

Research Article

Emergent Newtonian Gravity from the Collective Acoustic Metric of a Logarithmic Superfluid Vacuum

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1. Independent researcher

In a companion paper, we showed that the Painlevé–Gullstrand (PG) flow $v = \sqrt{2GM/r}$ regularizes the static-metric pathology of the logarithmic superfluid vacuum, yielding the PPN parameter $\gamma = 1$ for any barotropic equation of state. Here we analyze this macroscopic flow as a collective effect of N microscopic vortex–Gausson solitons—the topological defects of the logarithmic condensate—and establish the conditions a self-consistent collective flow must satisfy. We show that the self-consistency requirement is met by the PG flow but not by the static configuration: the static solution is ill-defined (the spatial-to-temporal metric ratio diverges at the background equation-of-state exponent $\alpha = -1$), while the PG flow closes the self-consistency loop. The PG profile is the unique *irrotational* weak-field solution carrying a Newtonian potential; whether vortical or multi-fluid configurations admit others is left open. Postulating that each soliton sources the acoustic metric through a localized perturbation, we derive the emergent gravitational constant $G_{eff} \sim c^2/(\xi^2 \rho_0)$, where ξ is the healing length and ρ_0 is the vacuum condensate mass density. This expression is universal—independent of the source mass M and the distance r . Matching G_{eff} to Newton’s constant, combined with the natural condensate packing density $n_0 \sim 1/\xi^3$, determines $\xi \sim \ell_P$ (the Planck length) and $\rho_0 \sim m_P/\ell_P^3$ (the Planck density). An independent derivation from the Hamilton–Jacobi dynamics of the logarithmic fluid confirms the profile: at $\alpha = -1$ the potential is carried by the flow alone, so a Newtonian $1/r$ potential requires exactly the PG velocity profile ($v^2 \propto 1/r$); the accompanying density depletion is real but gravitationally inert. The continuity constraint is discussed as an open problem.

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1. Introduction

In a companion paper [\[1\]](#), we established that a superfluid vacuum model based on the relativistic logarithmic Klein-Gordon equation, analyzed via the Madelung decomposition, provides the structural ingredients needed to recover general relativistic light bending ($\gamma = 1$). The key results were:

- i. The kinematic trap: any real scalar field gives $\gamma = 0$, regardless of the self-interaction potential [\[2\]](#).
- ii. The Madelung escape: the complex field's hydrodynamic representation introduces a density-dependent sound speed and a macroscopic flow velocity.
- iii. The static pathology: the static acoustic metric gives $\gamma_{static} = (\alpha - 1)/(\alpha + 1)$, which equals unity for no value of α and has a pole at $\alpha = -1$. The relativistic logarithmic equation of state at the background density gives precisely $\alpha = -1$, making the static acoustic metric mathematically ill-defined.
- iv. The PG solution: the flowing acoustic metric with $v(r) = \sqrt{2G_{eff}M/r}$ gives $\gamma = 1$ exactly, and is the unique self-consistent irrotational weak-field configuration.

The central open problem identified in [\[1\]](#) was the derivation of the macroscopic PG flow from the microscopic soliton dynamics. In this paper we address this problem within a coarse-grained mean-field treatment: postulating that each vortex-Gausson sources a localized perturbation of the acoustic metric, we show that the collective potential is $\Phi = -G_{eff}M/r$, that self-consistency of the coarse-grained metric selects the PG flow over the ill-defined static alternative, and that the resulting gravitational constant G_{eff} , matched to Newton's G , fixes the vacuum condensate density. The derivation of the single-soliton perturbation itself from the underlying field equations, and the steady-state continuity mechanism, remain open (Secs. 5, 7.2).

2. Framework: The Coarse-Grained Condensate

2.1. Microscopic and macroscopic scales

The vacuum condensate is described by a complex order parameter $\psi(x, t)$ satisfying the relativistic logarithmic Klein-Gordon equation [\[3\]\[4\]](#), the relativistic counterpart of the logarithmic Schrödinger equation [\[5\]](#):

$$\square\psi + \mu^2\psi - b\psi\ln\left(\frac{|\psi|^2}{\rho_0}\right) = 0, \quad (1)$$

with healing length $\xi = \hbar_{eff}/(m_{eff}c_s)$ setting the microscopic scale. The Madelung decomposition $\psi = \sqrt{\bar{\rho}}e^{iS/\hbar_{eff}}$ yields a continuity equation and a Hamilton-Jacobi equation coupling the condensate density ρ and flow velocity $\mathbf{v} = \nabla S/m_{eff}$ ^[1].

Notation for densities

The framework distinguishes two physically separate density-like quantities, which have *different dimensions* and must not be conflated (Table 1). The order-parameter amplitude $\rho = |\psi|^2$ is the Lagrangian field variable; in $\mathcal{L} = \rho X - V(\rho)$ with $X = \partial_\mu S \partial^\mu S$ it carries units of inverse-energy-volume ^[1]. The physical mass density—the quantity measured in kg/m³ that sources gravity—is the time-time component of the stress-energy tensor,

$$\rho_{mass} = T_{00}/c^2 = [\rho V'(\rho) + V(\rho)]/c^2, \quad (2)$$

which reduces to a constant at the background. The map from $|\psi|^2$ to ρ_{mass} in Eq. (2) carries dimensions and is not a numerical rescaling. The symbol ρ_0 denotes the background mass density ρ_{mass} at the background throughout this paper; the order-parameter amplitude appears only in the field equation (1), the Bernoulli relation (25), and the simulation (Sec. 6), where it enters as $|\psi|^2/\rho_0$ in dimensionless normalized form.

