

Research Article

Poisson Bracket Formulation of Spacetime Continuity in Sagnac-Type Effects: Implications for Simultaneity, ADM Gravity, and Quantum Field Theory

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The conventionalist thesis asserts that all internal clock synchronization procedures are equivalent to Einstein's, making the one-way speed of light conventional and the Lorentz transformations (LT) and Lorentz transformations with absolute simultaneity (LTA) empirically equivalent. This paper investigates these claims through a novel canonical phase-space analysis of Sagnac-type optical effects. Using the Poisson bracket as a consistency check, we show that treating the Mansouri-Sexl synchronization parameter ϵ as a free gauge variable is incompatible with a phase-space description of light propagation on non-simply connected topologies. The bracket $\{2L, c'\}$ vanishes—restoring commutativity between loop geometry and local light speed—only if the conventional parameter is fixed and vanishes ($\epsilon = 0$), corresponding to the LTA; for the LT ($\epsilon = v$), the bracket is nonzero, indicating that $2L$ and c' are not independent observables and that spacetime continuity is breached. The promotion of ϵ to a conjugate momentum is introduced here as a test hypothesis to examine the internal consistency of the conventionalist framework; it is not assumed to be an established Hamiltonian result. This does not mean that nature "forces" $\epsilon = 0$, but rather that the conventionalist assumption that ϵ is a free parameter is internally inconsistent for Sagnac-type effects. The inconsistency is resolved by adopting a synchronization procedure not equivalent to Einstein's—such as the classical entanglement synchronization described in Ref. ^[1]—which naturally corresponds to $\epsilon = 0$ and renders the one-way speed of light observable in principle. We also present a heuristic extension to the Arnowitt-Deser-Misner (ADM) formulation of general relativity, suggesting that the shift vector field—the GR analog of ϵ —may lose its gauge freedom on non-simply connected topologies. A speculative quantization of the phase-space commutator yields an uncertainty relation

$\Delta(2L) \cdot \Delta c' \gtrsim (\hbar/2)|c/(c+v)|$, implying that imposing an invariant coordinate light speed ($\Delta c' \rightarrow 0$) forces divergent spatial fluctuations ($\Delta(2L) \rightarrow \infty$). Tentative remarks on topological vacuum anomalies in quantum field theory suggest that the LTA may eliminate the coordinate time dislocation on closed loops. We further examine Faraday induction from moving electrified bodies and equilibrium paradoxes in relativistic electromagnetism, showing that the LTA resolve these paradoxes naturally. Our analysis indicates that the claimed LT-LTA equivalence is not general, and that experimental investigations of the one-way speed of light may be feasible.

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1. Introduction

A physical theory evolves and is modified or even abandoned when one of its basic postulates is invalidated by experiment. The primary goal of this article is to reassess the status of Special Relativity (SR) in light of recent theoretical and experimental advances. We assess the merits and limitations of the Lorentz and alternative relativistic transformations, address paradoxes, and examine experimental interpretations and their impact on modern physics.

Following the null Michelson–Morley experiment (1887), various tests have probed light-speed invariance. Optical experiments—including the Sagnac effect (1913)^[2], the Michelson–Gale experiment (1925)^[3], the linear Wang et al. effect (2005)^[4],^[5], and Earth’s rotation measurements by Schreiber (2014)^[6]—show non-null results often interpreted as favoring light-speed variance. These are generally viewed as Sagnac-type effects^{[7][8]}.

Theoretical developments include reevaluating SR’s second postulate and the rise of “conventionalism” through the generalized relativistic transformations of Mansouri and Sexl (1977)^[9],^{[1][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27][28][29][30][31][32][33][34][35][36][37][38][39][40][41][42][43][44][45][46][47][48][49][50][51][52][53]}. This debate, initiated by Sagnac, has been advanced by Selleri^{[13][14][15]}, Gift^{[16][17]}, Kipreos^{[18][19][20]}, Lundberg^{[21][22]}, Field^{[23][24]}, Spavieri et al.^{[1][7][8][25][26][27][28][29][30][31][32]}, and Engelhardt^[33]. Recent contributions include Spavieri and Haug’s analysis of the reciprocal linear Sagnac effect (2023)^[31] and their examination of light propagation in the linear Sagnac effect (2019)^[25]. The classical entanglement synchronization procedure, central to this work, was introduced in the

International Journal of Theoretical Physics (2025)^[1]. Related work on optical effects with relativistic transformations has appeared in Open Physics (2025)^[7] and Heliyon (2023)^[8].

Conventionalism asserts that only the two-way speed of light c is observable, not the one-way speed^{[1][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27][28][29][30][31][32][33][34][35][36][37][38][39][40][41][42][43][44][45][46][47]}. Thus, transformations based on different but arbitrary synchronization procedures—such as the LT (based on relative simultaneity) and LTA (based on absolute simultaneity)—are claimed to be empirically equivalent. Then, conventionalism may adopt the LTA to “solve” the paradoxes or controversial difficulties that arise if the LT are used. Our aim is not to challenge SR’s extensively tested experimental foundation but to examine the limits of the equivalence assumed by conventionalism. We identify scenarios—loop-interferometric phenomena, the reciprocal linear Sagnac effect, Faraday induction from moving electrified bodies, and equilibrium paradoxes—where the LT and LTA diverge.

By including in our approach Poisson’s commutativity and possible connections to ADM gravity and quantum field theories, we extend our research to determine whether conventionalist equivalence holds generally or whether experimental discrimination is possible.

The present work is structured as an exploratory test-model comparison. We separate (i) established algebraic results derived from the Mansouri-Sexl transformations, (ii) interpretive assumptions (such as the promotion of the synchronization parameter to a phase-space variable), and (iii) heuristic hypotheses (the ADM and QFT extensions). This separation is intended to make the logical status of each claim transparent. In particular, we emphasize that the “spacetime continuity breach” predicted by the LT on closed contours is a physical prediction of the LT framework, not a coordinate-dependent artifact: the conventionalist dismissal of this discontinuity presupposes the empirical equivalence of the LT and LTA, which is precisely what the present work calls into question. Where our conclusions depart from the standard account of Sagnac-type effects, we present them explicitly as our interpretation and contrast them with the conventionalist reading, so that readers can situate the argument within the existing debate. We note that this interpretation is not held by the present authors alone: it aligns with the position sustained over more than a century by a substantial body of physicists who, on the basis of published theoretical and experimental work on Sagnac-type optical effects, have argued that the evidence indicates a non-invariant one-way light speed (Refs.^{[1][2][4][5][7][8][13][14][15][16][17][18][19][20][21][22][23][24][31][33]}).

The background discussion in Section 2 and parts of the Discussion build on, and partly reproduce, material from our earlier preprint^[34].

2. Revision of the Controversy: One-Way Light Speed Invariance

This section reviews results previously discussed in several articles. The emerging scenario is complex but necessary to introduce here, as it provides the foundation for understanding the role of Poisson commutativity within the relativistic context, which is examined in the next section. For the reader's convenience, we summarize below the key findings and the main arguments for and against conventionalism.

The Foundations: Einstein and Conventionalism vs. conservation of simultaneity.

- **Standard SR (1905):** Assumes light propagates at a constant one-way speed c in all inertial frames via Lorentz Transformations (LT).
- **Einstein Synchronization:** For clocks A and B separated by L , Einstein sets the one-way speed equal to the average round-trip speed: $c = 2L/T$. Clock B is set to $t = L/c$ upon signal arrival.
- **Conventionalism in Modern Mainstream View:** Many argue that the one-way light speed is conventional since only the round-trip speed is directly observable^[11]. Some favor coordinate transformations based on conservation of simultaneity.

