Review of: "Fidelity of quantum blobs"

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This is an interesting article. And for context, about 15 -20 or so years ago, a number of us in the field of chemical dynamics were very interested in quantum phase-space representations as a way to develop better quantum and semi-classical approximation methods.

However, I believe many of the problems addressed in this article are somewhat well-understood and addressed by others over the past 20 years.

Can't question of fidelity of a quantum state in phase space can be expressed in terms of the over-completeness relation of coherent states?

Further, the question of Liouville equation representation can be easily be derived using the de Broglie-Bohm representation. This was discussed by Dias and Prata [1] and by a number of others, I don't think they were the first to discuss this and it's probably discussed in Holland's book on the Bohm trajectory approach. I'll reiterate the main points here.

Without going into the philosophical discussions surrounding the Bohm approach, we will stick to the pure mathematical analysis. Suppose we write an arbitrary 1D quantum state in polar form as

$$\psi = Re^{iS/\hbar}$$

were both R and S are real-valued functions of x and t. This is the Bohm/Madalung form of the wavefunction. It's straightforward, then to derive the equations of motion for both

$$\frac{\partial\rho}{\partial t} = -\nabla\left(\frac{p}{m}\right)\nabla S$$

where $\rho = R^2$and

$$\partial_t S + \frac{1}{2m}(\nabla S)^2 + V(x) + Q(x,t) = 0$$

where $Q(x,t)$ is the "quantum potential" as given by

$$Q(x,t) = -\frac{\hbar^2}{2m}\frac{1}{R}\nabla^2 R$$

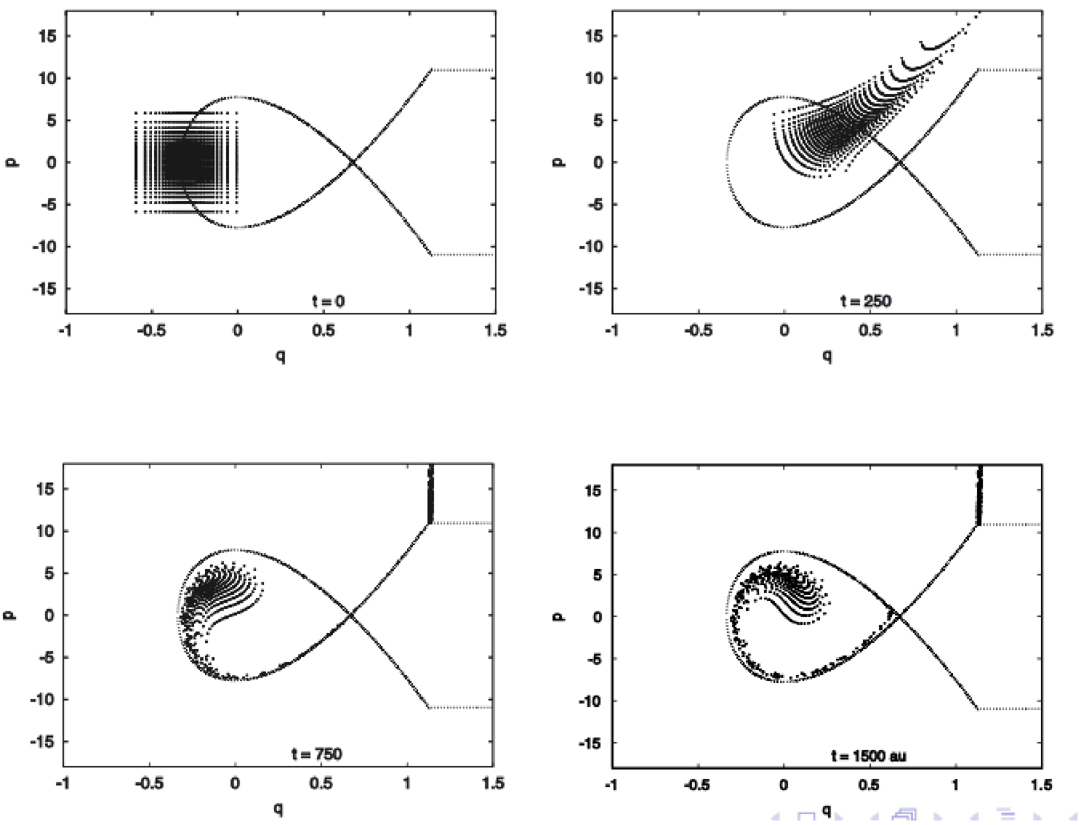
Neglecting this term, or setting $\hbar \to 0$ gives the *classical* Hamilton-Jacobi equation for a single *classical* trajectory and we identify S as the “action”. These two equations are trivial to derive from the time-dependent Schrodinger equation. Since the momentum is related to the gradiant of the action S, via $p = \nabla S$, it's straightforward then to conclude that the dynamics of an ensemble of Bohm trajectories described by a phase-space distribution written exactly as one would expect:

$$\partial_t P(x, p) = \{H(p, x) + Q(x, t), P\}$$

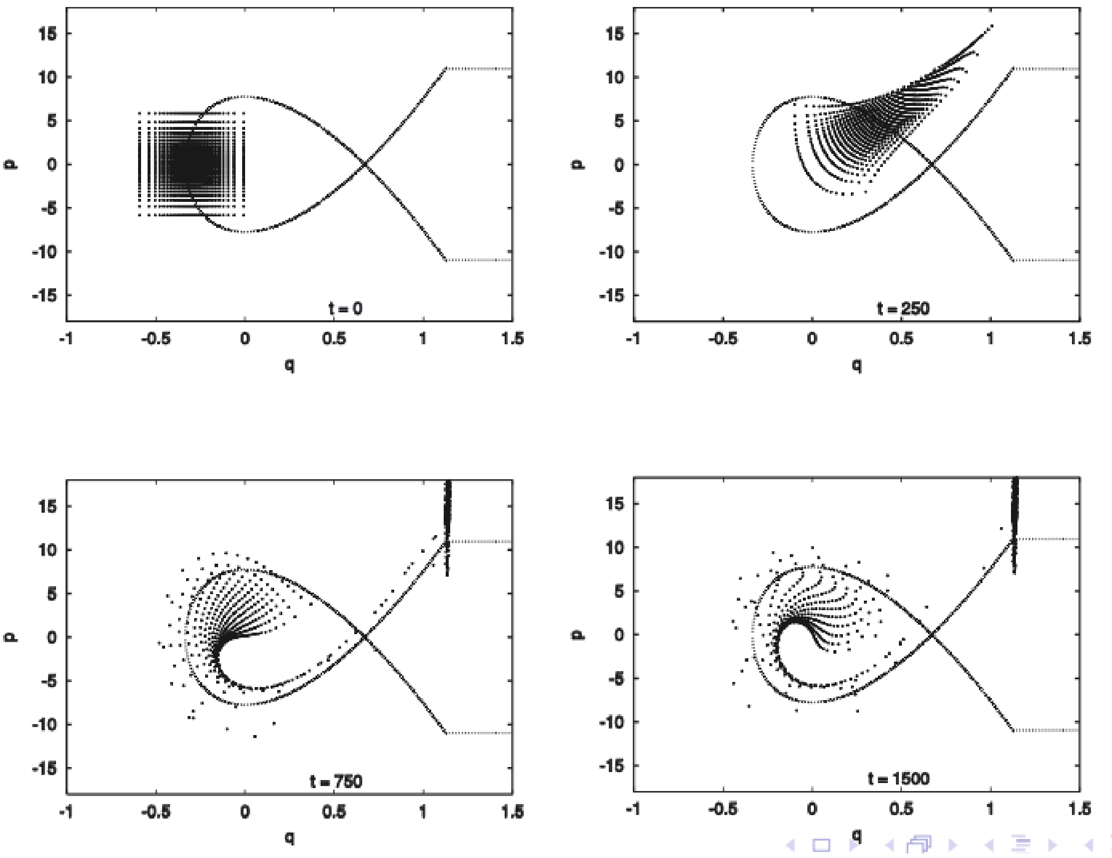
where H(p,x) is the classical hamiltonian and Q is the additional Bohm quantum potential. So, by Liouville theorem, phase-space volume is conserved just as in classical mechanics. The point of departure, however, is that the “quantum” trajectories are entangled with each other (via Q) where as their classical counterparts are independent. For numerical comparison, here are some results by Craig Martens for the escape of an ensemble of particles from a qubic potential of the form

$$V(x) = \frac{1}{2} m\omega^2 x^2 - \frac{1}{3} bx^3$$

These are the classical simulations:



and these are the quantum simulations (again by Craig Martens).



You can see similarities between the two in terms of how the quantum “blob” distorts and becomes highly extended in phase space.

Getting back to the paper at hand, I think that those of us interested in quantum dynamics will find some of the ideas expressed in this paper as interesting and perhaps insightful and I hope the author takes my comments as kind critique.

1	Bohmian trajectories and quantum phase space distributions N Dias and J.N. Prata. Phys. Lett. A 302 (2002) 261-272