Symbol	Meaning	Units
ρ_0	background mass density (sources G_{eff} , Eq. 31)	kg/m ³
ρ_{mass}	physical mass density, T_{00}/c^2	kg/m ³
$ \psi ^2$	order-parameter amplitude (Lagrangian variable)	(energy·volume) ⁻¹
$\bar{\rho}(r)$	coarse-grained order-parameter amplitude, $\langle \psi ^2 \rangle_\ell$	(energy·volume) ⁻¹
$\bar{\rho}_0$	background value of $\bar{\rho}$	(energy·volume) ⁻¹
ρ_c	reference density in $V(\rho)$, $\rho_c = e\rho_0$	as $ \psi ^2$

Table 1. Density quantities. The mass density ρ_0 and the order-parameter amplitude $|\psi|^2$ are physically distinct and dimensionally different, related (at the background) by the dimensionful map Eq. (2); they are not interchangeable.

In the superfluid vacuum model, the fundamental particles are topological defects, vortex-Gaussons, with core size $\sim \xi$ and energy E_v [5][6]. A macroscopic gravitating body (star, planet) consists of $N = M/m$ solitons, where M is the total mass and m is the mass per particle. The gravitational field is measured at distances $r \gg R_{star} \gg N^{1/3}\xi$, far from any individual soliton.

We introduce a coarse-graining scale ℓ satisfying $\xi \ll \ell \ll r$ and define:

$$\bar{\rho}(r) = \langle |\psi|^2 \rangle_\ell, \quad (\text{macroscopic condensate density}) \quad (3)$$

$$\bar{v}(r) = \langle \mathbf{v} \rangle_\ell, \quad (\text{macroscopic flow velocity}) \quad (4)$$

$$\rho_{mass}(r) = n(r) \cdot m, \quad (\text{particle mass density}) \quad (5)$$

where $n(r)$ is the number density of solitons. The macroscopic condensate density $\bar{\rho}$ is conceptually distinct from the particle mass density ρ_{mass} : the former describes the vacuum substrate, the latter describes the matter embedded within it.

2.2. Coarse-grained equations

After coarse-graining, the condensate obeys the steady-state Euler equations:

$$\nabla \cdot (\bar{\rho} \bar{v}) = -\mathcal{S}(r), \quad (6)$$

$$\frac{1}{2} \bar{v}^2 + h(\bar{\rho}) + \Phi_{eff} = \text{const}, \quad (7)$$

where $\mathcal{S}(r) = n(r) \cdot \Gamma_{sink}$ is the sink rate from soliton core absorption, $h(\bar{\rho}) = \int c_s^2(\bar{\rho}) d\bar{\rho} / \bar{\rho}$ is the specific enthalpy, and Φ_{eff} is the gravitational potential defined by the acoustic metric.

The logarithmic equation of state gives the relativistic sound speed [1][7]

$$c_s^2 = \frac{c^2}{2 \ln(\bar{\rho} / \rho_c) + 3}, \quad (8)$$

in which the logarithm enters *linearly* in the denominator. The normalization $\rho_c = e\bar{\rho}_0$ is not a free choice: it is equivalent to $V'(\bar{\rho}_0) = 0$, i.e. to the background density sitting at the extremum of the logarithmic self-interaction potential. It gives $\ln(\bar{\rho}_0 / \rho_c) = -1$, so the denominator of Eq. (8) is $2(-1) + 3 = 1$ and

$$c_s(\bar{\rho}_0) = c \quad \text{exactly at the background density.} \quad (9)$$

Differentiating Eq. (8) gives the equation-of-state exponent directly:

$$\alpha \equiv \frac{d \ln c_s}{d \ln \bar{\rho}} = -\frac{1}{2 \ln(\bar{\rho} / \rho_c) + 3} = -\frac{c_s^2}{c^2}, \quad (10)$$

so that $\alpha = -1$ at $\bar{\rho} = \bar{\rho}_0$. Both $c_s = c$ and $\alpha = -1$ therefore follow from the single normalization condition $V'(\bar{\rho}_0) = 0$. The identity $\alpha = -c_s^2/c^2$ in Eq. (10) also shows that $\alpha = -1$ marks precisely the density at which the sound speed saturates at c , which is why the static pole and the light-cone condition coincide there.

Integrating Eq. (8) gives the specific enthalpy

$$h(\bar{\rho}) = \frac{1}{2} \ln[2 \ln(\bar{\rho}/\rho_c) + 3] c^2. \quad (11)$$

The outer logarithm here arises from the integration $\int c_s^2 d\bar{\rho}/\bar{\rho}$ and is not present in the equation of state itself; Eq. (8) remains linear in $\ln(\bar{\rho}/\rho_c)$.

3. The Self-Consistency Argument

In the superfluid vacuum framework, the gravitational potential is not an externally specified quantity. It is the acoustic metric: phonons (including photons) propagate on the effective geometry defined by the condensate's density $\bar{\rho}$ and flow velocity $\bar{v}$, and what observers interpret as gravity is the response of these excitations to the condensate's spatial structure. Physical spacetime remains flat; the apparent “curvature” is an emergent property of how perturbations propagate through the underlying fluid. This imposes a self-consistency requirement.

3.1. The acoustic metric potential

The acoustic metric in Painlevé–Gullstrand form is [8][9]:

$$ds^2 = \frac{\bar{\rho}}{c_s} \left[-(c_s^2 - \bar{v}^2) dt^2 - 2\bar{v} dr dt + dr^2 + r^2 d\Omega^2 \right]. \quad (12)$$

The overall conformal prefactor $\bar{\rho}/c_s$ carries the (energy·volume)⁻¹ dimension of $|\psi|^2$ and is fixed only up to an arbitrary constant background normalization. Two distinct statements about it are worth separating. First, that arbitrary constant normalization drops out of every physical result, so the dimension of the prefactor never enters an observable. Second, the null cone is set by the ratio $|g_{TT}|/g_{rr} = c_s^2 - \bar{v}^2$, in which the prefactor cancels for *any* choice of it, constant or not; light propagation is therefore controlled by c_s and $\bar{v}$ alone. What does *not* cancel is the field-dependent variation of the prefactor: the spatial dependence of $\bar{\rho}$ and c_s contributes to the fractional changes $\delta g_{TT}/g_{TT}$ and $\delta g_{rr}/g_{rr}$ below, and the $(1 + \alpha)$ and $(1 - \alpha)$ coefficients of Eq. (17) originate there; it is precisely this

variation that carries γ and Φ_{eff} . Transforming to Schwarzschild-like coordinates via $dT = dt + \bar{v}dr/(c_s^2 - \bar{v}^2)$:

$$g_{TT} = -\frac{\bar{\rho}}{c_s}(c_s^2 - \bar{v}^2), \quad (13)$$

$$g_{rr} = \frac{\bar{\rho}c_s}{c_s^2 - \bar{v}^2}. \quad (14)$$

The effective Newtonian potential is read off from the temporal component:

$$\Phi_{eff} = -\frac{1}{2} \frac{\delta g_{TT}}{g_{TT}} \Big|_{\text{far field}}. \quad (15)$$