Generalized Transformations (Mansouri & Sexl, 1977^[9]). To accommodate different synchronization choices, the arbitrary parameter ε is introduced:

$$\begin{aligned} t' &= \frac{t}{\gamma} - \frac{\varepsilon x'}{c^2} = \gamma \left[t \left(1 + \frac{\varepsilon v}{c^2} - \frac{v^2}{c^2} \right) - \frac{\varepsilon x}{c^2} \right] \\ x' &= \gamma(x - vt) \quad y' = y \quad z' = z \\ c' &= \frac{dx'}{dt'} = \frac{c}{1 + \frac{v}{c} - \frac{\varepsilon}{c}} \end{aligned} \tag{1}$$

Transformation	Parameter	Simultaneity	Light Speed
Lorentz(LT)	$\varepsilon = v$	relative	Invariant ($c' = c$)
LTA	$\varepsilon = 0$	absolute	Variant ($c' \neq c$)

The transformations (1) from S to S' with relative velocity v take different names based on ε . The LTA have been denoted as Tangherlini^[35], Selleri^[13], ALT^{[18][19][20]}, or LTA^{[1][21][22][23][24][25][26][27][28][29][30][31][32][33]}, where A indicates conservation of simultaneity.

The Controversy

1. **The Basis of Conventionalism:** Proponents claim LT and LTA are physically equivalent, differing only by the "arbitrary" parameter ε . They argue that any "internal" synchronization is conditioned by time dilation and length contraction and defaults to Einstein's results. Thus, Sagnac-type experiments indicating light-speed variance reflect a mathematical artifact, not a disproof of the LT.
2. **The Problems of the LT:** Under the LT, synchronizing clocks around a closed loop of length L yields a time jump $\Delta t' = -(vL/c^2)$ upon returning to the start.

Non-Integrability: For integrability, a closed-loop integral must be well-defined. The LT force a coordinate "step" at the closure point, creating a discontinuity. These difficulties appear in Sagnac-type optical effects involving non-simply connected topologies.

Concerning the alleged equivalence, we contend that LT and LTA represent different physical realities. Section 3 lists examples where they predict different observable results.

- **Alternative to Einstein Synchronization:** We use the recently proposed^[1] "preset classical entanglement"-based synchronization's simultaneous events at A and B without moving signals or clocks.

This bypasses time dilation and length contraction errors inherent in standard synchronization and, in principle, allows examining the "true" one-way speed and testing spatial isotropy.

2.1. Light propagation in the optical effects of the Sagnac type

In all Sagnac-type effects, the invariant proper time interval T measured by a clock for a counterpropagating photon leaving and returning after covering contour length $2\gamma L$ is:

$$T = \frac{2\gamma L(1 - v/c)}{c} = \frac{2\gamma L}{\gamma^2(c + v)}, \quad (2)$$

where c is the one-way speed in inertial frame S and v is the device velocity relative to S . T is independent of the clock's initial position and valid even when the clock changes velocity direction.

Result (2) is derived within a single inertial frame assuming one-way speed c , consistent with two-way Einstein synchronization. In the absence of evidence for anisotropy, the choice of the "preferred" frame is arbitrary. What differs between LT and LTA is the internal consistency when assessing local light speed along a closed contour with moving sections.

A schematic derivation for the linear Sagnac-Wang effect is in the Appendix.

The analysis results, discussed in the Appendix and summarized below, indicate:

1. Under the LT, the photon at local speed c in the return trip covers only a portion of the fiber, leaving a "missing" section of length $c\delta t' \simeq 2\gamma(v/c)L$ that must be covered in the past ($t' < 0$). This represents a spacetime continuity breach within the time interval measurement $\Delta t'$ for $t' > 0$. The missing section is the dotted line KB in Fig. 1-b.
2. Under the LTA, with local light speed $c' = \gamma^2(c + w)$, the photon covers the entire fiber length $2\gamma L$ in the observed interval T , preserving spacetime continuity.

2.2. Physical Situations where Conventionalism does not Hold

The conventionalist claims^{[9][10][11][48]}:

- Any internal synchronization is equivalent to Einstein's;
- Synchronization is arbitrary;
- The one-way speed of light is conventional; thus, LT and LTA are equivalent. Being equivalent to Einstein's, any internal synchronization yields local one-way speed c , and Sagnac-type experiments do not disprove the LT.

Our response:

a) Classical entanglement-based synchronization

A key element is a synchronization procedure based on a uniformly rotating rod developed in Ref.^[1]. A rod of length L is initially at rest in an inertial frame. Two marked points, A^* and B^* , at the rod's ends remain in phase during uniform rotation, crossing any transverse axis simultaneously. This built-in phase relationship can synchronize clocks at the rod's endpoints without exchanging signals.

This procedure is operationally independent of Einstein synchronization. Unlike Einstein synchronization—which imposes one-way light speed invariance by fiat—the rotating rod

synchronization can be applied separately in any inertial frame. The synchronization relationship between frames is not postulated but determined experimentally.

If this procedure in frame S yields the one-way speed c , then in a moving frame S' it will yield either $c' = c$ (Lorentz invariance) or $c' \simeq c \pm v$ (LTA variable speed). The outcome thus tests whether the one-way light speed is invariant or frame-dependent.

Because its preset instantaneous synchrony can be associated with the LTA, the rotating rod synchronization shows that not every internal synchronization is equivalent to Einstein's. The conventionalist claim thus rests on an unexamined presupposition.

b) Where and why the LT are considered to present difficulties

The difficulty of Einstein synchronization along closed moving contours has been discussed by several authors^{[1][2][13][14][15][16][17][18][19][21][22][23][24][25][26][27][28][29][30][31][32][33][36][37]}. Klauber^[36] showed that applying it to a closed moving contour amounts to a clock being out of synchronization with itself. Moreover, the Sagnac results favoring LTA over LT are not due to rotating frames being non-inertial—as also noted by Engelhardt^[33]. The difficulties emerge even with strictly inertial systems, as in the linear Wang-Sagnac effect (Wang et al.^{[4][5]}).

The problem emerging with Einstein synchronization when applied along a closed contour was noted over 50 years ago by Landau and Lifshitz^[38]:

“. . . However, synchronization of clocks along a closed contour turns out to be impossible in general. In fact, starting out along the contour and returning to the initial point, we would obtain for dx^0 a value different from zero . . .”

Instead, the LTA allows synchronizing all clocks without such difficulties. Thus, as shown in the linear effect in Fig. 1 in the Appendix, spacetime continuity for a photon covering the total fiber length $2\gamma L$ in interval T supports conservation of simultaneity (LTA) over relative simultaneity (LT).

c) Do the LT and LTA possess the same symmetry?

Symmetry provides an important theoretical argument that relative simultaneity (LT) is not generally interchangeable with absolute simultaneity (LTA). The Thomas-Wigner rotation occurs whenever non-collinear Lorentz transformations are composed. Jackson^[39] applies successive Lorentz transformations in deriving Thomas precession. In Ref.^[31], Spavieri and Haug consider the different symmetries of relative and absolute simultaneity and, following Jackson's derivation using the LTA, show that LT and

LTA yield different results. This confirms they are not generally physically equivalent: the LT form a Lie group of symmetries of Minkowski spacetime, while the LTA form a different symmetry group^[14].

d) The reciprocal linear Wang-Sagnac effect

In the standard linear (Wang-Sagnac) effect, the measuring device moves back and forth along a stationary contour. In the reciprocal effect, the device is stationary and the contour moves. The analysis by Spavieri and Haug^{[8][31]} indicates that LT and LTA predict different results for the invariant round-trip observable T . If X is the initial distance of clock C^* from pulley A, T is independent of X in the standard effect. However, for the reciprocal effect, they find $T = T(X)$ is X -dependent for the LT, while T is invariant and X -independent for the LTA. Thus, the two transformations are not equivalent and represent divergent physical frameworks.

