, which produces a cloud of electrons which eventually becomes macroscopic. The cloud of secondary electrons carries (x, y) and t information and the EVs persist throughout the measurement. Finally the EVs become classical numbers and one measurement is complete. In the MCP, electrons are set in an electrically metastable state and their cascade is triggered by the MIP.

Ensemble of EVs and comparison with the PD

A single measurement yields only one set of EVs. Measurements must be repeated until an ensemble of EVs is obtained for the entire two-dimensional surface as N(x, y), where N means the number of photon events. In other words, data must be integrated in time t. This ensemble should be compared with the PD, P(x, y) or the interference pattern.

Persistence of EVs and absence of an eigenstate

The EVs persist throughout the measurement. First the EVs are carried by the MIP, then they are carried by the cloud of secondary electrons, and they finally become c-numbers.

How about the persistence of an eigenstate after a quantum jump? In Section II, we found that state vectors are used only to calculate the probability. We do not see any reason why eigenstates really exist as experimental entities.

B. Two-dimensional integrated photon detection by a CCD

We consider a double-slit experiment using an integration-type detector such as a CCD, which is described in detail in Paper I.

Microscopic quantum jump (MIJ)

An incident photon gets absorbed at a pixel (x, y) and a photoelectron as a MIP is excited by the internal photoelectric effect. The MIP is stored at a positive electrode within the pixel. This MIP does not trigger amplification. The EVs are the pixel location (x, y) itself.

Accumulation of MIPs

A pixel in the CCD can store a large number of excited photoelectrons as MIPs. The MIPs are accumulated until their number becomes macroscopic. The MIPs are accumulated at all pixels as N(x, y) to directly obtain an integrated real ensemble.

Persistence of EVs and absence of an eigenstate

Since each photoelectron stays at the same pixel, (x, y) as the EVs, the EVs persist. There is no reason to imagine the persistence of an eigenstate after a quantum jump.

Photon counting vs integration-type detection

Is there any difference between photon-counting detection and photon detection by a CCD in terms of a MIJ? We considered this question in Paper II and our conclusion is that there is no essential difference up to the stage of emission of a MIP.

C. α decay in a Wilson cloud chamber

We now consider α decay of U^{238} placed in a Wilson cloud chamber (WCC). An U^{238} nucleus decays into a Th^{234} nucleus which stays in the source, and an α particle which is emitted from the source. Theoretically the α decay is described by an S wave and it was once considered that the linear tracks seen in the WCC were contradiction to the S-wave wave function, because ionization would appear randomly in space for the S wave. This was an issue raised by A. Einstein in 1927 at the Fifth Solvay Conference. It was once considered to be a major challenge to QM.

In Paper II, we discussed this problem and showed that this is not a contradiction from the point of view that a wave function does not collapse. Here we outline how to solve this problem by a thought experiment and some issues are discussed afterwards. The details of the solution are given in Paper II.

1. Spherical vacuum chamber. We imagine a spherical vacuum chamber whose inner surface is sensitive to the point of arrival of an α particle. α-particle source is placed at the center of the spherical chamber. We will not see any ionization track. Instead we will see individual points of arrival on the inner surface of the chamber. As statistics improve, the distribution of the points will approach a uniform distribution.
2. Spherical Wilson cloud chamber (WCC). The spherical WCC is filled with air and supersaturated water vapor. The kinetic energy of an α particle emitted from the source is about 5 MeV, while the ionization potential of an air molecule is about 15 eV. The α particle collides with an air molecule, ionizes it, and generates a molecular ion of the air and an electron. Since the electron is light and driven away, the molecular ion keeps the three-dimensional position at the time of collision as EVs. The collisional ionization is a MIJ and the molecular ion is a MIP carrying the information of the EVs. Since the water vapor is supersaturated, the condensation of water molecules occurs around the molecular ion, which is the trigger of amplification. The result of the condensation is the formation of a macroscopic water droplet carrying the EVs as c-numbers. At this point, one

measurement of the position of ionization is complete. Since the kinetic energy of the α particle is much greater than the ionization potential of the air molecule, successive ionization occurs.

3. Linear tracks. For standard temperature and pressure, the mean free path of the α particle before one ionization occurs is $x = 7 \times 10^{-5}$ cm. The ionization cross section is $\sigma = 5 \times 10^{-16}$ cm^2[7]. We see a long linear track because of the following two numbers. First the ratio of $\sqrt{\sigma}/x$ is $10^{-4} \ll 1$ and therefore the track is linear. Second the ratio of the ionization potential of an air molecule (15 eV) and the kinetic energy of the α particle (5 MeV) is $3 \times 10^{-6} \ll 1$ and therefore the track is very long.
4. Isotropic ejection of α particles and radial tracks. Since the S-wave wave function does not collapse for each ejection of an α particle the direction of ejection is isotropic. Since the tracks are long and linear, they are seen radially. This isotropic ejection is the manifest of the S wave.
5. Thought experiment vs reality. Our thought experiment of the spherical WCC made isotropic ejection clear. In reality, a WCC is not spherical and this fact probably hampered a correct interpretation of this phenomenon.