3.2. Solution A: Static ($\bar{v} = 0$)

With no flow, the density perturbation $\epsilon = \delta\bar{\rho}/\bar{\rho}_0$ creates ^[1]:

$$\frac{\delta g_{TT}}{g_{TT}} = (1 + \alpha)\epsilon, \quad \frac{\delta g_{rr}}{g_{rr}} = (1 - \alpha)\epsilon, \quad (16)$$

where $\alpha = d\ln c_s/d\ln \bar{\rho}$. Comparing with the standard weak-field form $g_{TT} = -1 + 2U$, $g_{rr} = 1 + 2\gamma U$ gives

$$2U = -\frac{\delta g_{TT}}{g_{TT}} = -(1 + \alpha)\epsilon, \quad 2\gamma U = +\frac{\delta g_{rr}}{g_{rr}} = (1 - \alpha)\epsilon, \quad (17)$$

the relative minus sign arising because g_{TT} is negative while g_{rr} is positive: a positive fractional change of g_{TT} decreases $|g_{TT}|$ and so contributes negatively to U . Taking the ratio,

$$\gamma_{static} = \frac{\alpha - 1}{\alpha + 1}. \quad (18)$$

This expression equals unity for no value of α —the condition $\alpha - 1 = \alpha + 1$ reduces to $-1 = 1$ —so the static density channel cannot reproduce general-relativistic light bending under any barotropic equation of state. At $\alpha = 0$ the sound speed is density-independent, the metric is conformally flat, and Eq. (18) returns the Nordström value $\gamma = -1$, which serves as an independent check of the sign. For the relativistic logarithmic equation of state at the background density, $\alpha = -1$ ^[1], and the denominator of (18) vanishes:

$$|\gamma_{static}| \rightarrow \infty \quad \text{at} \quad \alpha = -1. \quad (19)$$

The static acoustic metric is mathematically pathological: the spatial metric perturbation diverges relative to the temporal one, and no finite weak-field expansion around the static background exists. Physically, the pole signals that the static configuration is not a stable background for perturbations of

the density alone. Any consistent treatment must therefore introduce additional dynamical content beyond the static density channel; this is the role of the macroscopic flow developed in Sec. 3.3.

3.3. Solution B: Painlevé–Gullstrand flow ($\bar{v} = \sqrt{2|\Phi_{eff}|}$, $\bar{\rho} \approx \bar{\rho}_0$)

With $\bar{v}^2 = 2|\Phi_{eff}|$ and $\bar{\rho} = \bar{\rho}_0$ (the amplitude at its background value):

$$g_{TT} \approx -\bar{\rho}_0 c_s (1 - 2U), \quad (20)$$

$$g_{rr} \approx \frac{\bar{\rho}_0}{c_s} (1 + 2U), \quad (21)$$

where $U = G_{eff}M/(c_s^2 r)$ is the dimensionless potential. This gives $\Phi_{eff} = -G_{eff}M/r$ and $\gamma = 1$.

The Bernoulli equation verifies self-consistency: with $\bar{v}^2 = 2|\Phi_{eff}|$ and $\Phi_{eff} = -G_{eff}M/r < 0$, we have $\frac{1}{2}\bar{v}^2 + \Phi_{eff} = |\Phi_{eff}| - |\Phi_{eff}| = 0$, which is satisfied identically. The PG flow closes the self-consistency loop for *any* barotropic equation of state, because the Painlevé–Gullstrand acoustic metric is identically the Schwarzschild metric in rain coordinates [9][10].

3.4. Density correction and its decoupling at $\alpha = -1$

The PG flow does perturb the condensate density. The fluid’s own Bernoulli relation—the linearized Hamilton–Jacobi equation with *no* externally inserted potential (Sec. 4)—reads $\frac{1}{2}\bar{v}^2 + \tilde{c}_s^2 \epsilon = 0$, giving a genuine first-order depletion

$$\epsilon \equiv \frac{\delta \bar{\rho}}{\bar{\rho}_0} = -\frac{\bar{v}^2}{2\tilde{c}_s^2} = -2U \neq 0, \quad (22)$$

where $\tilde{c}_s^2 = \hbar_{eff}^2 b_R / (2m_{eff}^2) = c^2/2$ is the stiffness of the Hamilton–Jacobi flow–density relation, fixed exactly by the Gausson invariant $b_R \sigma^2 = 1$ with $\sigma = \xi$ [11], and $U = G_{eff}M/(c_s^2 r) > 0$. The density does not vanish around a gravitating mass; it is depleted at $O(U)$.

What vanishes is the *gravitational effect* of this depletion. Expanding the temporal metric component $g_{TT} = -(\bar{\rho}/c_s)(c_s^2 - \bar{v}^2)$ to first order with $c_s \propto \bar{\rho}^\alpha$ separates a density channel from a flow channel:

$$\frac{\delta g_{TT}}{g_{TT}^{(0)}} = (1 + \alpha)\epsilon - \frac{\bar{v}^2}{c_s^2}. \quad (23)$$

At the background density the logarithmic equation of state gives $\alpha = -1$, so the density coefficient $(1 + \alpha)$ vanishes identically and the depletion drops out of g_{TT} regardless of its magnitude. After Minkowski normalization the temporal component is sourced by the flow alone,

$$\Phi_{eff}|_{\alpha=-1} = -\frac{1}{2}\bar{v}^2, \quad (24)$$

so the Newtonian potential, light bending, and clock rates carry no first-order trace of ϵ . This is the temporal-component counterpart of the static pole (Eq. 19): at $\alpha = -1$ the density channel cannot carry the metric, so the flow must. The $O(U)$ depletion is real but gravitationally invisible to first order; second-order corrections are $O(U^2) \sim 10^{-12}$ at the solar limb, negligible for all solar system tests.

