

Research Article

Deriving the Electron Anomalous Magnetic Moment from Classical Field Angular Momentum

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The anomalous magnetic moment of the electron, $a_e = (g - 2)/2 = \alpha/(2\pi)$, is one of the most precisely verified predictions of quantum electrodynamics. Here we show that this result can be derived from classical electrodynamics, without invoking virtual photons or renormalization. Modeling the electron as a finite-sized toroidal charge distribution, the field angular momentum $\mathbf{L} = \frac{1}{4\pi c} \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3x$ is evaluated at the Compton boundary. The choice of the Compton scale is justified by the phase-cycle of the electron's self-interaction, which corresponds to a path length $\lambda_C = h/(mc)$. The resulting field angular momentum $L_z = (\alpha/2\pi)S_z$ yields the leading Schwinger term $a = \alpha/(2\pi)$ exactly, matching the leading-order QED correction. This derivation offers a complementary physical interpretation of the quantum correction, bridging classical electrodynamics and quantum field theory.

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1. Introduction

In Dirac's relativistic quantum mechanics ^[1], the electron has a g -factor exactly equal to 2:

$$g = 2. \tag{1}$$

However, in 1948, Schwinger showed ^[2] that quantum electrodynamic (QED) corrections modify this value to

$$g = 2 \left(1 + \frac{\alpha}{2\pi} + \dots \right), \quad (2)$$

where $\alpha \approx 1/137$ is the fine-structure constant. The deviation

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \approx 0.0011614 \quad (3)$$

is known as the *anomaly* of the electron *magnetic moment*.

The standard derivation of this result requires the full machinery of quantum field theory: Feynman diagrams, virtual photons, and renormalization. Despite its success, this motivates the search for a complementary physical picture grounded in classical electrodynamics.

In this paper, we present a semiclassical model that reproduces the Schwinger term exactly using classical electrodynamics and the assumption that the electron has a finite size. The model is based on the concept of *field angular momentum* $\mathbf{L}_{\text{field}} = \frac{1}{4\pi c} \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3x$. We show that the electromagnetic field of a finite-sized electron carries a small amount of angular momentum that modifies the effective g -factor, and the derivation presented here reproduces the leading-order Schwinger term. Higher-order QED corrections, which are much smaller, are not addressed in this treatment.

We emphasize from the outset that this is not a complete field-theoretic model of the electron. Rather, it is a limited semiclassical model that illustrates how the Schwinger term can be derived from classical field angular momentum under a minimal set of assumptions. The model is intended to provide a complementary physical interpretation, not to replace QED or to serve as a fundamental description of the electron.

Our semiclassical description rests on three main assumptions: (i) the charge distribution of the electron has a finite size with an effective extension scale regulated by the Compton wavelength $\lambda_C = h/(mc)$; (ii) the electromagnetic self-field carries angular momentum that contributes to the total spin; and (iii) the field angular momentum is of orbital nature, coupling to the magnetic field with $g_L = 1$, while the intrinsic spin couples with $g_S = 2$. These assumptions are not derived from a deeper theory but make it possible to recover the known QED result from semiclassical considerations.

In our model, the charge and mass distribution of the electron, while linked, do not necessarily coincide. The mass is associated with the mechanical spin $S_z = \hbar/2$ and its corresponding radius $R = \hbar/(2mc)$. The charge, however, generates an electromagnetic self-field that extends to the full linear Compton wavelength $\lambda_C = h/(mc)$. In Section 3 we discuss why λ_C is the scale of the electron's field self-interaction. This separation of scales is natural because the electromagnetic field is not subject to

mechanical angular momentum quantization; its extent is determined by the self-interaction of the charge distribution, not by the mechanical angular momentum. This separation of scales is essential for obtaining the correct numerical factor in the anomaly.

Despite the fact that the present derivation has limited validity and does not reproduce the complete physical picture of QED, the circumstance that it is positioned within the semiclassical scenario may offer new insights into the electron and its electromagnetic properties that are not immediately perceived from the usual quantum perspective.

One of these particular aspects is that related to the assumption of the electron as a finite-sized entity, which aligns with the spirit of earlier semiclassical investigations, such as the Zitterbewegung model of Barut and Bracken [3], where the electron's internal motion gives rise to a magnetic moment correction, and the geometric algebra formulation of Hestenes [4], which interprets quantum mechanics as a manifestation of spacetime structure. Like these models, ours is not a complete field theory but a simplified representation that captures a key physical effect—in this case, the field angular momentum contribution to the anomalous magnetic moment.