Below we break down the entire measurement into individual stages.

Microscopic quantum jump (MIJ)

An incident α particle collisionally ionizes an air molecule, generating an molecular ion and an electron. The resultant ion of the air molecule is the MIP carrying the position information at the time of the collision. This three-dimensional position is the EVs.

Amplification as the condensation of water molecules

The ion of the air molecule is the trigger and water molecules condense around it. There is the amplification process which leads to the formation of a macroscopic droplet whose three-dimensional position is the EVs as c-numbers. Since water vapor is supersaturated, water molecules are set in a thermodynamically metastable state and their condensation is triggered by the MIP.

Persistence of EVs and absence of an eigenstate

The microscopic position of the ion of the air molecule is the initial EVs. After the condensation of water molecules and formation of a droplet, the EVs become classical numbers, but the EVs persist throughout the measurement process. There is no need to consider the presence of an eigenstate.

Mott problem

This problem of α decay in a WCC is also known as the Mott problem. N. F. Mott solved it in 1929[8], but his approach was totally different from ours. His work was considered to be a precursor to the decoherence theory developed later. In Paper I, however, we rejected the decoherence theory together with other unsuccessful theories in the past, because these theories adopted the PCA implicitly.

IV. Many-particle quantum mechanics

A. Difference between single-particle and many-particle systems

In Paper II, we abruptly started the discussion of many-particle QM with the occupation number of a quantum state. Although, we consider that the occupation number is a proper starting point, now we feel that we took a shortcut in Paper II without giving sufficient background. Here we discuss the difference between single-particle and many-particle QM and attempt a smooth transition between the two.

In QM, there is a gap between a one-body system and a two-body system composed of two similar particles for three reasons.

1. In the two-body problem, the symmetry of state must be considered. It is symmetric for bosons and antisymmetric for fermions.
2. In the two-body problem, there can be a two-body interaction, which makes Schrödinger equation nonlinear. Because of this, it took a long time to analyze a He^4 atom (two-electron system) after the successful analysis of a hydrogen atom.
3. In the one-body problem, there is no distinction between first and second quantization. But from the two-body problem, the equivalence between the two becomes nontrivial.

We now move on to a general many-body problem.

1. This problem is treated in the chapter 9, “Systems containing several similar particles”, and chapter 10, “Theory of radiation” in Dirac’s textbook [3]. Dirac inconspicuously introduces second quantization in chapter 10. He takes a practical approach in that electrons as representatives of fermions are treated by first quantization in chapter 9 and photons as representatives of bosons are treated by second quantization in chapter 10. Therefore the occupation number representation appears for the first time for photons.

2. Dirac was first to discover second quantization [9] in 1927 in his theory of radiation. A single photon is special in that it does not have a probability amplitude, because it is a relativistic particle [10]. Therefore first quantization does not exist for photons. Dirac quantized classical electromagnetic waves by second quantization. One can discuss the PD of an ensemble of photons, but one cannot discuss the probability amplitude of a single photon. We will treat many-photon systems in detail in Section IV.E.
3. In case of a non-relativistic many-body system, both first and second quantization exist. First quantization gives a symmetric state for similar bosons and second quantization quantizes material waves of bosons. First quantization gives an antisymmetric state for similar fermions and second quantization quantizes material waves of fermions. The equivalence of first and second quantization was proven for bosons by P. Jordan and O. Klein in 1927 [11] and for fermions by P. Jordan and E. Wigner in 1928 [12] in the presence of two-body interaction between a pair of particles. In Section IV.F, we will discuss Bose-Einstein condensation in which two-body atomic interaction is not negligible. In Paper I, we showed Jordan's contribution to the MP. Jordan was apparently one of the most important figures in the creation period of QM.

B. Occupation number of a quantum state

The essential point of single-particle QM is that an observation cannot be complete only from a single measurement that yields only one set of EVs. Now we examine the situation in many-particle QM by considering the occupation number of a quantum state. A similar consideration is given in Paper II.

Fermions

The occupation number for a fermion state is either 0 or 1. Therefore ordinary fermions do not have macroscopic number of fermions in one quantum state and an ensemble of EVs cannot be obtained from a single measurement. It must be obtained from repeated measurements or as a virtual ensemble. Formation of a macroscopic quantum state is prohibited by the Pauli exclusion principle.