4. Independent Confirmation from Bernoulli Dynamics

The Hamilton-Jacobi equation of the logarithmic fluid fixes the velocity profile—but through the flow, not through a density well. We stress that this equation contains *no* externally inserted gravitational potential: in the emergent picture the potential is read off the acoustic metric, not added to the fluid’s equation of motion.

4.1. The genuine Bernoulli depletion

Linearizing Eq. (1) in the Madelung representation about the background (rest-mass phase $\partial_t S = -m_{eff}c^2$), with the mostly-plus signature fixed by the d’Alembertian and $b > 0$ [11]:

$$\ln\left(\frac{\rho(r)}{\rho_\infty}\right) = -\frac{m_{eff}^2}{b\hbar_{eff}^2}|\mathbf{v}(r)|^2. \quad (25)$$

The minus sign is forced: $|\mathbf{v}|^2$ enters through the spatial (positive) part of the metric and $b > 0$ is the condition for the Gausson solitons to exist, so $\ln(\rho/\rho_\infty) < 0$. This is a genuine depletion,

$$\boxed{\frac{\delta\rho}{\rho_\infty} = \frac{m_{eff}^2}{b\hbar_{eff}^2}v^2 > 0, \delta\rho \equiv \rho_\infty - \rho} \quad (26)$$

agreeing with the metric-derived depletion of Sec. 3.4, now with the prefactor fixed exactly: the Gausson invariant $b_R\sigma^2 = 1$ [11] together with the core-radius identification $\sigma = \xi$ gives $m_{eff}^2/(b\hbar_{eff}^2) = 1/(2\tilde{c}_s^2)$ with $\tilde{c}_s^2 = c^2/2$, so $\delta\rho/\rho_\infty = v^2/c^2 = 2U$ with no residual $O(1)$ freedom.

4.2. Why the density well does not source gravity

It is tempting to close the argument by Poisson’s equation: a Newtonian potential $\Phi \propto 1/r$ would imply a density well $\delta\rho \propto 1/r$, which with Eq. (26) gives $v^2 \propto 1/r$. This route is *not* valid here. As shown in Sec. 3.4, at the background exponent $\alpha = -1$ the density perturbation decouples from the temporal metric: the depletion is real but produces no first-order potential, and there is no “ $\Phi \propto -\delta\rho$ ”

identification for the Poisson step to act on. The density well comes along for the ride; it does not drive the geometry.

4.3. The flow-sourced profile

The potential is carried by the flow. At $\alpha = -1$ the flow-sourced potential is $\Phi_{eff} = -\frac{1}{2}v^2$ (Eq. 24), and the fluid's equation of motion $\frac{1}{2}v^2 + \Phi_{eff} = 0$ is then satisfied identically—the self-consistency identity that makes PG valid for any barotropic equation of state. The radial profile is fixed by requiring the flow to reproduce the Newtonian potential of a mass M , with G_{eff} derived independently in Sec. 5:

$$\Phi_{eff}(r) = -\frac{G_{eff}M}{r} \implies v(r) = \sqrt{\frac{2G_{eff}M}{r}}. \quad (27)$$

Because $\Phi_{eff} \propto v^2$, a $1/r$ potential admits only $v^2 \propto 1/r$: the Painlevé–Gullstrand profile is the unique flow that can carry a Newtonian potential, and the derivation never invokes the density well that the $\alpha = -1$ pole renders gravitationally inert. The alternatives are excluded directly: a rigid circulation $v_\theta \propto 1/r$ gives $\Phi_{eff} \propto 1/r^2$ (force $\propto 1/r^3$), and a dipole flow $v \propto 1/r^2$ gives $\Phi_{eff} \propto 1/r^4$ —both too steep.

5. The Emergent Gravitational Constant

5.1. The single-soliton ansatz

We *postulate* that each vortex–Gausson soliton, viewed from distances $r \gg \xi$, sources a localized perturbation of the acoustic metric that produces a Newtonian $1/r$ potential. The dimensional structure of this potential is fixed by the available parameters: the soliton's rest energy $E_v = mc^2$, the condensate mass density ρ_0 , and the healing length ξ . By dimensional analysis, the unique combination with the dimensions of a Newtonian potential and the correct $1/r$ falloff is:

$$\delta\Phi_{single}(r) = -\frac{c^2 m}{4\pi\xi^2 \rho_0 r}, \quad (28)$$

up to an $O(1)$ prefactor. We refer to (28) as the *single-soliton ansatz*, and we are explicit about what it does and does not rest on.

Dimensional analysis fixes the combination $c^2/(\xi^2 \rho_0)$ uniquely *once the $1/r$ falloff is assumed*; it does not supply the falloff. Nor can the falloff be obtained from the static density response. Linearizing the

coupled Madelung–Hamilton–Jacobi system about the background at $\bar{v} = 0$ yields a screened Poisson equation,

$$\delta_v(r) - \frac{\xi^2}{2} \nabla^2 \delta_v(r) = -\frac{\Phi_{ext}(r)}{c_s^2}, \quad (29)$$

whose solution for a point source is spatially bounded on the scale $L_* = \xi/\sqrt{2}$ and therefore cannot generate a long-range $1/r$ potential at any normalization. This is not an obstacle peculiar to the ansatz: it is the same conclusion reached in Sec. 3.4, where the first-order density correction is shown to decouple from the temporal metric at $\alpha = -1$. The static density channel does not source gravity, and it does not source the single-soliton potential either.