There exists a vast class of phenomena for which the relativity principle and LT are compatible, and LT and LTA are equivalent. Notably, in the reciprocal linear Sagnac effect, reciprocity is preserved under the LTA but not under the LT. Reciprocity—the symmetry that exchanging "moving" and "stationary" frames should yield equivalent descriptions—is fundamental to the relativity principle. The fact that the LT breaks reciprocity while the LTA preserves it indicates a conceptual advantage of the LTA.

e) The predictions of the LT and LTA for the law of induction

An examination of moving charged bodies reveals a clear distinction between LT and LTA predictions. Under the LT, an electrified body at rest in frame S , when observed from moving frame S' , produces a time-varying magnetic field due to space-time mixing in $t' = \gamma(t - vx/c^2)$. The electrostatic field $\mathbf{E}$ ($\nabla \times \mathbf{E} = 0$ in S) acquires a non-zero curl in S' , yielding $\nabla' \times \mathbf{E}' = -c^{-1} \partial_t \mathbf{B}' \neq 0$, predicting an observable electromotive force and induced current in a closed conducting wire at rest in S' .

By contrast, under the LTA, $t' = t/\gamma$ has no space-time mixing, preserving simultaneity. Puccini^[40] and Bonaura^[41] derived electromagnetic field transformations under LTA from Maxwell's equations. For an electrified body at rest in S , their results yield $\nabla' \times \mathbf{E}' = 0$, so there is no magnetic induction, and Faraday's law predicts no induced *emf* or current in S' . A detailed derivation is in Appendix B.

f) The predictions of the LT and LTA for the equilibrium paradoxes

"Paradoxes" of equilibrium date back to the Tolman-Lewis right-angle lever paradox^[43]. It consists of a lever in equilibrium under external forces in its rest frame S . In a moving frame S' , the force

transformation introduces an extra non-vanishing external torque, making the lever seem forced to rotate.

One viable solution was developed by Cavalleri et al.^[44] in 1979. They recognized that the standard approach using the Maxwell tensor $\Theta^{\mu\nu}$ does not solve equilibrium paradoxes: Maxwell's theory is incomplete^[44]. However, using the total stress-energy tensor $T^{\mu\nu} = \Theta^{\mu\nu} + S^{\mu\nu}$ and the continuity equation $\partial_\nu T^{\mu\nu} = 0$ reflecting equilibrium, they showed the paradoxes are solved within standard SR if Maxwell's electromagnetic tensor $\Theta^{\mu\nu}$ is complemented with the stress tensor $S^{\mu\nu}$. The internal stresses now balance the external torque in S' , and no net torque acts.

However, following Bonaure^[41] and Puccini^[40], using LTA instead of LT, the force transformation introduces no extra external torque, and the lever remains naturally in equilibrium in every inertial frame without the need for $S^{\mu\nu}$. Consequently, with LTA, relativistic electromagnetism is complete and there are no equilibrium paradoxes^[17].

Conclusions of this Section:

Conventionalism's equivalence does not hold generally, and the LTA are not interchangeable with the LT because they represent distinct ontological descriptions.

3. Canonical Non-Commutativity and the Failure of Conventionalist Gauge Freedom on Closed Contours

To examine the foundational validity of the conventionalist thesis along non-simply connected topologies, we construct a classical phase-space framework as a consistency check. Conventionalist narratives (such as those by Reichenbach and Grünbaum) assert that the synchronization parameter ϵ is a completely free, unconstrained gauge choice. We test this assumption by asking:

Is a phase-space description of the Sagnac effects consistent for arbitrary ϵ ?

3.1. Poisson Bracket Evaluation on an Open Track Contour

Let us evaluate the explicit Mansouri-Sexl formulation regulating the coordinate one-way speed of light c' for a counterpropagating wavefront inside both linear and circular Sagnac loops:

$$c'(v, \epsilon) = \frac{ds'}{dt'} = \frac{c}{1 - \frac{v}{c} + \frac{\epsilon}{c}}, \quad (3)$$

where s' represents the curvilinear path length $2L = \int ds' = \int c'(\epsilon) dt'$ of an optical fiber (index of refraction $n = 1$) traversed by light.

For the purpose of this consistency check, we treat the spatial path length as the independent canonical coordinate $q = 2L = s'$ and the Mansouri-Sexl parameter as the conjugate momentum $p = \epsilon$. We emphasize that this is an assumption: we are testing whether such a phase-space description is consistent for arbitrary ϵ . For an open contour we set:

- **Generalized Coordinate**

$$\mathbf{q} = 2\mathbf{L} = \mathbf{c}'(\epsilon)\mathbf{T}(\epsilon), \quad (4)$$

the spatial length of the open track contour, a basic kinematic relationship representing the physical metric constraint of the system geometry.

- **Generalized Conjugate Momentum**

$$\mathbf{p} = \epsilon, \quad (5)$$

the Mansouri-Sexl directional synchronization parameter, acting as the infinitesimal generator of translations in the local coordinate time slices across space.

A priori we do not know whether a physical quantity depends on ϵ . However, we may apply Poisson's approach to check whether the ϵ -dependence is meaningful. Assuming that v and ϵ are constant along s' , we have $q = s' = c'(\epsilon)T(\epsilon)$ and the directional coordinate speed of light is functionalized through the open track boundary condition as:

$$c'(q, \epsilon) = \frac{q}{T(\epsilon)}, \quad (6)$$

where $T(\epsilon)$ represents the synchronization-dependent propagation time interval along the segment measured by local clocks co-moving with the optical fiber. In principle, the quantity $q = s'$ is observable and independent of ϵ . In fact, as a classical entity, a photon starting from $s' = 0$ can be seen to reach the position corresponding to the length s' traversed, which can be measured with meter sticks placed locally along s' . Yet, the interval $T = T(\epsilon)$ measured by spatially separated clocks is arbitrary as the clock synchronization depends on ϵ . Then, the path $2L = c'(\epsilon)T(\epsilon)$ can be traversed at arbitrarily different one-way light speeds depending on the procedure adopted to synchronize the spatially separated clocks that yield the interval $T(\epsilon)$.

These assumptions allow us to apply our classical phase-space framework to the optical effects of the Sagnac type considered in this paper.

To evaluate the dynamic compatibility of these observables, we expand the Poisson bracket $\{2L, c'\}$ via the canonical chain rule:

$$\{2L, c'\} = \frac{\partial(2L)}{\partial q} \frac{\partial c'}{\partial p} - \frac{\partial(2L)}{\partial p} \frac{\partial c'}{\partial q}. \quad (7)$$

Since $2L$ represents the rigid coordinate q , its phase-space gradients satisfy $\partial(2L)/\partial q = 1$ and $\partial(2L)/\partial p = 0$. The expression simplifies, yielding the final relation:

$$\{2L, c'\} = \frac{\partial c'}{\partial \epsilon} = \frac{\partial}{\partial \epsilon} \left(\frac{q}{T(\epsilon)} \right) = -\frac{2L}{T(\epsilon)^2} \frac{dT(\epsilon)}{d\epsilon}. \quad (8)$$

3.2. The Inconsistency of Conventionalist Gauge Freedom

The derived relation proves that the classical non-commutativity between the metric length and the speed of light is entirely conditional. The Poisson bracket vanishes identically ($\{2L, c'\} = 0$), restoring algebraic compatibility between q and c' as independent observables, if and only if:

$$\frac{dT(\epsilon)}{d\epsilon} = 0. \quad (9)$$

This condition requires the total elapsed propagation interval T along the open track to be invariant with respect to changes in the coordinate clock synchronization convention. Consequently, an open contour admits a phase-space description with q and c' as independent, commuting observables if and only if the physical transit time is decoupled from the choice of the gauge parameter ϵ .