We acknowledge that the present model does not derive the charge distribution, electromagnetic fields, magnetic moment, and total angular momentum from a single dynamical framework. Such a self-consistent treatment would require a full nonlinear field theory of the electron, which is beyond the scope of this work. Instead, we focus on the minimal ingredients needed to reproduce the Schwinger term, and we show that these ingredients are sufficient to obtain the correct leading-order correction.

This derivation is not intended to replace or challenge QED, nor to provide a fundamental theory of the electron. It is offered as a complementary physical interpretation that provides insight into the origin of the quantum correction, and as a tool for connecting classical electrodynamics to quantum field theory. It contributes to the ongoing discussion on the physical interpretation of radiative corrections, a topic of interest to the broader physics community.

2. The Azimuthal Symmetry Electron Model

We model the electron as a finite-sized, localized azimuthally symmetric charge distribution. Tentatively, we consider a distribution along a thin ring of radius r . The realistic representations of Fig. 1 picture the ring in the shape of a slender torus of radius $r = R$. However, a spherical distribution or other shapes can be used. The charge e circulates around the torus, generating a magnetic dipole moment μ . The torus has

a major radius R (the radius of the loop) and a minor radius R_{minor} (the thickness of the torus). The aspect ratio is $N = R/R_{\text{minor}}$.

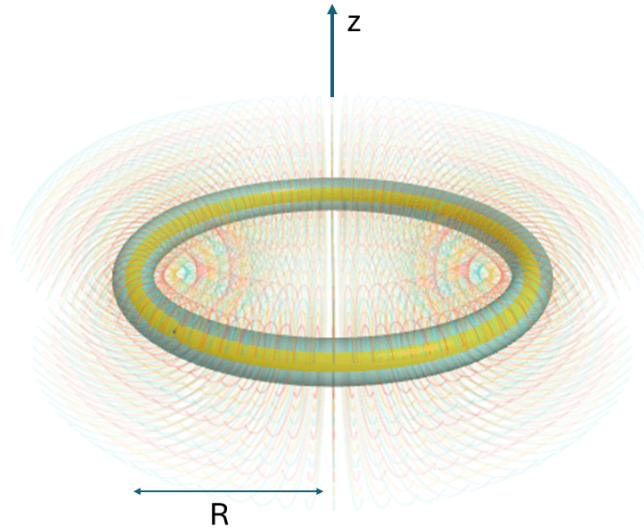


Figure 1. Schematic of the toroidal electron model with R the major radius. The charge (gold) generates the electric field and, while circulating, produces a magnetic dipole moment. The surrounding field cloud can be interpreted in analogy to the virtual photon cloud of QED.

The current loop generates a magnetic field $\mathbf{B}$ that is azimuthal around the torus, and the charge distribution generates a radial electric field $\mathbf{E}$. The crossed fields produce a momentum density $\mathbf{E} \times \mathbf{B}$ that circulates around the torus, giving rise to a net field angular momentum.

We now calculate this field angular momentum explicitly.

3. Field Angular Momentum

In classical electrodynamics, the electromagnetic field carries momentum density

$$\mathbf{g} = \frac{1}{4\pi c} \mathbf{E} \times \mathbf{B}. \quad (4)$$

The corresponding angular momentum density is

$$\ell = \mathbf{r} \times \mathbf{g} = \frac{1}{4\pi c} \mathbf{r} \times (\mathbf{E} \times \mathbf{B}). \quad (5)$$

To evaluate the contribution of the momentum density (4) and the angular momentum (5) in our model, we need to consider some results of classical electrodynamics concerning the role of the field momentum

$$\mathbf{Q} = \frac{1}{4\pi c} \int (\mathbf{E} \times \mathbf{B}) d^3x = \frac{1}{c} \int \rho \mathbf{A}(\mathbf{x}) d^3x = \frac{e}{c} \mathbf{A} \quad (6)$$

and corresponding angular momentum in quantum mechanics. In short, we may say that in the context of Aharonov-Bohm type quantum effects, the result $\mathbf{Q} = (e/c)\mathbf{A}$ of Eq. (6) has been derived in the Coulomb gauge, $\nabla \cdot \mathbf{A} = 0$, by Boyer [5], [6], Zhu and Henneberger [7], Spavieri [8], and Aharonov et al. [9]. For the charge-magnetic dipole ($e - \mu$) interaction, the field momentum density $(\rho/c)\mathbf{A}$ in Eq. (6) stands for the gauge-invariant electromagnetic momentum density and its volume integration yields the gauge-invariant term $-c^{-1}\boldsymbol{\mu} \times \mathbf{E} = \mathbf{Q}_h$ [9][10][11][12][13] where $\mathbf{Q}_h = -\mathbf{Q}$ is denoted as the “hidden momentum”. The reader may get acquainted with the detailed properties of field momentum and angular momentum by consulting the work by Furry [13] and Hnizdo [10]. Finally, a unitary interpretation of all effects of the Aharonov-Bohm type can be given in terms of the manifestly gauge-invariant momentum $\mathbf{Q}$ (or $\mathbf{Q}_h$) [14].