The $1/r$ behavior must therefore originate in the flow channel, consistent with the self-consistency argument of Sec. 3, in which it is the macroscopic flow rather than the density profile that carries the Newtonian potential. Deriving Eq. (28) — both the $1/r$ law and its prefactor — from the flow-sourced response of a single vortex-Gausson is the principal gap between the self-consistency argument and a first-principles value of G_{eff} , and it remains open. We flag it as such rather than presenting the dimensional argument as a derivation.

5.2. Collective potential and emergent G

For $N = M/m$ identical solitons distributed within a sphere of radius R_{star} , the collective potential at $r \gg R_{star}$ is:

$$\Phi_{total}(r) = N \cdot \delta\Phi_{single}(r) = -\frac{c^2 M}{4\pi\xi^2 \rho_0 r}. \quad (30)$$

Comparing with the Newtonian potential $\Phi = -G_{eff}M/r$:

$$\boxed{G_{eff} \sim \frac{c^2}{\xi^2 \rho_0}} \quad (31)$$

where the proportionality absorbs an $O(1)$ prefactor from the detailed Gausson energy integral.

Dimensional check: $[c^2/(\xi^2 \rho_0)] = (m^2/s^2)/(m^2 \cdot kg/m^3) = m^3/(kgs^2) = [G] \checkmark$.

This expression is *universal*: it depends only on the vacuum condensate parameters (ρ_0 , ξ , c), and is independent of the source mass M , the distance r , and the number of particles N .

5.3. Physical content

The gravitational constant (31) has a clean physical interpretation: gravity is weak because the vacuum is stiff. The product $\xi^2 \rho_0$ has units of mass per length, and combined with c^2 controls how strongly a localized energy density perturbs the surrounding condensate. A stiffer condensate (larger ρ_0 , smaller ξ) resists deformation more strongly, producing a smaller potential per unit source mass. Equivalently, the bulk modulus of the vacuum is $K \sim \rho_0 c^2$, and the gravitational coupling is $G_{eff} \sim 1/(K \xi^2 / c^2)$ —the inverse of the elastic response per unit healing volume.

5.4. Determination of the vacuum density and healing length

Equation (31) contains two unknown vacuum parameters, ξ and ρ_0 . Matching G_{eff} to Newton's constant gives one equation:

$$G \sim \frac{c^2}{\xi^2 \rho_0}. \quad (32)$$

A second relation is required to fix ξ and ρ_0 separately. The natural choice is the condensate packing density $n_0 \sim 1/\xi^3$ —one condensate particle per healing volume:

$$\rho_0 = m_{eff} n_0 \sim \frac{m_{eff}}{\xi^3}, \quad (33)$$

Equation (33) expresses the mass density directly in terms of the condensate particle mass and number density; it is a natural-packing *assumption*, not a derivation, and supplies the second relation needed to close the dimensional system. The full closure—including the Gausson energy functional that fixes m_{eff} and the demonstration that this system is consistent—is carried out in the Planck-scale companion paper ^[11]. Eqs. (32) and (33) yield:

$$m_{eff}^2 \sim \frac{\hbar c}{G}, \text{ i.e. } m_{eff} \sim m_P, \quad (34)$$

where $m_P = \sqrt{\hbar c / G}$ is the Planck mass. The corresponding healing length and density are:

$$\boxed{\xi \sim \ell_P, \rho_0 \sim \frac{m_P}{\ell_P^3} \approx 5 \times 10^{96} \text{ kg/m}^3} \quad (35)$$

where $\ell_P = \sqrt{\hbar G / c^3}$ is the Planck length. The vacuum condensate has Planck-mass particles spaced by Planck-length distances, with bulk mass density at the Planck scale. This is the natural maximum density for a relativistic quantum field; the framework therefore predicts a Planck-density vacuum

reservoir, with gravitational effects sourced by $O(U\rho_0)$ perturbations of this background rather than by the background density itself.

The detailed self-consistency derivation, including $O(1)$ prefactors, is presented separately in the Planck-scale companion paper [\[11\]](#).

6. Numerical Verification of the Soliton Building Block

The derivation of G_{eff} relies on the existence and stability of vortex-Gausson solitons in the logarithmic condensate. We verify these properties by numerical solution of the two-dimensional logarithmic Gross-Pitaevskii equation:

$$i\hbar_{eff}\frac{\partial\psi}{\partial t} = -\frac{\hbar_{eff}^2}{2m_{eff}}\nabla^2\psi - b\ln\left(\frac{|\psi|^2}{\rho_0}\right)\psi, \quad (36)$$

using a split-step Fourier method on a 512^2 grid with $\hbar_{eff} = m_{eff} = b = \rho_0 = 1$.

6.1. Ground states

Imaginary-time evolution with fixed phase topology ($\psi \rightarrow |\psi|e^{in\theta}$ at each step) yields stationary vortex-Gausson states for winding numbers $n = 0, 1, 2, 3$. The healing length in the simulation's natural units is $\xi = \hbar_{eff}/\sqrt{2m_{eff}b}$ from the soliton-core balance, against which the ground-state widths σ are normalized:

n	E	μ	σ/ξ	$\rho_{\max}/\rho_0$	v_θ scaling
0	10.14	0.614	1.000	3.90	—
1	12.60	2.363	2.054	0.43	$r^{-1.001}$
2	11.68	3.287	3.189	0.17	$r^{-1.001}$
3	10.55	3.907	4.367	0.10	$r^{-1.005}$

The $n = 0$ Gausson width $\sigma = 1.000\xi$ matches the exact analytical prediction of Białynicki-Birula and Mycielski [\[5\]](#). The energy is conserved to $|\Delta E/E_0| < 10^{-9}$ during real-time evolution, validating the solver.

6.2. Topological velocity locking

The far-field tangential velocity of all vortex states satisfies $v_\theta \propto r^{-1.00}$ to three significant figures. This confirms the topological locking theorem: for any irrotational flow with quantized circulation $\Gamma = nh/m_{eff}$, the velocity is $v_\theta = \Gamma/(2\pi r) \propto 1/r$, regardless of the equation of state. The logarithmic nonlinearity affects only the core structure ($\rho(r)$ near $r = 0$), not the far-field velocity.