From the Mansouri-Sexl time transformation (1), the propagation interval $T = \Delta t'$ along the moving optical fiber is related to ϵ by:

$$t' = \frac{t}{\gamma} - \frac{\epsilon x'}{c^2} \Rightarrow T(\epsilon) = \Delta t' = \frac{\Delta t}{\gamma} - \frac{\epsilon \Delta x'}{c^2}. \quad (10)$$

Hence:

$$\frac{dT(\epsilon)}{d\epsilon} = -\frac{\Delta x'}{c^2} \neq 0, \quad (11)$$

since the photon traverses a nonzero path length $\Delta x' \neq 0$.

Accordingly, with Einstein synchronization and for any value of ϵ , we are met with the obstruction (11) that prevents the commutativity $\{2L, c'\} = 0$.

This is the crucial point: The condition $dT(\epsilon)/d\epsilon = 0$ is incompatible with (11) unless the space-time mixing term is absent from the time transformation. This suggests that the meaningful expression of T in (10) should be independent of ϵ , requiring that the coefficient of $\Delta x'$ in (10) vanishes, which corresponds to:

$$\epsilon = 0, \tag{12}$$

a condition that cannot be derived from (11).

Physical status of the discontinuity. We emphasize that the “continuity breach” predicted by the LT is not a coordinate artifact in the sense claimed by conventionalism. Within conventionalism, the LT and LTA are assumed to be empirically equivalent, so the missing section $c\delta t'$ can be dismissed as a non-observable theoretical artifact removable by a change of synchronization convention. That dismissal, however, presupposes that the one-way speed of light is unobservable and that the LTA can always be substituted for the LT. As shown in Section 2 and in Refs. [\[1\]](#), [\[31\]](#), [\[8\]](#), this presupposition fails: the classical entanglement synchronization procedure provides a non-Einstein synchronization in which the one-way speed is, in principle, observable, and the LT and LTA are not equivalent but represent different physical realities. Consequently, the discontinuity implied by the LT on closed contours—the impossibility of covering the full contour $2\gamma L_0$ at the invariant local speed c in the observed interval T —is interpreted here as a physical prediction of the LT framework, not as a coordinate-dependent description that can be removed at will. Under the LTA, by contrast, no such discontinuity arises: the photon covers the entire contour at the local speed $c' = \gamma^2(c + v)$. The two frameworks therefore differ in their physical predictions, and the LT’s discontinuity is a genuine feature of the description based on relative simultaneity. In the standard account of Sagnac-type effects, the discontinuity is regarded as a coordinate artifact removable by a change of synchronization convention, and the LT and LTA are taken to be empirically equivalent. Departing from that standard account, with our interpretation the proper analysis treats the one-way speed of light as observable in principle (via the classical entanglement synchronization of Ref. [\[1\]](#)), which removes the premise on which the standard dismissal rests. Readers may therefore situate our conclusion within the broader debate on the conventionality of simultaneity. As in the Introduction, we note that this is not a minority position held only by the present authors: it is supported by the substantial body of physicists who, since Sagnac, have argued that Sagnac-type optical effects indicate a non-invariant one-way light speed (Refs. [\[1\]](#)[\[2\]](#)[\[4\]](#)[\[5\]](#)[\[7\]](#)[\[8\]](#)[\[13\]](#)[\[14\]](#)[\[15\]](#)[\[16\]](#)[\[17\]](#)[\[18\]](#)[\[19\]](#)[\[20\]](#)[\[21\]](#)[\[22\]](#)[\[23\]](#)[\[24\]](#)[\[31\]](#)[\[33\]](#)).

The logical status of this conclusion:

The Poisson bracket analysis with the result (11) does not "force" $\epsilon = 0$ as a physical requirement. Nevertheless, it reveals that the conventionalist assumption—that the quantities $c'(\epsilon)$ and $T(\epsilon)$ are functions of the free gauge parameter ϵ —is incompatible with a phase-space description of optical effects on non-simply connected topologies. To remove the obstruction, we need to justify the choice of the only value of ϵ for which the bracket vanishes: the value $\epsilon = 0$ where the quantities c' and T become independent of ϵ .

We stress that $\epsilon = 0$ is not derived from physical first principles; it is the unique value that removes the algebraic obstruction within the proposed phase-space formulation. The physical interpretation of this result—that the one-way speed becomes observable under a non-Einstein synchronization—is an interpretive step supported by the classical entanglement procedure of Ref.^[1].

Our justification is as follows:

Physically, conventionalism is based on Einstein synchronization, where the constant c represents the average two-way light speed from A to B and back to A. The one-way speed from A to B, or from B to A, is indeterminate ($c' = c'(\epsilon)$) and ϵ is a variable. However, the inconsistency we have met with Poisson's commutativity suggests that ϵ is a fixed parameter. The analysis made indicates an incompatibility between unrestricted ϵ -freedom and the imposed closed-contour continuity condition. The solution to this inconsistency is to go beyond conventionalism and Einstein synchronization, and pass to a different scenario. What Poisson's commutativity requires is that, with $\epsilon = 0$, the c' one-way speed is observable. This is possible if, at least in principle, we can adopt a synchronization procedure not equivalent to that of Einstein, e.g., the classical entangled synchronization of Ref.^[1].

Hence, within standard SR based on two-way Einstein synchronization, conventionalism with $c' = c'(\epsilon, v)$ is valid even though it is met with the Poisson commutativity obstruction. By setting $\epsilon = 0$ and removing the obstruction, we are implicitly assuming that the one-way $c' = c'(v)$ is observable as predicted by the classical entangled synchronization. The condition $\epsilon = 0$ corresponds to the LTA time transformation $t' = t/\gamma$, which eliminates the spacetime mixing responsible for the non-commutativity. With $\epsilon = 0$, the time interval T along the moving path becomes independent of the synchronization convention, and the Poisson bracket $\{2L, c'\}$ vanishes identically. Thus, within this phase-space formulation, consistency is obtained for $\epsilon = 0$, corresponding to absolute simultaneity. The choice $\epsilon = 0$ is confirmed by determining ϵ in the case of closed contours, as shown below.

3.3. The special case of the closed-loop Sagnac effect

For light propagating along the closed-loop Sagnac effect, the proper time interval T is measured by a single clock that needs no synchronization. We wish to check the implications due to ϵ if we set:

$$q = 2L = c'(\epsilon)T. \quad (13)$$

Since, from Eq. (13), $c' = q/T$ and the ratio q/T is independent of ϵ , we must have:

$$c'(v, \epsilon) \Rightarrow c'(v), \quad (14)$$

which is consistent with the result that, in the phase-space description, commutativity requires $\epsilon = 0$ (Eq. 12). Then, the Poisson commutativity condition of Eq. (8) gives:

$$\{2L, c'\} = \frac{\partial c'}{\partial \epsilon} = \frac{\partial}{\partial \epsilon} \left(\frac{q}{T} \right) = 0. \quad (15)$$

We turn our attention to the condition of spacetime continuity together with the invariant proper time interval.

The closure condition of spacetime continuity requires:

$$2L = \gamma 2L_0 \simeq 2L_0, \quad (16)$$

where $2L_0$ is the closed contour measured in the lab frame ($\gamma 2L_0$ is the closed contour measured along the moving optical fiber).

Moreover, according to Eq. (2), for the Sagnac effect, the invariant proper time interval T is:

$$T = 2\gamma L_0 / [\gamma^2(c + v)]. \quad (17)$$

Then, on account of Eqs. (13), (16), and (17), we are met with the consequences of the constraint:

$$2L = c'(\epsilon) \frac{2L_0}{\gamma(c + v)}. \quad (18)$$

Nonconservation of spacetime continuity:

If, with the LT, we assume $\epsilon = v$ and $c'(\epsilon) = c$ invariant, according to (18):

$$2L = c \frac{\gamma 2L_0}{\gamma^2(c + v)} = \gamma 2L_0 (1 - v/c) < 2\gamma L_0, \quad (19)$$

inducing the topological continuity breach discussed in Section 3 because, at the local speed c , the propagating photon cannot cover the whole closed contour $2\gamma L_0$ in the observed interval T [\[1\]\[7\]\[8\]\[16\]\[18\]\[19\]\[21\]\[22\]\[25\]\[26\]\[27\]\[28\]\[29\]\[30\]\[31\]\[32\]\[46\]](#).