Bosons

The occupation number for a boson state is 0, 1, 2, ..., ∞. Therefore for light bosons, one-particle state can have a large number of bosons and an ensemble of EVs can be obtained from a single measurement.

As representative examples of many-boson systems, many-photon systems and He^4 systems are discussed in detail in Sections IV.E and IV.F respectively. In particular, liquid He^4 in superfluid at low temperature is an example of Bose-Einstein condensate (BEC)[13].

Bosons composed of Cooper pairs of fermions

Exceptions for fermions in general are Cooper pairs in a BEC system. They are Cooper pairs of electrons in superconductor[14] and those of liquid He^3 in superfluid[15]. We will discuss superconductivity in detail in Section IV.G.

C. Measurements of many-particle systems

In Paper II, we emphasized following points. Previously, theories of quantum measurements have focused on explaining how EVs become classical and have not covered an ensemble of measurements. The MIJ interpretation made the MP simple and allowed the analysis of an ensemble of measurements.

In case of bosons, a large real ensemble of similar bosons can be in one-particle state. Such one-particle state may be regarded effectively as classical as we will discuss later. Here we can introduce a new terminology, macroscopic quantum jump (MAJ), which is a QJ from macroscopic to macroscopic. A MAJ in a sense is a classical measurement.

D. Connection between many-boson world and classical world

The physical theory that describes a many-boson system is quantum field theory (QFT) [16]. We would like to see how this QFT behaves when the number of bosons increases from a finite number to a huge number and if a quantum state with a huge number of bosons can be regarded as a classical state. Since this transition is governed by QFT alone, QFT describes the transition seamlessly and there is no change in mathematics from a finite number of bosons to an infinite number of bosons.

In what follows, we will present three concrete examples of many-boson systems. Now our goal is two-fold. First, we would like to show the equivalence of a many-particle system with a huge number of bosons and a classical system. Second, we comment on the occupation numbers or the fractions of bosons in boson states.

E. Many-photon system and classical electromagnetic waves

For a quantum state specified by the number of photons N and phase ϕ, N and ϕ can be regarded as canonical conjugate variables. N and ϕ are related to the particle nature and wave nature of light respectively. There is an uncertainty relation,

$$\Delta N \Delta \phi \geq 1, \tag{3}$$

where ΔN is the fluctuation of photon number and $\Delta\phi$ is the phase fluctuation. If the fluctuation of N satisfies the condition,

$$N \gg \Delta N \gg 1, \tag{4}$$

the condition for phase fluctuation,

$$\Delta \phi \sim (\Delta N)^{-1} \ll 2\pi, \tag{5}$$

becomes possible and in this case, a classical monochromatic plane electromagnetic wave is realized. What we have done is an inverse process of field quantization, in which classical electromagnetic fields exist first and we quantize them to obtain photon fields by introducing commutation relations. At high light levels corresponding to classical waves, individual photons are not noticeable, while at low light levels, individual photons become obvious. Since a classical wave is obtained in one measurement, the corresponding quantum state has a huge number of photons.

Photons are simple to handle because there is no interaction between a pair of photons, unless vacuum polarization occurs at very high energy. They are also easy to handle because they are massless and their chemical potential is zero. Therefore they are freely created and annihilated regardless of their energy.

The same argument is applicable to phonons in metals. The relation between phonons and sound waves is analogous to that between photons and classical electromagnetic waves.

Now we comment on the occupation number of photons depending on the frequency, ν. The number of states,$D(\nu)d\nu$ goes as

$$D(\nu)d\nu \sim \nu^2 d\nu, \tag{6}$$

where $D(\nu)$ is the state density for unit frequency. The energy of a single photon is $h\nu$.

At radio frequency, both $D(\nu)$ and ν are low, and the occupation number per unit frequency is very high. Therefore the light behaves as radio waves.

On the other hand, at optical, both $D(\nu)$ and ν are high and photons are only sparsely distributed. A laser uses a coherent state or the eigenstate of annihilation operator. The occupation number for the laser is not an eigenvalue, but an expectation value. But this occupation number is still finite and the nature of laser light is still particle.

One interesting application of optical technology is optical quantum computers. Their operating frequency is in the range of GHz to THz. At GHz, the occupation number is high and the nature of light is wave, while at THz, the occupation number is order of unity and the nature of light is particle. If we focus on a readout or a measurement, the lower frequency appears advantageous, since a single readout yields a classical result.