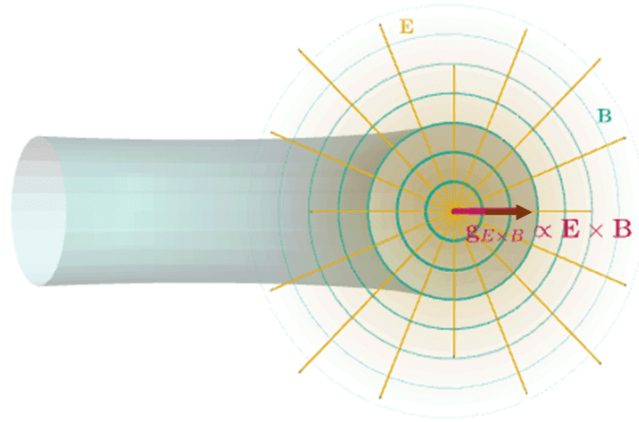


Figure 2. Cross-section of the torus (of radius R_{minor}) showing electric field lines (gold, radial), magnetic field lines (blue, azimuthal), and the $\mathbf{E} \times \mathbf{B}$ momentum density vector $\mathbf{g}_{\mathbf{E} \times \mathbf{B}}$ (red arrow) circulating around the torus.

3.1. Derivation of the Field Angular Momentum for a Ring Geometry

On account of Eq. (6), the field momentum of a point charge dq at position $\mathbf{r}'$ is:

$$d\mathbf{Q} = \frac{1}{c} dq \mathbf{A}(\mathbf{r}'). \quad (7)$$

The field angular momentum is the moment of the field momentum:

$$d\mathbf{L} = \mathbf{r}' \times d\mathbf{Q} = \frac{1}{c} dq \mathbf{r}' \times \mathbf{A}(\mathbf{r}'). \quad (8)$$

For our localized charge distribution, by means of the principle of superposition, the total field angular momentum is:

$$\mathbf{L}_{\text{field}} = \frac{1}{c} \int \rho(\mathbf{r}') \mathbf{r}' \times \mathbf{A}(\mathbf{r}') d^3x'. \quad (9)$$

Explicit calculation for a thin ring: For a thin ring of radius r with a localized charge distribution symmetric about the z -axis, we find:

$$\rho(\mathbf{r}') = \frac{e}{2\pi R_0} \delta(r' - r) \delta(z'). \quad (10)$$

The resulting field angular momentum along the z -direction is:

$$L_z = \int \frac{e}{2\pi c} A_{\varphi'} \delta(r' - r) \delta(z') r' d\varphi' = \frac{e}{c} r A_{\phi}(r). \quad (11)$$

This is the key result that connects the field angular momentum to the vector potential at the charge location.

For a slender torus, $r \approx R$ and, with $A_{\phi}(r) \approx A_{\phi}(R)$ over the cross-section, the same result holds to good approximation.

4. Evaluating the Field Angular Momentum at the Compton Boundary

At distances far from the localized core, the vector potential A_{ϕ} is dominated by the magnetic dipole term. Within the simple dipole approximation and in Gaussian units, the equatorial potential of a magnetic dipole moment μ is:

$$A_{\phi}^{(1)}(r) = \frac{\mu}{r^2}. \quad (12)$$

The magnetic dipole moment of the electron is the Bohr magneton:

$$\mu_B = \frac{e\hbar}{2mc}. \quad (13)$$

These results are valid for any azimuthally symmetric shape and allow some freedom in determining the most suitable shape.