Conservation of spacetime continuity:

If, instead, we assume spacetime continuity (16), according to (18):

$$\begin{aligned}
 2L &= \frac{c}{1 - \frac{v}{c} + \frac{\epsilon}{c}} \frac{\gamma^2 L_0}{\gamma^2 c(1 + v/c)} = 2\gamma L_0 \\
 1 &= \frac{c}{1 - \frac{v}{c} + \frac{\epsilon}{c}} \frac{(1 + v/c)(1 - v/c)}{c(1 + v/c)} \\
 1 &= \frac{(1 - v/c)}{1 - \frac{v}{c} + \frac{\epsilon}{c}} \Rightarrow \epsilon = 0
 \end{aligned} \tag{20}$$

in agreement with Eq. (12).

Hence, to conserve spacetime continuity, we must have conservation of simultaneity and $\epsilon = 0$, as implied with the use of the LTA. In this case, for a counterpropagating photon, the variant light speed is:

$$c'(\epsilon = 0) = \frac{c}{1 - \frac{v}{c}} = \gamma^2(c + v). \tag{21}$$

Note that, within conventionalism, any internal synchronization procedure defaults to Einstein synchronization. However, in general, the local differential one-way light speed $dx'/dt' = c'(\epsilon)$ is no longer necessarily the invariant $c' = c$ obtained with Einstein synchronization, because we may adopt the non-equivalent classical entanglement synchronization described in Ref.^[1]. The conventionalist thesis is that—by virtue of the alleged LT-LTA equivalence—we can meet the difficulties inherent to the LT by adopting the LTA to interpret the Sagnac optical effects^{[9][10][11][12][36][47][48][49]}. However, this thesis is untenable: The difficulties inherent to the LT cannot be solved with the LTA because—as shown in Section 3—the LT and LTA represent different physical realities.

Summary of the Poisson Bracket Analysis

The Poisson bracket analysis establishes the following:

1. The conventionalist assumption that ϵ is a free, unconstrained gauge parameter is incompatible with a phase-space description of Sagnac-type effects on non-simply connected topologies.
2. The phase-space description leads to Poisson's commutativity only at $\epsilon = 0$ (the LTA) in the context of a synchronization procedure not equivalent to that of Einstein.
3. This does not mean that nature "forces" $\epsilon = 0$; rather, it means that if one wishes to maintain a Hamiltonian/phase-space description of the phenomenon in the context of Einstein synchronization and the LT, the conventionalist freedom in choosing ϵ is topologically constrained.
4. Therefore, the claimed LT-LTA equivalence is not general: it fails for non-simply connected topologies where global loop closure imposes continuity conditions that the LT's relative simultaneity cannot satisfy without introducing discontinuities.

Mathematically, the approach presented above based on Poisson's commutativity proves that treating ϵ as a free, unconstrained gauge variable while simultaneously demanding a continuous, closed spatial manifold is a category mistake.

4. Exploratory Framework: Extensions to Canonical Gravity and Field Theory

Disclaimer: The analytical extensions presented in this section serve as preliminary conceptual mappings intended to stimulate future theoretical research. The underlying arguments are heuristic and structured to explore potential structural interfaces between the classical phase-space constraints derived above and more comprehensive physical frameworks; they should not be interpreted as rigorous, complete proofs. These extensions are included to suggest possible structural analogies, not necessarily to establish results. They should be regarded as hypotheses for future investigation.

The non-vanishing Poisson bracket established in Section 4 exposes a fundamental algebraic incompatibility when treating the synchronization parameter ϵ as an unconstrained phase-space gauge choice over non-simply connected topologies. The spatial loop boundary ($2L$) and the directional coordinate speed of light (c') exhibit non-trivial algebraic coupling, indicating that global topology fundamentally restricts coordinate gauge freedom over closed trajectories. To investigate whether this topological constraint reflects intrinsic properties of the gravitational fabric and quantum vacuum fields, we extend this non-commutative framework to the Arnowitt-Deser-Misner (ADM) formulation of General Relativity and Quantum Field Theory (QFT).

4.1. Canonical ADM Analogy and the Shift Vector Field

In the canonical ADM formulation of general relativity, the four-dimensional spacetime metric $g_{\mu\nu}$ is foliated into a family of three-dimensional spatial hypersurfaces Σ_t , yielding the standard line element:

$$ds^2 = -(N^2 - N_i N^i) dt^2 + 2N_i dt dx^i + q_{ij} dx^i dx^j \quad (22)$$

where q_{ij} represents the induced spatial metric, N is the lapse function, and N^i defines the shift vector field. The non-diagonal components $g_{0i} = N_i$ govern the local clock synchronization metric across the spatial slice, acting as the general-relativistic analog to the Mansouri-Sexl parameter ϵ .

In standard canonical gravity, lapse and shift functions are treated as Lagrange multipliers enforcing the Hamiltonian and momentum constraints rather than dynamical variables possessing canonical

conjugate momenta. However, within an extended or parameterized phase-space framework—where lapse and shift are promoted to dynamical entities¹—a macroscopic loop length observable along a closed spatial contour γ , defined by $L[\gamma] = \oint_{\gamma} \sqrt{q_{ij} dx^i dx^j}$, yields a non-vanishing heuristic bracket:

$$\{L[\gamma], N_i(x)\} \sim -\frac{1}{c^2} q_{ij} \frac{dx^j}{ds} \delta^{(3)}(x - x_{\gamma}) \quad (23)$$

This relationship suggests that global topology disrupts the unconstrained gauge freedom traditionally attributed to the shift vector field. Mirroring the classical Sagnac mechanism, maintaining a non-vanishing global shift flux ($\oint_{\gamma} N_i dx^i \neq 0$) introduces coordinate time dislocations that breach macro-continuity. Spacetime continuity is preserved exclusively under a vanishing shift flux configuration ($\oint_{\gamma} N_i dx^i = 0$), mapping directly to the LTA framework where absolute simultaneity restores a single-valued time coordinate over arbitrary spatial manifolds.

4.2. Quantum Commutators and Spatial Uncertainty Relations

Canonical quantization of the extended phase space mapping converts the classical bracket from Eq. (8) into a quantum commutator:

$$[\hat{2L}(\epsilon), \hat{c}'] \sim -i\hbar \frac{(\hat{c}')^3}{c^2(c+v)} \quad (24)$$

Evaluating this commutator at the conventional Einstein synchronization limit ($\epsilon = v$, where $c' = c$) yields the uncertainty relation:

$$\Delta(2L) \cdot \Delta c' \gtrsim \frac{\hbar}{2} \left| \frac{c}{c+v} \right| \quad (25)$$

This expression suggests that imposing a rigid, non-fluctuating coordinate speed of light ($\Delta c' \rightarrow 0$) forces divergent spatial fluctuations in the conjugate loop boundary ($\Delta(2L) \rightarrow \infty$). This quantum fuzziness represents a microscopic manifestation of the Sagnac continuity breach, implying that light-speed invariance and smooth manifold boundaries may become mutually exclusive at the Planck scale.

4.3. Topological Vacuum Anomalies in Field Quantization

In Quantum Field Theory, microcausality mandates that the commutator of two field operators vanishes at spacelike separations: $[\hat{\Phi}(x), \hat{\Phi}(y)] = 0$. On a non-simply connected topology, field operators counterpropagating along a closed loop interact directly with the global boundary conditions.