F. Bose-Einstein condensation

A He^4 atom is a boson with spin 0. Its major difference from a photon or phonon is that He^4 atoms are massive and not created or annihilated under normal circumstances and their total number is conserved. In real life, atomic interaction is not negligible in helium gas or helium liquid, but here we tentatively consider perfect helium gas without atomic interaction. In this situation, the total energy of the gas is the sum of kinetic energies of all the atoms. The total energy takes the minimum when all the atoms are in the lowest one-particle state. This phenomenon of bosons is called Bose-Einstein condensation (BEC). Even in the presence of atomic interaction, BEC occurs if appropriate conditions are satisfied. In this case, a finite fraction of the total number of atoms condenses into the lowest one-particle state.

The system of He^4 atoms can be seen as waves in the sense of material waves of de Broglie. Of course these are quantized waves and they should be regarded as linear operators. However, when BEC occurs, the number of bosons in the lowest state is huge and the phase is well determined. Therefore the wave associated with the BEC system can be regarded as classical. On the macroscopic scale, the BEC is seen in superfluidity, which is a flow without friction. This BEC system is specified by a complex wave function called an order parameter, which quantifies the macroscopic quantum coherence and characterizes the superfluid phase.

Now we comment on the occupation number of the lowest state of liquid He^4 at 0K. The estimation of the exact value of the occupation number is difficult, but the fraction of the atoms in this BEC has been estimated to be 5% to 10% of the total atoms both experimentally and theoretically[17]. This small number is due to the presence of atomic interaction. This is the maximum value and the fraction decreases at higher temperature.

G. Superconductivity

The microscopic theory of superconductivity is the BCS theory[14]. Describing the BCS theory is beyond the scope of this paper, and here we only point out the similarity of a superconducting state to a BEC state and that of supercurrent to superfluid.

In a metal between a pair of electrons only Coulomb repulsion exists at normal temperature. However at certain low temperature, attraction by phonon exchange slightly overcomes the Coulomb repulsion and a pair of electrons forms a bound state called a Cooper pair. The electron spins are antiparallel and the spatial wave function is an S wave. Therefore the Cooper pair is in a singlet state and it behaves as a boson with spin 0.

At 0K, in the absence of attraction by phonon exchange, electrons would fill from the lowest one-particle state to the Fermi level. In the presence of phonon exchange in a metal, there exists the superconducting state slightly below the Fermi level into which Cooper pairs condense. The energy difference between the Fermi level and the superconducting state is called the superconducting gap Δ. To break up a Cooper pair by a photon, its energy must be greater than 2Δ. At finite temperature, but below the superconducting transition temperature, a finite fraction of the total Cooper pairs still condenses into the superconducting state. For instance, this transition temperature is 1.2K for aluminum.

Once the BEC of Cooper pairs occurs, as a macroscopic phenomenon, supercurrent flows which is analogous to superfluid of bosons. There is no dissipation in supercurrent unlike Ohm current. We can see the equivalence of superconductivity and classical electromagnetism from this phenomenon.

There is not much comment on the occupation number of Cooper pairs in a superconducting state. It is apparent that the fraction of Cooper pairs in the superconducting state is less than 100% even at 0K, but actual number depends on material properties and other factors. We cannot say anything definite about the occupation number at this point.

We briefly comment on quantum computers based on superconductivity. Their classical readout process may turn out to be a real advantage, since multiple readouts are performed for the purpose of quantum error correction. The final form of quantum computers may be “quantum mechanical” and “classical”.

V. Consideration of quantum field theory

In Section II, we argued against the reality of the wave function and the collapse of the wave function of non-relativistic quantum mechanics (NRQM). Here we reconsider these issues from a different

perspective of relativistic quantum field theory (RQFT), which can be regarded as a more fundamental theory than NRQM.

A. Motivation

The field operator in RQFT is a dynamical variable resulting from the quantization of classical de Broglie wave in three dimensional space. Therefore, it is natural to regard it as reality. Now we ask the following question: how is it related to the wave function in NRQM? If the wave function is the direct descendant in the derivation of NRQM from RQFT, it can also be regarded as reality. In other words, if Schrödinger equation is an approximate field equation, the wave function can be regarded as a kind of dynamical variable.

The outline of the derivation is the following. We start from a relativistic field equation such as Dirac equation. We take the low velocity limit and low energy limit. In taking the low energy limit, particle and antiparticle components are decoupled, and creation and annihilation are suppressed. This way the particle number is conserved in NRQM. The wave function appears as a matrix element of the field operator and it is a complex number. Since the field operator itself is a dynamical variable, the part of it may also be a dynamical variable. In that sense, the wave function can be real.