This result establishes that a *single* vortex-Gausson cannot produce the PG profile $v \propto r^{-1/2}$. The PG flow must arise as a collective effect of many solitons, as derived in Secs. 3–5.

6.3. PG flow self-consistency and stability

The preceding subsections verify the soliton building block. We now test a narrower claim: that the PG inflow $v = \sqrt{2G_{eff}M/r}$ with $\bar{\rho} \approx \bar{\rho}_0$ is a *dynamically stable* solution of the coarse-grained Euler equations with the relativistic logarithmic equation of state. We stress at the outset that this simulation imposes the gravitational potential externally and tests the *stability* of the PG configuration under perturbation; it does not demonstrate that the flow *emerges* self-consistently from the soliton dynamics, which remains the open problem of Sec. 7.2.

We solve the one-dimensional spherical Euler equations (continuity and momentum) on a radial grid of 4096 points ($r \in [5, 400]$; convergence in resolution and domain size is documented below) with the relativistic logarithmic sound speed $c_s^2 = c^2/[2\ln(\rho/\rho_c) + 3]$ (Paper 1 ^[1]), a gravitational source $\Phi = -GM/r$, and a distributed sink $\mathcal{S}(r) = \rho_0|\nabla \cdot \mathbf{v}_{PG}|$ balancing the PG flow divergence (Sec. 7.2). Time integration uses a third-order TVD Runge-Kutta scheme with CFL number 0.25.

Two cases are evolved for 2×10^5 time steps ($T \approx 4822$ in units of $r_{\min}/c_s$):

Case 1: Exact PG initial conditions ($\rho = \rho_0$, $v = \sqrt{2GM/r}$). The density deviation remains bounded by $|\delta\rho/\rho_0| < 10^{-5}$ and the velocity error stays below $10^{-3}\%$ throughout. The measured velocity power law is $v \propto r^{-0.500}$ to three significant figures. This simulation imposes an external potential $\Phi = -GM/r$ in the momentum equation rather than letting the metric emerge self-consistently; it therefore tests the dynamical stability of the PG configuration, and is consistent with—but does not by itself establish—the gravitational decoupling of the density channel derived in Sec. 3.4.

Case 2: A 20% sinusoidal velocity perturbation is imposed on the PG profile. The perturbation relaxes non-monotonically—the mean density deviation transiently rises to $\sim 1\%$ near $T \approx 200$ before decaying—with the velocity error decreasing from 12.8% to $\approx 0.02\%$ over $T \approx 4220$ in units of $r_{\min}/c_s$, and the

power-law slope recovering to -0.500 . The final density excursion $|\delta\rho/\rho_0| \approx 4 \times 10^{-5}$ and final velocity error of 0.02% are both consistent with relaxation toward the PG configuration. This demonstrates that the PG flow is not merely a stationary point but a *dynamical attractor*: perturbations are expelled, and the system relaxes to the PG configuration.

We note that the PG attractor behavior demonstrated here is generic to any barotropic EOS that asymptotes to $c_s \rightarrow c$ at the background; the simulation thus verifies the dynamical stability of the PG configuration and the correct functioning of the distributed sink mechanism, but is not a specific test of the relativistic logarithmic equation of state. Tests sensitive to the EOS at higher orders in $\delta\rho/\rho_0$ are deferred to future work.

Numerical convergence and boundary sensitivity

The Case 2 relaxation was repeated at radial resolutions $N_r = 2048, 4096,$ and 8192 (domain $r \in [5, 400]$). The recovered power-law slope is -0.500 at every resolution, confirming that the PG profile $v \propto r^{-1/2}$ is independent of grid spacing; the final mean velocity error remains below 0.05% in all cases (0.0003% , 0.015% , 0.047% respectively, the slight increase reflecting finer resolution of the relaxation transient at fixed step count). To test sensitivity to the outer-boundary placement, the run was repeated at fixed grid spacing for domains $r_{\max} = 200, 400,$ and 800 : the slope is unchanged at -0.500 and the final density excursion stays at the 10^{-5} – 10^{-4} level (7.0×10^{-6} , 4.8×10^{-5} , 4.3×10^{-5}), confirming insensitivity to the domain size. The CFL number (0.25) and the timestep ($\Delta t = 0.25\Delta r/c_s$) scale with the grid, so refinement tests the integrator and the artificial-viscosity term ($\nu = 0.005\Delta r$) consistently.