Under standard Lorentz transformations (LT), an observer traversing a moving closed track encounters a coordinate time dislocation $\Delta t'$ upon loop closure. Attempting to enforce an invariant local light speed over this path introduces a temporal jump that breaks the continuity of the field propagator:

$$\langle 0 | \mathcal{T} \hat{\Phi}(t', \mathbf{x}) \hat{\Phi}(0, \mathbf{x}) | 0 \rangle_{LT} \neq \langle 0 | \mathcal{T} \hat{\Phi}(t' + \Delta t', \mathbf{x}) \hat{\Phi}(0, \mathbf{x}) | 0 \rangle_{LT} \quad (26)$$

This inequality points to a potential topological anomaly in the quantum vacuum expectation value, which conventionally requires artificial counterterms to restore mathematical consistency.

Conversely, the LTA framework eliminates the space-time mixing term ($t' = t/\gamma$), ensuring the time coordinate remains globally single-valued ($\oint dt = 0$). Field operators natively satisfy smooth, periodic boundary conditions $\hat{\Phi}(t', \mathbf{x} + 2\gamma\mathbf{L}) = \hat{\Phi}(t', \mathbf{x})$, preventing topological synchronization anomalies and protecting the global invariance of the quantum vacuum continuum across non-simply connected spatial topologies.

5. Discussion

As discussed in Section 2, one of the merits of conventionalism has been to show that, owing to the claimed equivalence between the Lorentz transformations (LT) and the Lorentz transformations with absolute simultaneity (LTA), most experiments corroborating the LT corroborate the LTA equally well. After analyzing more than 40 basic experiments across different physical scenarios, Kipreos^{[19][20]} has shown that both the LT and LTA provide the same, or equivalent, meaningful interpretations. The only differences arise for optical effects of the Sagnac type^{[19][20]}, where the LTA are favored.

We do not dispute the extensive experimental support for special relativity. Rather, we argue that the claimed LT-LTA equivalence is not general: there exist specific physical scenarios—such as closed-contour experiments, the reciprocal linear Sagnac effect, and Faraday induction from moving electrified bodies—where the two frameworks diverge in their predictions or interpretations. Our canonical phase-space analysis reinforces this conclusion by showing that—within the proposed canonical construction—the consistency condition for spacetime continuity selects $\epsilon = 0$, corresponding to the LTA.

The heuristic extension to ADM gravity, presented with appropriate caveats, suggests a possible deep structural analogy: the shift vector field N_i , which governs local clock synchronization in general relativity, might not be pure gauge on closed spatial contours in an extended phase-space formulation. A heuristic Poisson bracket $\{L[\gamma], N_i(x)\}$ would suggest that loop geometry and the synchronization field might be canonically conjugate. Spacetime continuity would then be preserved only when the global shift

flux vanishes—a condition mapping heuristically to the LTA's absolute simultaneity. This tentatively suggests that the conventionalist freedom in choosing synchronization might be fundamentally topologically constrained.

Quantization of a heuristic phase-space commutator yields a speculative uncertainty relation that might expose an incompatibility between light-speed invariance and smooth manifold boundaries at the quantum scale. Forcing $\Delta c' \rightarrow 0$ would cause divergent spatial fluctuations $\Delta(2L) \rightarrow \infty$, providing a possible quantum signature of the Sagnac continuity breach. In quantum field theory, the LTA framework might prevent topological synchronization anomalies by eliminating the space-time mixing that produces coordinate time dislocations on closed loops. Field operators would satisfy smooth periodic boundary conditions, potentially protecting the invariance of the quantum vacuum across arbitrary topologies. However, we emphasize that these QFT considerations are highly speculative and require substantial further development.

The LTA have applications in electromagnetism, resolving equilibrium paradoxes that have challenged standard relativistic formulations. In this context, the LTA can be tested against the LT in experiments involving Faraday's law of induction applied to electrified bodies in motion.

By utilizing internal synchronization procedures, such as the one based on "classical entanglement" described in Ref.^[1]—which is a priori not equivalent to Einstein synchronization because it does not require physical transport of information—it becomes theoretically possible to test Lorentz invariance and light-speed invariance. Such tests could confirm or refute the contended equivalence between relative and absolute simultaneity.

The primary distinction between the LT and the LTA lies in their divergent light speed predictions in the frame of the measuring clock, as evidenced in the case of the GPS^{[45][46]}, the circular Sagnac effect^[2], the linear Wang-Sagnac effect^{[4][5]}, and particularly the reciprocal linear Sagnac effect^{[8][31]}. The result that in the interpretation of the linear Wang-Sagnac effect the LT imply a spacetime continuity breach that is not reflected by the LTA represents a strong argument favoring the LTA versus the LT. Our Poisson bracket analysis formalizes this result, showing that treating ϵ as a free gauge variable while demanding a continuous closed manifold is mathematically inconsistent.

A broader conceptual implication merits consideration. The Galilean transformations are recognized as having a restricted domain of applicability, valid in the limit of low velocities compared to the speed of light ($v \ll c$), but fundamentally superseded by the Lorentz transformations for the full range of

relativistic phenomena. In an analogous manner, the difficulties encountered by the LT in interpreting light propagation along closed moving contours—particularly the spacetime continuity breach and the failure to preserve reciprocity in the reciprocal linear Sagnac effect^{[8][31]}—suggest that the LT, despite their mathematical consistency and empirical success in simply-connected topologies, may likewise possess a restricted domain of physical applicability. Specifically, the LT appear ill-suited for describing phenomena on non-simply connected manifolds where global loop closure conditions impose topological constraints that the LT's relative simultaneity cannot satisfy without introducing discontinuities. The LTA, by contrast, accommodate such global constraints naturally. Then, a possible implication of the proposed model suggests that the LTA may represent a more general framework whose domain includes, but extends beyond, that of the LT. This perspective recasts the LT-LTA relationship not as a matter of conventional equivalence, but as a possible hierarchy of theoretical validity analogous to the Galilean-Lorentzian relationship.

The heuristic ADM and QFT extensions, speculative as they are, hint that this topological constraint might be an intrinsic feature of spacetime geometry rather than an artifact of coordinate choice. They are presented as directions suggested by the canonical construction rather than as consequences established by it.

Coordinate transformations form a fundamental aspect of any physical model or theory. They should be valid generally, including in the treatment of optical effects. The interpretational challenges of optical effects, the Selleri paradox^[13], and the advances made within the conventionalist discourse are seldom mentioned in papers dealing with the Standard Model—either because they are not widely known or because they are assumed to have been historically resolved. While modern science has made significant progress in prominent mainstream research, it is important to recognize that the challenges facing the Standard Model extend beyond dark energy and dark matter to include the fundamental viability of the Lorentz transformations and the different symmetry of the LT and LTA.

6. Conclusions

This paper shows that the claimed empirical equivalence between the Lorentz Transformations (LT) and Lorentz Transformations with Absolute Simultaneity (LTA) fails for non-simply connected topologies. While both frameworks yield identical predictions for standard, simply connected experimental configurations, they diverge fundamentally in scenarios governed by global loop closure conditions, such

as Sagnac-type effects, the reciprocal linear Sagnac effect, and Faraday induction from moving electrified bodies.

Our canonical phase-space formulation formalizes this divergence. The analysis indicates that treating the Mansouri-Sexl synchronization parameter ϵ as an unconstrained gauge variable on a continuous, closed spatial manifold is mathematically inconsistent within the proposed canonical construction. Global spacetime continuity requires the conventional parameter ϵ to be fixed at the value $\epsilon = 0$, a value that corresponds to the absolute simultaneity of the LTA. Under this configuration, the coordinate speed of light naturally absorbs the system kinematics ($c' \neq c$), preserving both the continuity of the manifold and the fundamental principle of reciprocity.

Furthermore, exploratory extensions into canonical gravity and quantized field theories indicate that this topological constraint may reflect intrinsic geometric properties of spacetime. The heuristic ADM and QFT mappings suggest that absolute simultaneity naturally prevents coordinate time dislocations and quantum vacuum anomalies on closed topologies, whereas the invariant light speed of the LT introduces spatial uncertainty divergences.