B. Current status

The above outline may appear reasonable, but the situation surrounding the derivation of NRQM from RQFT seems complicated. There is no textbook to describe it explicitly and there are researchers like T. Padmanabhan who published a fifty page article titled “Obtaining the Non-relativistic Quantum Mechanics from Quantum Field Theory: Issues, Folklores and Facts”[18]. First of all, the derivation is mathematically highly complex. In addition, there appears to be at least two issues. One is the treatment of antiparticles and the other is the difficulty in the treatment of multiple particles. If we trust Padmanabhan, it may be too early to say that the derivation is complete. As for the second issue, we will present a possible solution in the next subsection. Then the remaining issue may be a conceptual one related to antiparticles.

C. Two-step approach

Here we present a different approach to relate RQFT and NRQM. This procedure may bypass the mathematical complexity involved in deriving NRQM from RQFT directly. It is a two-step procedure

composed of relating RQFT and non-relativistic quantum field theory (NRQFT) first, and then further relating NRQFT and NRQM second.

It is well known that NRQFT is derived from RQFT by taking a non-relativistic limit of RQFT. NRQFT is well established in the fields of condensed matter physics and also in particle physics. As we have discussed in Section IV.A, the equivalence of first and second quantization was proved in late 1920's. First quantization means standard NRQM and second quantization means NRQFT.

From the above consideration, we have found that the problem has been solved without resorting to complex mathematics. Since we are interested in the nature of wave function, we can only focus on its relation to the field operator in NRQFT. Similarly to the field operator in RQFT, the field operator in NRQFT can be regarded as a dynamical variable in three dimensional space and as reality. The wave function is a matrix element of the field operator in NRQFT and it has the meaning of probability amplitude. We have finally reached the point of examining the reality of probability amplitude, not philosophically, but physically and mathematically.

1. Being a matrix element of the field operator, the wave function as a probability amplitude will be real. Otherwise, the matrix itself cannot be real.
2. If we judge from the role of the wave function in NRQM, the wave function cannot be a dynamical variable. We cannot discuss the EV of the wave function for instance.
3. When a particle interacts with a detector, its associated field operator will disappear. This indicates that the wave function actually collapses upon measurement.

In Section II, our arguments were based on probability consideration within NRQM and we reached the conclusions that the wave function is not real and that it does not collapse. In this section, we started the analysis from RQFT and reached the above conclusions. This reversal of fortune indicates physics prevails over mathematics. It also indicates that NRQM is not a complete theory.

D. A half solved problem

After publishing v2. of this paper, we found a recent work by M. Bhaumik[19]. He derives the relation between QFT and NRQM and that between the field operator matrix and the wave function as a matrix element. He has already done what we discussed in the previous subsection. His logic is that the particle picture of Standard Model is real and therefore its descendant the matrix element is also real. His work is mathematically rigorous.

His claim of the reality is not unanimously accepted, probably because the reality of the field operator needs clarification. Here we propose one possible amendment of his theory.

In Section II, we have seen that a dynamical variable as an observable must have a real eigenvalue. So it may be natural to require that a true dynamical variable is a function of the field operators with mathematically real number. We first consider NRQFT. Instead of the field operator Ψ itself, we may be able to start from the number density operator, $\rho = \Psi^\dagger \Psi$. It has a property that the volume integral of ρ, $\int \rho dV = 0, 1, 2, ...$, since this integral is the number of particles. Therefore it is natural to consider that ρ is real mathematically and it is real (ontic). Now we ask: can we derive $p = \psi^* \psi$ from ρ? Here p means the probabilty density and ψ denotes the wave function as a probability amplitude. The answer is obviously "yes", because this is just a reverse problem of second quantization[10].

Within the framework of NRQM, the wave function itself is not real (not ontic), but the probability density is real. So there is another reversal of fortune.

1. The wave function and a state vector are not real as claimed in Section II.
2. The probability density and probability are real, contrary to the claim in Section II. However they are not real in the sense that they cannot be obtained from a single measurement as we discussed before.
3. The relation between NRQFT and NRQM corresponds to that between an ensemble of many particles and a probability amplitude of a single particle.
4. A deeper problem of the relation between RQFT and the reality/unreality of the wave function still remains.

This is our current status.

VI. Conclusion

Physicists might have believed that the entire physical world was described by mathematics. However, this belief may have hindered the solution of the MP. Now we know that between the microscopic and macroscopic worlds, there is a gap called MP which must be described by an ordinary language. This MP requires more refined interpretation of mathematical quantities in QM.

Statements and Declarations

Data Availabilitty

N/A.

Author Contributions

T.N. conceived the idea, performed the analysis, and wrote the manuscript.

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