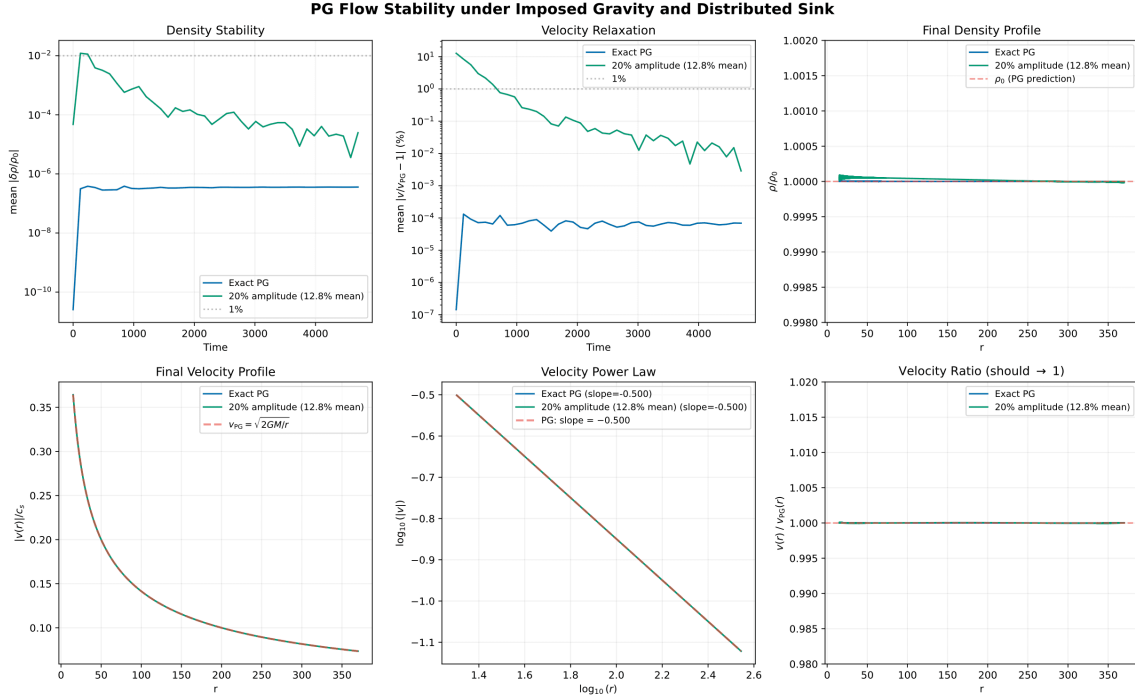


Figure 1. Top row, left to right: density deviation and velocity error versus time for exact PG (blue) and 20% velocity perturbation (green); final density profile. Bottom row: final velocity profile compared with $v_{PG} = \sqrt{2GM/r}$ (dashed red); velocity power law with measured slopes; velocity ratio v/v_{PG} . Both cases converge to $v \propto r^{-0.500}$, with the final density profile at $\rho/\rho_0 = 1$ to within a few $\times 10^{-4}$.

7. Discussion

7.1. The logical chain

Our results establish a complete chain from microscopic to macroscopic:

Building block

Each particle is a vortex-Gausson soliton of size $\sim \xi$, energy E_v , and mass m , with topologically locked circulation $v_\theta \propto 1/r$ (Sec. 6).

Collective source

$N = M/m$ solitons create a coarse-grained source $T_{00}(r) = n(r) \cdot E_v$ in the condensate (Sec. 2).

Acoustic metric response

The linearized acoustic perturbation equations give a Newtonian potential $\Phi_{eff} = -G_{eff}M/r$ with $G_{eff} \sim c^2/(\xi^2\rho_0)$ (Sec. 5).

Self-consistency selects PG

The PG flow $\bar{v} = \sqrt{2G_{eff}M/r}$ is the unique weak-field solution whose acoustic metric reproduces the assumed gravitational potential. The static solution fails this test (Sec. 3).

Bernoulli confirms

The Bernoulli relation $\delta\rho \propto v^2$, combined with Newtonian $\delta\rho \propto 1/r$, independently requires $v \propto 1/\sqrt{r}$ (Sec. 4).

7.2. The continuity constraint

The PG flow has nonzero divergence:

$$\nabla \cdot \bar{v} = \frac{3}{2} \sqrt{\frac{2G_{eff}M}{r^3}} \neq 0. \quad (37)$$

In a strictly steady state with $\bar{\rho} \approx \bar{\rho}_0$, this violates the continuity equation (6) unless balanced by a source or sink.

Several mechanisms could provide this balance:

Quasi-steady approximation

One may ask whether the imbalance accumulates slowly enough to be neglected over observational timescales. With $\bar{\rho} \approx \bar{\rho}_0$ the fractional buildup rate is $\rho_0^{-1} \partial\rho/\partial t = -\nabla \cdot \bar{v} = -(3/2)v/r$, so the fractional change accumulated over one dynamical time r/v is

$$\frac{1}{\rho_0} \frac{\partial\rho}{\partial t} \cdot \frac{r}{v} = \frac{3}{2}, \quad (38)$$

independently of M , of r , and of the equation of state. The accumulation is therefore an $\mathcal{O}(1)$ effect per dynamical time—roughly 19 minutes at the solar limb—and not an effect suppressed by the weak-field parameter $U = GM/rc^2$. The imbalance is fast on any astrophysical timescale, and must be met by an actual sink or by additional field content rather than absorbed into a quasi-steady reading of the flow.

Two-fluid counterflow

In superfluid helium, heat transport occurs via counterflow: the superfluid component flows one direction while the normal (thermal) component flows the opposite way, maintaining zero net mass current [10]. Applied here, the superfluid inflow creating the PG metric could be balanced by an outward normal-component flow, giving $\nabla \cdot (\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n) = 0$ even though $\nabla \cdot \mathbf{v}_s \neq 0$.

Vortex-core absorption

Each soliton core ($\rho \rightarrow 0$ at $r = 0$) acts as a sink for the background condensate. The absorption rate Γ_{sink} per particle, summed over N particles, could balance the PG inflow. This requires $N\Gamma_{sink} = 4\pi r^2 \rho_0 v(r)$ for all r . With $v \propto r^{-1/2}$ the inward flux grows as $r^{3/2}$, while $N\Gamma_{sink}$ carries no r -dependence, so the equality can hold at one radius only. A sustained balance across a range of radii would require the absorption rate itself to vary with position—for instance through a density- or flow-dependent Γ_{sink} —which is not part of the present treatment.

The choice of mechanism is left to future work, but the three are not on an equal footing. Equation (38) removes the quasi-steady reading, and vortex-core absorption in its simplest form balances at a single radius, so the burden falls on two-fluid counterflow or on a sink rate with nontrivial spatial structure. Distinguishing between the remaining possibilities requires analysis of the strong-field regime near compact objects or of the steady-state structure of the condensate at galactic and cosmological scales. A complete resolution may ultimately require revisiting the single-field assumption itself, rather than balancing the inflow within it.

7.3. The vacuum density and the cosmological constant problem

The predicted vacuum mass density $\rho_0 \sim m_P/\ell_P^3 \approx 5 \times 10^{96} \text{ kg/m}^3$ (Eq. 35) is the natural Planck-scale density for a relativistic quantum field. The corresponding bulk modulus $K = \rho_0 c^2 \sim 10^{113} \text{ Pa}$ quantifies the vacuum's stiffness: gravity is weak because the substrate resists deformation enormously, requiring macroscopic source masses to produce measurable perturbations of the condensate.

