Review of: "Incorrect conclusions drawn for plausible looking diagrams"

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I hope that Professor Eleftherios will take this comment as a compliment: I wish that I would have thought to have written this article. The examples are superb and the overall analyses are well done.

Furthermore, the author has cited some literature from journals such as *Dogus University Journal, Mediterranean Journal for Research in Mathematics Education, and Quaderni di Ricerca in Didattica* that I was not aware of.

However, there are two directions that I feel would significantly improve the manuscript. (1) Philosophy of Mathematics. (2) Mathematics Education with Dynamic Geometric software.

In terms of Philosophy of Mathematics, I frankly was surprised that there was no reference to Imre Lakatos' *Proofs and Refutations* and many subsequent articles that responded to it. For example, I would suggest D. A. Anapolitanos' (1989) reassessment of the work and P. Ernest's (1997) article in *Philosophia Mathematica*. For a relevant educational article, I recommend B. Sriraman (2006) "An Ode to Imre Lakatos: Quasi-thought experiments and actual mathematical classrooms." *Interchange* (37 (1-2): 151-178.

Professor Eleftherios asserts: "Teachers must implement paradoxes in their teaching, because the strange result increases student's interest, leading to new learning paths in order to gain knowledge." While I do include paradoxes in my teaching, I think it is important to cite the actual educational literature that has elucidated some of the values of doing so. I recommend two particularly insightful pieces: (1) E. Paul Goldenberg and Albert A. Cucoco's chapter on Rich Lehrer's and Daniel Chazan's book: *Designing Learning Environments for Understanding Geometry and Space;* and, (2) Molly Brown, Michael J. Bosse', and Kayla Chandler's "Student Errors in Dynamical Mathematical Environments" because they differentiate between syntactic and semantic language and use multiple diagrams of student work to define 6 kinds of errors: domain, process, interpretation, activity, evaluation, and interaction. I believe that a meta-analysis like theirs would provide an excellent framework for the examples laid out by Professor Eleftherios and support his claim of "must."

Finally, because the diagrams are developed in educational "Dynamical Mathematical Environments," I believe that some reference should be made to the literature of developers and users of popular software. Personally, this reviewer uses *Geometer's Sketchpad* and *Cinderella*. I have been particularly influenced by Bill Finzer, the developer of *Geometer's Sketchpad* and feel that he has insight into many of the issues raised by Professor Eleftherios.