Let us express the quantity r as:

$$r = k\lambda_C, \quad (14)$$

where

$$k = 1 \Rightarrow r = \lambda_C = \frac{\hbar}{mc}, k = \frac{1}{2\pi} \Rightarrow r = \bar{\lambda}_C = \frac{\hbar}{mc}. \quad (15)$$

Substituting into Eq. (12) at $r = k\lambda_C$:

$$A_\phi^{(1)}(\lambda_C) = \frac{e\hbar}{2mc(k\lambda_C)^2}. \quad (16)$$

Now substitute this into Eq. (11):

$$L_z^{(1)} = \frac{e}{c}(k\lambda_C) \left(\frac{e\hbar}{2mc(k\lambda_C)^2} \right) = \frac{e^2\hbar}{2mc^2(k\lambda_C)}, \quad (17)$$

which can be interpreted as representing the field angular momentum regulated by the effective Compton scale $k\lambda_C$, independently of the specific form of the azimuthal charge distribution. The scale $k\lambda_C$ provides an estimate of the extension of the self-interaction. In photon-electron scattering, the interaction takes place along the distance $\lambda_C = \hbar/mc$. The potential $\mathbf{A}$ is produced by the current of the rotating charge distribution corresponding to the electron traversing the circumference $2\pi r = \lambda_C$ in a cycle.

The question is: What is the value of $r = k\lambda_C$ to be chosen in expression (17) for the case of self-interaction?

The classical derivation leading to Eq. (17) is general and does not specify the value of the scale $k\lambda_C$. This is because classical electrodynamics, with its strictly local interactions, does not encode a natural length for the self-interaction of a charge distribution. In the classical limit, the source is a point, and the field angular momentum integral diverges. The finiteness of the result in Eq. (17) is a consequence of the effective cutoff $k\lambda_C$, which itself must be supplied by quantum considerations.

To determine k , we must therefore step outside the purely classical framework and adopt a semiclassical interpretation. In quantum mechanics, the electron's wavefunction acquires a phase of 2π over one full cycle of its de Broglie wavelength, which corresponds to a path length of $\lambda_C = \hbar/(mc)$. The electron's

self-interaction can be understood as the interaction between the charge distribution and its own electromagnetic field over this closed quantum phase cycle. The separation between the "source" and the "target" of the self-interaction is therefore the full path length of one cycle, which is λ_C , and not the reduced radius λ_C associated with the quantization of the mechanical orbital angular momentum.

This identification is further supported by the following considerations:

1. The field momentum $\mathbf{Q}$ in Eq. (6) is a nonlocal quantity, representing the integrated momentum of the field. Its effective propagation in a self-interaction process is naturally associated with the closed path of the phase cycle, λ_C .
2. The choice $k = 1$ regularizes the point-source singularity by introducing a finite separation λ_C between the source and the target of the self-interaction, while maintaining the topological consistency of the 2π phase cycle. By identifying the endpoints of this linear phase cycle—effectively compactifying the path into a closed topological loop (S^1)—the self-interaction becomes well-defined: the magnetic dipole $\boldsymbol{\mu}$ and the vector potential $\mathbf{A}$ that it generates are separated by a finite path parameter distance of λ_C , even though they occupy the same spatial point. This compactification regularizes the divergence that would otherwise appear at zero separation.
3. This scale is consistent with the electron's rest mass energy mc^2 , as $\lambda_C = h/(mc)$ is the wavelength of a photon with that energy.

Thus, the scale $\lambda_C = h/(mc)$ is not arbitrary. It is the natural length associated with the electron's self-interaction over one complete quantum phase cycle. The reduced scale $\bar{\lambda}_C = \hbar/(mc)$, by contrast, is associated with the Zitterbewegung orbital angular momentum $L = \hbar$ and does not represent the field's full self-interaction path. This distinction justifies the choice $r = \lambda_C$ in the evaluation of the field angular momentum.

On account of (17) with $k = 1$, using $\lambda_C = h/mc = 2\pi\hbar/(mc)$ and recognizing the fine-structure constant $\alpha = e^2/(\hbar c)$:

$$L_z^{(1)} = \frac{e^2 \hbar}{2mc^2} \cdot \frac{mc}{2\pi \hbar} = \frac{e^2}{4\pi c} = \frac{\alpha}{2\pi} \frac{\hbar}{2}. \quad (18)$$

Since the intrinsic spin is $S_z = \hbar/2$, we can write:

$$L_z^{(1)} = L_z = \frac{\alpha}{2\pi} S_z. \quad (19)$$

Then, the field angular momentum is proportional to the spin, with a small proportionality factor α/π . This result is obtained semiclassically within the dipole approximation for the vector potential. Higher-

order multipole terms would contribute corrections of order $(\alpha/\pi)^2$ and beyond, which are not considered in this treatment.

