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Peer Review

Review of: "Quantum Probability for Statisticians: Some New Ideas"

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This is an excellent review article on the author's theory of theoretical variables and how it can be applied to statistical decision theory. I particularly enjoyed reading the section involving replacing conditional probability distributions of a random variable given a parameter by an operator involving the spectral integration of the conditional probability with respect to the spectral measure of the parameter in the quantum context. The author's idea of replacing a maximal set of non-commuting dynamical variables of a quantum system by a maximal set of inaccessible variables and then deriving families of commuting variables (that can be simultaneously measured owing to commutativity) from this maximal set by forming functions of them is a very new idea. The author has also applied his theory to maching learning problems. What I would like to see now is the following: Given a family of non-commuting density matrices in the quantum theory, after assigning apriori probabilities to the occurrence of these densities, we wish to construct a POVM that would minimize the average Bayesian risk involving in deciding which density has occurred based on given cost values involved in making a decision that the ith density has occurred when actually the jth one is true. For the case of two densities, this is the binary quantum hypothesis testing problem and its solution is known. For more than two densities, heavy optimization algorithms require to be written down to determine the optimum POVM. I would like to see whether the author can use his approach in getting a partial solution to this nonlinear optimization problem. The second aspect is the author's treatment of decision theory using a semiclassical approach, ie, one dynamical variable, the parameter, is defined by its spectral measure while the other observed dynamical variable has a spectral measure defined by applying the spectral theorem to to the integral of the conditional probability with respect to the spectral measure of the parameter. This would then imply that the two spectral measures, namely that of the parameter and that of the observation mutually commute. Since they commute, it would reduce to a classical likelihood problem. The real essence of quantum theory that would have no

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analogue in the classical context would be when one can speak of a conditional density of the measurement given the parameter observable when the two do not commute. Such a density cannot be constructed in general but maybe one can use the theory of the Wigner distribution to construct approximate conditional quantum densities and make an analysis of the quantum corrections involved in computing quantities like the Cramer Rao lower bound. Finally, I would like to remark that in the Hudson-Parthasarathy theory of quantum stochastic processes with values in the space of operators acting in Boson Fock space, one constructs processes that behave in different coherent states like classical Brownian motion and classical Poisson processes. In general coherent states, such processes behave like Levy processes, namely superpositions of classical Brownian motion and classical Poisson processes. In some other coherent states, these processes behave as noncommutative Levy processes that have no classical analogue. My question is that can

the author generalize his decision theory to stochastic processes at least in discrete time ?

For example, if a stochastic process has noncommutative samples, we measure it at successive times, with each measurement causing a state collapse on which the next measurement is made. I also enjoyed reading through the initial sections of the author's work where he talks about POV measures defined by a positive operator valued measure on a Borel sigma field and a density operator. It is a well-known theorem due to Naimark that POVMs can be replaced by PVMs by enlarging the Hilbert space and allowing a unitary operator to act on the tensor product between the PVM and the identity, followed by a partial trace of this multiplied by an auxiliary density operator. If then, all POVMs can be derived from PVMs on enlarged Hilbert spaces, why do we use POVMs at all?

In summary, I feel that the author's contributions are very good, but I would like to see how stochastic processes can be included in the quantization framework. One point that many physicists, including the legendary Roger Penrose, have made is that quantum theory is an incomplete theory because dynamics being unitary is reversible, but to make any sense of the dynamics, we require a measuring apparatus which is classical, and the process of measurement is irreversible. The author's idea of theoretical variables may be able to partially sort out this paradox by replacing classical measuring apparatuses with the actor's mind, where non-commutative, i.e., inaccessible variables live, but during the process of communication between two actors, only accessible functions of these variables are involved.

As regards how the author's ideas can be used to solve the quantum m-ary hypothesis problem, we could replace POVMs for decision by measurement of accessible variables and construct decisions based on the outcomes of this measurement.

As regards non-commutative processes, we consider the set of all time samples of the process as a maximal set of inaccessible variables and the individual samples of the process as accessible variables.

Consider the canonical position and momentum operators for a one-dimensional quantum particle. We form the Wigner distribution of this pair using the classical Fourier transform. We can then construct the conditional Wigner distribution of position given momentum using this Wigner distribution and carry out the maximum likelihood algorithm for extremizing this complex conditional density of position given momentum. Complex values of the resulting position estimate are then made real by taking the real part.

Declarations

Potential competing interests: No potential competing interests to declare.