Review of: "Incorrect conclusions drawn for plausible looking diagrams"

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It is a commonly accepted fact that productive struggle requires expending effort when solving a challenging problem. The author builds on assuming that mathematical modelling is a productive struggle which becomes motivating for learners. Author argues that “The struggle reduces whenever you can draw a diagram, in order to recognize the correct mathematical path for the mathematical proof.”

The author states that it is crucial for the mathematical description to be flawless. But then concludes that “mistakes occur from flawed reasoning, arithmetic errors or faulty logic.”

In concluding that, the author approaches the work exclusively from the point of view of mathematics. He assumes that the geometric figures drawn in the diagrams are geometrically valid, and therefore cannot be misinterpreted.

The author forgets that human beings interpret graphic representations through a perception process that necessarily, always, has biases. The classical book by Stephen Palmer (Vision Science: Photons to Phenomenology) is the most recommended reference for the author to introduce himself to the different implications of the vision process.

In particular, psychology studies the principles of visual perception. The Gestalt rules, which emerged in the early twentieth century, may be particularly useful in the context of this study. Although they are currently criticized for being incomplete, these laws are an excellent starting point for understanding key aspects of the problem of perception.

Besides, the foundations of Semiology of Graphics were brilliantly defined by Jacques Bertin more than half a century ago. Therefore, it would be important for the author to rethink his work taking these principles into account.

Those principles guided the successful book by Edward Tufte, who defends minimalism in the graphical representation of data, by removing any type of attribute that hinders its understanding. Seeing the numerous examples it provides, it is easy to come to the conclusion that humans have a strong tendency to "spoil" graphic representations. Some recent publications in the scientific literature abound in this idea (DOI: 10.1109/MCG.2021.3132004).

The conclusion that the fallacies described in the examples “occur taking into account only the diagrams” misses the critical point that diagrams are perceived, and perception has its own rules, which do not always match geometrical rules.

Section 2 is supposed to talk about paradoxes derived from diagrams. However, diagram of the Koch Snowflake is not perceived by humans as having infinite length. This is analogous to sampling an irregular closed perimeter like a coastline at increasing resolution, it will yield a longer measurement than when sampling the perimeter at a lower resolution. But humans will never perceive the coastline as infinite (or nor more infinite than, say, a circle). The paradox (or the unexpected property of fractal curves) is in the mathematical notation, not in the graphical representation.

Regarding the second example, I guess that people would perceive both the area and the volume as infinite (because the figure “does not have a cover on its right side”). In passing, the three-dimensionality of the second figure is very poor. Appropriate use of projections would allow for better quality illustrations.

In the case of the missing square and the bizarre areas, the visual paradox only appears if the false expectation is created that a human being can clearly perceive small differences in slope or alignment.

I can agree with the assertions that (1) “Mistakes generate curiosity, increase motivation, create a learning environment and show that faulty logic is common in Mathematics, but can be useful for Mathematical progress “, and (2) “teachers must implement paradoxes in their teaching, because the strange result increases student’s interest”. But they are just hypotheses here, since the paper does not provide evidence on their validity.