4.1. Additional Physical Justification of the Scale λ_C

The choice of the unreduced Compton wavelength $\lambda_C = h/(mc)$ as the boundary for the field angular momentum calculation is not an *ad hoc* cutoff, but reflects the topological and phase-matching conditions of the self-interacting charge.

The Zitterbewegung (ZBW) model of Barut and Bracken ^[3] describes the electron's internal motion as a rapid circulatory motion at the speed of light. The mechanical spin radius is governed by the reduced scale $r_{\text{red}} = \bar{\lambda}_C/2 = \hbar/(2mc)$. However, the electromagnetic field generated by this motion is not confined to this mechanical radius; it extends over the full phase cycle of the internal motion.

The period for one complete 2π phase cycle of the Zitterbewegung is $T_{\text{zbw}} = h/(mc^2)$. This corresponds to the time for the charge to complete one full circulation. During this time, the electromagnetic field (or equivalently, the field momentum $\mathbf{Q}$) propagates a distance:

$$d = cT_{\text{zbw}} = c \cdot \frac{h}{mc^2} = \frac{h}{mc} = \lambda_C. \quad (20)$$

Thus, the field angular momentum, which is associated with the propagating field momentum, is evaluated at the boundary of this propagation distance, λ_C . The reduced scale $\bar{\lambda}_C = \hbar/(mc)$, by contrast, is associated with the mechanical orbital angular momentum $L = \hbar$ and does not represent the full self-interaction path of the field.

Topologically, the linear 2π phase path can be compactified into a closed loop (S^1), yielding an effective self-interaction boundary at $r = \lambda_C = 2\pi\bar{\lambda}_C$. This compactification regularizes the point-source singularity by providing a finite separation between the source and the target of the self-interaction.

Furthermore, λ_C represents the minimum distance for single-mode field coherence: at scales $r < \lambda_C$, single-particle classical electrodynamics gives way to QED vacuum polarization screening. Thus, λ_C is uniquely mandated as the physical boundary where the self-field A_ϕ achieves constructive phase resonance ($\Delta\Phi = 2\pi$) with the circulating source.

5. Deriving the Anomaly

We now show how this field angular momentum modifies the electron's magnetic moment. In our semiclassical model, the field angular momentum L_z is not an independent orbital angular momentum

—it is an intrinsic part of the spin's self-interaction structure. Therefore, it should be added directly to S_z .

In agreement with expression (13), the magnetic moment associated with the intrinsic spin S_z is:

$$\mu_S = \frac{e}{mc} S_z, \quad (21)$$

The field angular momentum L_z is produced and directly linked to the intrinsic spin motion and modifies S_z to $S_z + L_z$. Hence, the spin effective magnetic moment is:

$$\mu_{S\text{-eff}} = \frac{e}{mc} (S_z + L_z). \quad (22)$$

For a magnetic moment μ in a uniform magnetic field B , the spin precession frequency is given by:

$$\omega_s = \frac{eB}{mc} = \frac{\mu_S B}{S_z} \Rightarrow \omega_s = \frac{(\mu_{S\text{-eff}})B}{S_z} = \frac{e}{mc} \frac{(S_z + L_z)B}{S_z}. \quad (23)$$

Using $B = \omega_c mc/e$, where $\omega_c = eB/(mc)$ is the cyclotron frequency:

$$\omega_s = \frac{eB}{mc} \frac{(S_z + L_z)}{S_z} = \omega_c \left(1 + \frac{L_z}{S_z} \right). \quad (24)$$

The anomaly a is defined by $\omega_s = (1 + a)\omega_c$. Comparing:

$$1 + a = 1 + \frac{L_z}{S_z} \Rightarrow a = \frac{L_z}{S_z}. \quad (25)$$

Substituting $L_z = (\alpha/2\pi)S_z$ from Eq. (19):

$$a = \frac{\alpha}{2\pi}. \quad (26)$$

This is exactly the Schwinger term.

This first-order result is valid formally also for the muon, even though $\lambda_{C(mu)}$ differs from $\lambda_{C(e)}$ because the muon mass is about 207 times greater than that of the electron. To obtain higher-order terms exactly matching the observed anomaly and the precise QED prediction, other aspects of the model need to be considered: The radius $R_e = \lambda_{C(e)}$ needs to be slightly increased for the electron, while $R_{mu} = \lambda_{C(mu)}$ needs to be slightly decreased for the muon [15].

