

Research Article

A New ‘Instrumental’ Empirical Structure for Assessing the Qualitative (Welfare) Effects of any Monetary Policy Ideology

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This study’s formal theory derives an ‘instrumental’ empirical structure which facilitates inferences in respect of the welfare effects of any monetary policy ideology. Implementationally, the new structure facilitates a sequestering of the evolution of a macroeconomy’s *primal quantity problem* from the evolution of its *price dual equivalent*. Using the macroeconomic quantities of the USA, with the monetary authority’s *Monetary Policy Actions (MPA)* latent in the data, the evolution of prices in the USA is shown to have been optimal. Regardless, with the selfsame menu of macroeconomic quantities in tow, the evolution of money growth is shown to have been constrained to be low-growth, time inconsistent, and revolving circularly around a relatively adverse equilibrium. The finding that the evolution of money growth has, uniquely interpretation as the gutting of employment to fund low growth rates then a rehiring of labor as growth increases (a strategy which then does not translate into high growth rates) and the finding that a risk averse Federal Reserve tends to sacrifice future growth potential for ‘safety’ help rationalize study findings. Also relevant is the formal theoretical evidence that the adoption of a ‘*rational-risk incentivization instrument*’ – a fiscal policy instrument – towards the management of a macroeconomy Pareto dominates the adoption of the *MPA*. The robustness of the endogenously embedded forecasting structure – the stylized facts in respect of the perturbations to the U.S. economy that emerged in the years, 1987-1988, 1999-2000, and 2007-2008, are robustly anticipated at least three-years ahead – lends additional credence to the robustness of the new empirical structure.

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1. Introduction

Whereas the studies that either infer or simulate the quantitative impacts of a monetary authority's *Monetary Policy Actions (MPA)* yield insights as to the immediate or predicted future impact of the *MPA* on a macroeconomy, simultaneously all such purely quantitative insights do not yield any unambiguous inferences in respect of the welfare effects - *beneficence* or *harm* - of the *MPA*. The rationale is straightforward, namely since severally any of an output contraction (whenever an economy is overheating), maintenance of the current output (at the supremum of rational risk), or an increase to output (whenever an economy is underperforming) feasibly is optimal, in of themselves, none of an output contraction, stable output, nor an increase to output suffices as evidence of the welfare effects of the *MPA*. Applying the insight, namely that there do not exist any unequivocally beneficent quantitative directions for the outcomes of the *MPA*, the conditions that facilitate the most robust inferences in respect of the welfare effects of the *MPA* are restricted to be structurally qualitative - that is, intertemporally distributional - not directionally quantitative; in stated respect, note that simulations whose path are spanned by some parametric algorithm lack characterization as structural qualitative outcomes. The highlighted dichotomy has illustration as follows. Whereas there exists a plethora of empirical evidence for the existence of a systemic relation between unemployment and money growth, regardless said evidence is not directly linked to either harm or beneficence to agents' welfare. To see this, since ideally there exists some optimal level of money supply, absent the parameterization, separately of a quantitative optimum, the existence of a relation between unemployment and money growth says nothing about either harm or beneficence to agents' welfare. The studies that examine the relation between money growth and unemployment are inclusive of, Sargent and Wallace^[1], Barro^{[2][3][4]}, Barro and Rush^[5], Rush^[6], or Rush and Waldo^[7], and Jefferson^[8]. Whereas an alternate set of empirical studies premise inferences on the sign restrictions that are imposed on the directionality of the *quantitative effects* of monetary policy, regardless typically such models are non-robust to inferences in respect of the welfare effects - harm vis-a-vis beneficence - of monetary policy; rather typically said models facilitate an assessment as to whether the directionality of the quantitative effects of monetary policy align with either some prior or some monetary policy rule (see for example, ^[9]). As such, none of such studies juxtapose the '*actual*' path of a macroeconomy with some exogenously inferred benchmark. Said studies, which are too numerous to list exhaustively, are inclusive of, Malikane and Semmler^[10], Kapetanios et al.^[11], Schenkelberg and Watzka^[12], Ma^[13], Gambacorta et al.^[14], Falagiarda and Saia^[15],

Weale and Wieladek^{[16][17]}, Mazier et al.^[18], Salem and Labidi^[19], Yongwu et al.^[20], Hua et al.^[21], and Guindo^[22]. Each of Borio and Zhu^[23] and Baumeister and Hamilton^[24] concur on the highlighted weakness of the enumerated studies, affirm that the non-flexibility of the models that embed ‘*financial accelerators*’ (see for example, ^{[25][26]}), such as, the *Dynamic Stochastic General Equilibrium (DSGE)* class of models, to the incorporation of exogenously formulated benchmarks that are robust to general equilibrium is a significant weakness of all such models. For a rich and extensive criticism of the DSGE class of models and the conclusion that the DSGE class of models are not useful starting points for the modeling of monetary policy, see Stiglitz^[27] or Stiglitz^[28]. Bauer, Bernanke, and Milstein^[29] concur on the desirability of a search for new approaches to a study of the effects of the *MPA*, recommend a study of the interactions that feasibly subsist between the *MPA* and the risk channel of a macroeconomy. In aggregate, there is arrival at the following inference, namely absent this study, there exists a dearth of empirical structures whose parameters facilitate unambiguous inferences as to the beneficence, or otherwise of the *MPA* that are introduced into an economy by a monetary authority.