Ultimately, these findings indicate that the LT and LTA should not be viewed as mere conventional variants, but as distinct physical descriptions with different domains of validity. The LTA accommodate global topological constraints that the LT cannot satisfy without introducing structural discontinuities, offering a more comprehensive framework for relativistic phenomena across arbitrary spatial configurations.

The present work should therefore be read as an exploratory test-model comparison: the algebraic results are rigorous, the interpretation within the LTA framework is well-supported by previously published work, and the extensions to gravity and field theory are presented as hypotheses requiring further development. We recall that this interpretation is not held by the present authors alone: it aligns with the substantial body of physicists who, since Sagnac, have argued that Sagnac-type optical effects indicate a non-invariant one-way light speed (Refs. [\[1\]\[2\]\[4\]\[5\]\[7\]\[8\]\[13\]\[14\]\[15\]\[16\]\[17\]\[18\]\[19\]\[20\]\[21\]\[22\]\[23\]\[24\]\[31\]\[33\]](#)).

Appendix A. Interpretation of the Linear Wang-Sagnac Effect

We focus on the linear Sagnac effect of Fig. 1, where an optical fiber (with refractive index $n = 1$) slides at speed v on the two pulleys A and B with $AB = L = L_0$. The clock C^* is fixed to the fiber and moves from the lower to the upper section while a counter-propagating photon performs the round trip in the time

interval T given by (2). C^* is denoted by C'' while on the fiber lower section and by C' while on the upper section. The result (2) is based on the assumption that the one-way light speed is c in the lab frame S where the pulleys are at rest. Because of the symmetry, the lower and upper sections of the optical fiber move at the same speed v relative to S , and the transformations (1) can be applied. The light speed $c'(v, \epsilon)$ along the fiber is given by (3), and Poisson's commutativity can be derived as discussed in Section 4.

Alternatively, we can assume that the frame where the one-way light speed is c is the frame S'' co-moving with the lower fiber section. The advantage is that only two reference frames are involved, and we can apply the transformations (1) directly between frame S'' and the frame S' co-moving with the fiber upper section at the relative speed $w \simeq 2v$.

Considering that the detailed derivation is given in several articles^{[1][7][8][25][26][31]}, we present here schematically the results obtained using the LT and LTA and show where the spacetime continuity breach occurs if the LT based on relative simultaneity are adopted.

We determine the fiber distance covered by the photon traveling at the corresponding local speed in the time interval T given by (2) by measuring the intervals T''_{out} and T'_{ret} taken by the photon to traverse the lower and upper sections, respectively. For the out trip on the lower section, the clock $C^* \equiv C''$ is co-moving with S'' , and for the photon return trip on the upper section—as shown in Fig. 1-b—the clock $C^* \equiv C'$ is co-moving with S' .

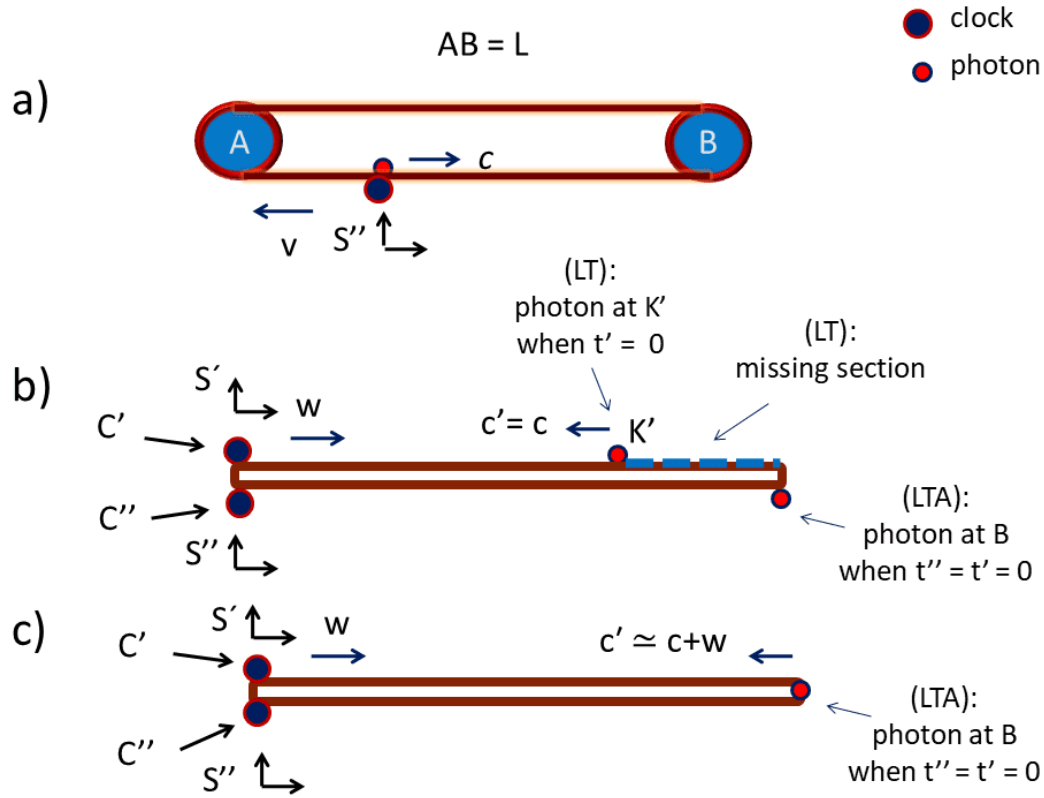


Figure 1. a) The linear Sagnac effect: a single photon travels along an optical fiber, which slides at speed v on the two pulleys A and B. The clock is co-moving with the fiber. The frames S' and S'' are co-moving with the upper and lower fiber sections, respectively, and with opposite velocities v relative to the lab frame AB. b) After being emitted by the clock C'' , co-moving with the lower fiber section at $t'' < 0$, the photon reaches B when A reaches C' at $t' = t'' = 0$ and the origins of S'' and S' coincide at point A. As observed from frame S'' , the photon has covered the distance L/γ in the interval $T_{out} = L/(\gamma c)$. According to the LT, in the return trip on the upper section, the photon is at K' at $t' = 0$. As seen from S' , it covers the shorter distance $\gamma L(1 - v/c)^2$ in the interval $T_{ret} = \gamma L(1 - v/c)^2/c$. The "missing section" $K'B = c\delta t' = 2\gamma(v/c)L$ has not been covered for $t' > 0$ in the observed interval T'_{ret} . c) According to the LTA, in the return trip on the upper section, the photon is at B at $t' = 0$ and covers the distance $c'T'_{ret} = \gamma_w L/\gamma$ in the interval T'_{ret} measured by clock C' . By traveling at the speed $c' \simeq c + w$, spacetime continuity is conserved with the LTA.

A.1. Summary of the Derivation

Starting from C^* (Fig. 1-a), the photon reaches B (Fig. 1-b) in the interval,

$$T''_{out} = T_{out} = \frac{L''}{c''} = \frac{L}{\gamma c}. \quad (A1)$$

As seen from frame S'' , the photon reaches B when, simultaneously, the pulley A reaches C^* (Fig. 1-b lower section).

With $w = 2v/(1 + v^2/c^2) \simeq 2v$ the relative velocity between S'' and S' , the photon return trip time interval from B to clock C' is:

$$T_{ret} = T'_{ret} = \frac{T''_{ret}}{\gamma_w} = \frac{\gamma L(1 - v/c)^2}{c}. \quad (A2)$$

Spatial distance covered by the photon according to the LT: Breach in spacetime continuity.