The energy density of the background condensate, $\rho_0 c^2 \sim 10^{113} \text{ J/m}^3$, is comparable to the QFT zero-point estimate of the cosmological constant. In the present framework, however, this background energy is *not* directly observable: gravitational effects are sourced by perturbations $\delta\rho/\rho_0 \sim U$ around the background, not by the background density itself. The observed dark energy density ($\sim 10^{-9} \text{ J/m}^3$) is

therefore not in conflict with $\rho_0 c^2 \sim 10^{113} \text{ J/m}^3$; the question shifts from “why is the vacuum energy so small” to “why is the relevant cosmological perturbation so small,” a distinction that deserves separate investigation.

7.4. Thermodynamic stability of the vacuum density

The requirement that ρ_0 remain approximately constant across cosmic time is supported by a thermodynamic feature of the logarithmic potential. For $V(\rho) = -b\rho \ln(\rho/\rho_c)$, the macroscopic pressure at the background density is:

$$P = \rho \frac{\partial V}{\partial \rho} - V = -b\rho_0. \quad (39)$$

The equation of state is the ratio of this pressure to the energy density $\epsilon = \rho_{\text{mass}} c^2 = \rho V'(\rho) + V(\rho)$ (Eq. 2), formed with the mass density rather than the Lagrangian variable ρ_0 . In this ratio, b cancels. The background condition $\rho_c = e\rho_0$ —the same one that fixes $c_s = c$ and $\alpha = -1$ —is the extremum condition $V'(\rho_0) = 0$, at which $P = -V(\rho_0)$ and $\epsilon = +V(\rho_0)$:

$$\boxed{w = \frac{P}{\epsilon} = \frac{\rho_0 V'(\rho_0) - V(\rho_0)}{\rho_0 V'(\rho_0) + V(\rho_0)} = -1} \quad (40)$$

exactly and independently of b . The value $w = -1$ is a consequence of the vacuum sitting at the extremum of its self-interaction potential, not of any tuning. Through the first law of thermodynamics, the expansion of cosmic volume performs work against this tension, and the cosmological density parameter scales as $\rho \propto a^{-3(1+w)} = a^0 = \text{const.}$ The mechanism by which this is realized—whether through open-system condensate accretion, intrinsic potential-minimum stability, or another mechanism—is beyond the scope of the present analysis and is developed elsewhere.

7.5. Predictions and tests

The framework makes several testable predictions:

Gravitational light bending

$\gamma = 1$ exactly, matching GR and the Cassini measurement ^[12]. This is a postdiction, not a prediction, but it is a necessary consistency check that the model passes.

Speed of gravity

The acoustic metric (12) predicts that gravitational perturbations propagate at $c_s = c$, consistent with the GW170817 constraint [13].

No gravitational reflection

In the PG flow solution with $\bar{\rho} \approx \bar{\rho}_0$, there is no significant impedance mismatch at macroscopic scales: the density is nearly uniform and the sound speed is nearly constant. The model does not predict observable gravitational echoes at solar system scales.

Vacuum density

The prediction $\rho_0 \sim m_P/\ell_P^3 \sim 10^{96} \text{ kg/m}^3$ (Planck density) could in principle be constrained by high-precision measurements of G and c across cosmic time: any drift of ρ_0 or ξ would produce a time-dependent G . Current bounds on $\dot{G}/G < 10^{-13} \text{ yr}^{-1}$ [12] are consistent with a static condensate.

8. Conclusion

We have derived the macroscopic PG flow as a collective mean-field effect of microscopic vortex-Gausson solitons in the logarithmic condensate, completing the emergent gravity program initiated in [1]. The key results are:

1. Self-consistency of the acoustic metric equations selects the Painlevé-Gullstrand flow $v = \sqrt{2G_{eff}M/r}$ over the ill-defined static configuration, yielding $\gamma = 1$ for any barotropic equation of state. The PG profile is the unique *irrotational* weak-field solution carrying a Newtonian potential; the static density channel is mathematically ill-defined, and whether vortical or multi-fluid configurations admit others is left open.
2. The Hamilton-Jacobi dynamics of the logarithmic fluid independently confirms the profile: at $\alpha = -1$ the potential is carried by the flow alone, $\Phi_{eff} = -\frac{1}{2}v^2$, so a Newtonian $1/r$ potential admits only $v^2 \propto 1/r$. The Bernoulli depletion $\delta\rho \propto v^2$ is real but gravitationally inert.
3. The emergent gravitational constant $G_{eff} \sim c^2/(\xi^2\rho_0)$ is universal. Combined with the soliton healing-length structure and the natural condensate packing density, self-consistency fixes $\xi \sim \ell_P$ and $\rho_0 \sim m_P/\ell_P^3$ (Planck scale).

4. The first-order density depletion from the PG flow, $\epsilon = -2U$, decouples from the temporal metric at the background exponent $\alpha = -1$, so the vacuum around a gravitating mass is gravitationally indistinguishable from free vacuum at the 10^{-12} level even though its density is perturbed at $O(U)$.

The apparent continuity constraint of the macroscopic PG inflow remains an open problem. Of the candidate mechanisms discussed in Sec. 7.2, the quasi-steady reading is removed by the $\mathcal{O}(1)$ buildup rate per dynamical time, and vortex-core absorption with a constant sink rate balances at a single radius; two-fluid counterflow and a spatially structured sink remain to be developed into a full steady-state treatment. A complete resolution may require revisiting the single-field assumption itself. The formal derivation of the macroscopic PG flow as a collective Bernoulli response of N microscopic vortex-Gaussons to the mean-field gravitational potential—the gravitational analogue of the Feynman-Onsager relation in rotating superfluids—is left to future work.

Statements and Declarations

Data and Code Availability

The simulation code for the Painlevé-Gullstrand stability, attractor, and grid-convergence study (Sec. 6, Figure 1) is available at https://github.com/kulangiev/kulangiev.github.io/tree/main/papers/paper_2 under the Creative Commons Attribution 4.0 (CC BY 4.0) licence. The repository contains the one-dimensional spherical Euler solver with distributed sink and the convergence-test driver, together with all numerical parameters, initial conditions, boundary treatments, and post-processing routines used to generate the figure.

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Declarations

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