Review of: "The edge rings of compact graphs"

Audace A.V. Dossou-Olory[1]

1 University of Abomey-Calavi

Potential competing interests: No potential competing interests to declare.

Many studies have been devoted to establishing connections between the combinatorial properties of simple graphs and the algebraic properties of their edge rings. For instance, it is known that the edge ring is a normal domain if and only if the graph satisfies the odd-cycle condition. The authors of the present paper defined a compact graph as a graph that satisfies both the odd-cycle condition and is even cycle-free. Their main results state that for a compact graph G, the projective dimension and Cohen-Macaulay type of the edge ring are both equal to one less the number of the induced cycles in G, and that the regularity of the edge ring coincides with the matching number of the graph derived from G by successively removing all vertices of degree one. In addition, they have determined the top graded Betti numbers of the edge ring.

To my opinion, the results are interesting but the development lacks of details on most the algebraic tools that are being used. Especially, it is usually recommended to support newly defined concepts with examples. The preliminary Section can be expanded further, so as to explain in details the concepts of graded R-module, graded free resolution, functor, etc which are crucial in understanding the graded Betti number. Also, I feel like the main characterization of compacts can be obtained without resorting to these advanced algebraic tools! Overall, the paper is suitable for publication in algebraic graph theory journals.