In the out trip, the distance covered is $L'' = \gamma^{-1}L$, as given by (A1). Due to the nonconservation of simultaneity of the LT, for S' in the return trip, the photon is already at point K' as shown in Fig. 1-b, and the return distance covered is,

$$L' = cT'_{ret} = \gamma L(1 - v/c)^2 \simeq L - 2(v/c)L < L, \quad (A3)$$

indicating that the photon does not cover the whole path $\simeq L$ on the return trip. The total path covered at speed c by the photon, L'' on S'' and L' on S' , is exactly,

$$L'' + L' = \gamma^{-1}L + \gamma L(1 - v/c)^2 = 2\gamma L - c\delta t' < 2L, \quad (A4)$$

where the term $\delta t' = \gamma_w \gamma^{-1} w L / c^2 = 2\gamma v L / c^2$ represents the "time gap" from S' to S'' due to the relative simultaneity predicted by the time transformation of the LT. Hence, at the local speed c , the photon cannot cover the whole fiber length $2\gamma L$ in the round-trip interval T . This result implies a breach in spacetime continuity by $c\delta t'$ because, at the local speed c , there is a "missing path" $2\gamma v L / c = c\delta t'$ that has not been covered in the interval $T = T_{out} + T_{ret}$.

Spacetime continuity is conserved with the LTA.

Using the LTA, which predicts a variant one-way speed, spacetime continuity is conserved. According to the LTA, the photon traverses at the speed c the distance $cT''_{out} = L/\gamma$ on the lower fiber section. In the return trip, the speed of the photon is $c' = \gamma_w^2(c + w)$, and it traverses the distance $c'T'_{ret} = \gamma_w L/\gamma$ on the upper fiber section (Fig. 1-c). The total fiber length traversed by the photon in the observed interval $T = T''_{out} + T'_{ret}$ is the contour length

$$\gamma_w L/\gamma + L/\gamma = 2\gamma L,$$

confirming that spacetime continuity is conserved with the LTA.

These results are independent of the form of the contour, of the initial position of C^* , and of whether it moves from the lower to the upper section in the invariant interval T .

Appendix B. Appendix B: Magnetic Induction from a Moving Electrified Body under the LTA

In this appendix, we provide a detailed derivation showing that, under the LTA, a moving electrified body produces no magnetic induction field. This derivation follows the approach of Puccini^[40] and Bonaura^[41], who derived the electromagnetic field transformations under the LTA starting from Maxwell's equations.

B.1. Field Transformations under the LTA

Starting from the same standard Maxwell equations, Puccini^[40] and Bonaura^[41] derived the electromagnetic field transformations under the LTA. For a stationary charge distribution producing a field $\mathbf{E}$ in frame S ($\mathbf{B} = 0$), assuming $\mathbf{v}$ in the x' direction (the relative motion of S' with respect to S), the fields in S' are the same as under the LT and given by:

$$E'_x = E_x, E'_y = \gamma E_y, E'_z = \gamma E_z, \quad (B1)$$

$$B'_x = 0, B'_y = \gamma \frac{v}{c} E_z, B'_z = -\gamma \frac{v}{c} E_y. \quad (B2)$$

Eq. (B2) shows that a magnetic field $\mathbf{B}'$ does appear in S' under the LTA, similar to the LT prediction. The difference lies in the Maxwell equations in S' , as shown below.

B.2. The Faraday-Maxwell Equation under the LTA

For an electrified body at rest in frame S , we have:

$$\nabla \times \mathbf{E} = 0, \mathbf{B} = 0. \quad (B3)$$

Using the LTA field transformations (B1)-(B2), the components of $\nabla' \times \mathbf{E}'$ are:

$$\begin{aligned} (\nabla' \times \mathbf{E}')_{x'} &= \partial'_{y'} E'_{z'} - \partial'_{z'} E'_{y'} \\ &= \gamma (\partial_y E_z - \partial_z E_y) = 0. \end{aligned} \quad (B4)$$

$$\begin{aligned} (\nabla' \times \mathbf{E}')_{y'} &= \partial'_{z'} E'_{x'} - \partial'_{x'} E'_{z'} \\ &= \partial_z E_x - \frac{1}{\gamma} \partial_x (\gamma E_z) \\ &= \partial_z E_x - \partial_x E_z = 0. \end{aligned} \quad (B5)$$

$$\begin{aligned}
(\nabla' \times \mathbf{E}')_{z'} &= \partial'_{x'} E'_{y'} - \partial'_{y'} E'_{x'} \\
&= \frac{1}{\gamma} \partial_x (\gamma E_y) - \partial_y E_x \\
&= \partial_x E_y - \partial_y E_x = 0.
\end{aligned} \tag{B6}$$

Thus, we obtain:

$$\nabla' \times \mathbf{E}' = 0, \tag{B7}$$

showing that, under the LTA, the curl of the electric field remains zero in the moving frame S' and no induced emf is generated.

Moreover, under the LTA, Puccini^[40] and Bonauro^[41] obtain the following Faraday-Maxwell equation in S' :

$$\nabla' \times \mathbf{E}' = -\frac{1}{c} \partial_{t'} \mathbf{B}'^*, \text{ with } \mathbf{B}'^* = \mathbf{B}' + \frac{1}{c} \mathbf{v} \times \mathbf{E}'. \tag{B8}$$

For the case of an electrified body at rest in S , we have from Eq. (B2):

$$\mathbf{B}' = -\frac{1}{c} \mathbf{v} \times \mathbf{E}'. \tag{B9}$$

Then:

$$\mathbf{B}'^* = \mathbf{B}' + \frac{1}{c} \mathbf{v} \times \mathbf{E}' = 0. \tag{B10}$$

Consequently, the right-hand side of Eq. (B8) vanishes, and we recover:

$$\nabla' \times \mathbf{E}' = 0, \tag{B11}$$

recovering the constraint of Eq. (B7). Hence, with the LTA, no emf due to magnetic induction is observed in S' .

B.3. Difference from the LT Case

- Under the LT, space and time are mixed $t' = \gamma(t - vx/c^2)$ and this causes the curl of the transformed electric field to become nonzero: the term $-c^{-1} \partial_{t'} \mathbf{B}'$ does not vanish in S' even when $\nabla \times \mathbf{E} = 0$ in S . This leads to a predicted observable emf and induced current in a closed conducting wire at rest in S' .
- Under the LTA, the time transformation $t' = t/\gamma$ contains no space-time mixing. As a result, $\nabla' \times \mathbf{E}' = 0$, and Faraday's law predicts no induced emf or current in S' .

The important point is that this result (valid for $\mathbf{B} = \mathbf{0}$ in S) is not to be confused with the case of a closed current loop stationary in S ($\mathbf{B} \neq 0$ in S), for which both frameworks give identical predictions for the

effective transformed magnetic field. The difference arises specifically for an electrified body with no intrinsic magnetic field in its rest frame.

The experimental verification of induction from moving electrified bodies has a contentious history. Indorato and Massotto^[42] revisited the early 20th-century controversy involving Poincaré, Crémieu, and Pender, concluding that the evidence remains inconclusive. The Rowland experiment, which attempted to detect the magnetic field from a rotating electrified disk, is unreliable because the predicted field is approximately 50,000 times smaller than the Earth's magnetic field^[42]. Thus, the distinct predictions of the LT and LTA in this scenario constitute an open experimental question.

Notes

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Data Availability

All data generated or analyzed during this study are included in this published article.

Author Contributions

The manuscript and the core derivation were conceived by G. S. and E.G.H. and developed together with the suggestions and help of A.D.G. The authors formed a team that drew the figures and revised the calculations of the derivations. A.D.G. revised the conceptual implications of Poisson's commutativity. All authors contributed to the writing and revision of the manuscript and have approved the final version.

Use of Generative AI

For checking English text only.

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Footnotes

¹ See, e.g., Kuchař, K. (1976) or Isham, C. J. (1992) for extended formulations of the ADM phase space.

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