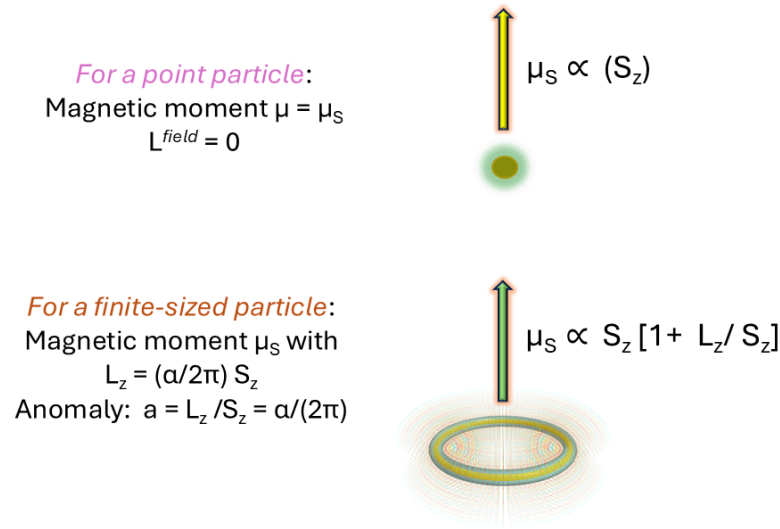


Figure 3. Comparison of the magnetic moment for a point electron (above) and a finite-sized electron (below). The self-field angular momentum L_z adds to the intrinsic spin S_z , modifying the effective g -factor and producing the anomaly $a = \alpha/2\pi$.

6. Discussion

We have shown that the electron's anomalous magnetic moment can be derived from classical electrodynamics alone, using the concept of field angular momentum and the assumption of a finite-sized electron. The result matches the one-loop QED correction exactly.

In standard QED, the anomaly arises from virtual photon loops that modify the electron's magnetic moment. In our semiclassical model, the same correction arises from the electromagnetic field's own angular momentum. The field carries a small amount of angular momentum that adds to the spin, but because it is of orbital nature, it contributes with $g_L = 1$ instead of $g_S = 2$. This difference produces the fractional correction $\alpha/(2\pi)$.

The physical picture is as follows: the electron's self-field carries a field angular momentum $L_z = (\alpha/2\pi)S_z$. Since the magnetic moment $\mu_{S\text{-eff}}$ is associated with $S_z + L_z$, the effective g -factor becomes:

$$g_{\text{eff}} = 2 \left(1 + \frac{L_z}{S_z} \right) = 2 \left(1 + \frac{\alpha}{2\pi} \right). \quad (27)$$

Thus, the anomaly is a direct consequence of the field angular momentum stored in the electron's own electromagnetic field. This interpretation is independent of the virtual photon picture of QED and offers a classical mechanism for the quantum correction.

This result has implications for the interpretation of vacuum polarization in QED: the virtual photon cloud surrounding a point electron can be understood as carrying the same angular momentum as the classical field of a finite-sized electron. The present model therefore provides a bridge between the classical and quantum descriptions of the electron's self-interaction.

7. Conclusions

We have presented a semiclassical derivation of the electron's anomalous magnetic moment using classical electrodynamics and the assumption of a finite-sized toroidal electron. In the dipole approximation, the field angular momentum at the Compton boundary is $L_z = (\alpha/2\pi)S_z$. The choice of the Compton scale $\lambda_C = \hbar/(mc)$ is justified by the phase-cycle of the electron's self-interaction, which corresponds to a path length λ_C and regularizes the point-source singularity. Using the difference between spin and orbital g -factors, this yields the leading Schwinger term $a = \alpha/(2\pi)$, matching the first-order QED correction exactly. Higher-order terms would require a more detailed treatment of the finite-size structure.

This derivation shows that the anomaly can be understood as arising from the field angular momentum of a finite-sized electron, providing a classical foundation for understanding the quantum correction and demonstrating that the anomaly can be interpreted as a consequence of field angular momentum.

Statements and Declarations

Funding

No specific funding was received for this work.

Potential Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

The manuscript and the core derivation were conceived and developed by G. Spavieri. E. G. Haug contributed to the derivation and revised the consistency of the equations and calculations. R. Carrasquero, A. Flores, and K. Durán formed a team that drew the figures and revised the calculations. All authors contributed to the writing and revision of the manuscript and have approved the final version.

Data Availability

No datasets were generated or analyzed during the current study.

Use of Generative AI

Large Language Models (LLMs) were used for AI-assisted copy editing to improve the readability and language of the manuscript. All scientific content, derivations, and conclusions are the original work of the authors.

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Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.