Imposing rational expectations on a monetary authority’s decision rubrics, the *MPA* serve to either spur or constrain growth (see for example, ^[16]). However, since always growth is desirable – because all rational agents are non-satiated with respect to wealth – in reality, monetary policy manages growth that is either deemed too risky (because the underlying risk is deemed irrational), or growth that is too low (a macroeconomy that is in the grip of a ‘growth-stasis’). Clearly, if the *MPA* attempt to redress either some irrationality or some stasis, equivalently sub-optimality, ideally they span new risks that are rational, respectively optimal. Applying the foregoing, there is arrival at a systemic link between growth and risk, an insight that is formalized in Obrimah^[30], which demonstrates that any and all increases to growth span new realizations of idiosyncratic risk and feasibly also span new realizations of systemic risk.¹

Consistent with the highlighted insight, each of Harberger (1998) or Christensen and Dib^[9] postulate or find that the *MPA* span new risks in an economy. For concreteness note that Obrimah^[30] does not infer that an increase to asset expected returns spans an increase to discount rates (a prediction that is a rational expectation); rather the study infers that the riskiness of a new higher growth rate is a determinant of the risk premiums that investors demand if they are to maintain investments in a firm.

Applying the inference, the study links changes to firms’ growth rates to the intertemporal evolution of risk, as such, antecedently to the finding, in the *Intertemporal Capital Asset Pricing Model (ICAPM)* of Merton^[31], that the intertemporal evolution of agents’ investment choice is parameterized by the intertemporal evolution of the *price of risk*, the factor that spans asset risk premiums^[32]. The enumerated

linkage directly implies the existence of a systemic relation between the aggregate of the growth rates that emerge in a macroeconomy and each of the resulting market-wide price of risk or market risk premium. With the *price of risk*, λ (see [32]) having formulation explicitly as, $\lambda = \mu/\sigma$ (where μ is asset expected returns and σ is the standard deviation of returns), there is arrival at a systemic formal theoretical structure for studies of the interactions that feasibly subsist between macroeconomic growth and either macroeconomic risk or uncertainty. Antecedently, there is arrival at the following important restriction, namely that conceptually the variables that proxy for the risk channel of the evolution of a macroeconomy ought to be robust to the spanning of λ , as such, feasibly *are parameters of the financial sector*, as opposed to parameters of the real sector.

Building on the foregoing, this study's formal theory infers that every robustly formulated risk channel for g is parameterized by a non-linear structure that embeds an inflexion point, equivalently that embeds a kink (or threshold) in the interior of its continuum. Whereas then *threshold* values have, hitherto primarily been associated with credit rationing^{[33][34][35][36][37][38]} (see for example, Shao 2010), noting that risk and returns are native to any and all economic or financial transactions, the formal theory infers that a kinked (threshold) structure is native to any robust representation of a composition of risk with returns. For a concurrence, namely that the evolution of an asset price, S , seems to be best described as a drift process that is interspersed with jumps, equivalently kinks, see Mandelbrot^[39], Fama^[40], Press^[41], or Merton^[42]. Rather stylizedly, Bakshi et al.^[43] furnish the supporting empirical evidence, infer that the evolution of a market traded S is most robustly parsed by a drift process, which is interspersed with jumps that, themselves alter the path of the drift process. Studies of monetary policy whose study structures allow for non-linearity, but not necessarily for a kink, which find the said allowance to be critical to study inferences are inclusive of, De Graeve et al.^[44], Laseen et al.^[45], Jin and De Simone^[46], or Benchimol et al.^[47]. The economics of the kink that is inferred to reside in the interior of the continuum of risk, equivalently growth has rationalization as follows. Since always monetary policy attempts to either rein in or spur risk, always it '*leans against the wind*' (see for example, [48]). However, since it is irrational to reject an increase to risk which is not accompanied by any increase to the severity of the underlying uncertainty as to the investment outcomes, ideally a monetary authority attempts to rein in risk only whenever the severity of the underlying uncertainty is deemed irrationally high. Contrarily, since rational expectations exclude the following possibility, namely that a low risk is spanned by an irrationally high severity of uncertainty (for a formal proof of the exclusion, see [32]), it is the violation of some incentive compatibility investment criterion that induces agents not to progress risk and returns.

Since a low-growth domain within which agents are unwilling to progress risk is natively segmented from a necessarily high-growth domain in which feasibly a progression to risk is irrational, imposing rational expectations, there exists a kink between the two domains.

Applying the foregoing, plus noting that the study outcomes are robust to a sequestering of the effects of fiscal policy from the effects of monetary policy, this study derives a new instrumental empirical structure which facilitates inferences as to the welfare effects of the aggregate of a government's efforts towards the management of a macroeconomy. Importantly, this study does not model the timing of a monetary authority's policy actions (for alternate studies which do not model the timing of actions, see either [49] or [44]). In stated respect, the empirical structure is 'instrumental' in the sense that it does not require any parameterization of the timing of the actions of a monetary authority, rather derives inferences from a menu of structural (qualitatively distributional) conditions that are uniquely specified. Let a *msv*, e.g. gross domestic investments, denote a periodic macroeconomic stock variable whose periodic realization is not independent of, equivalently is reference-dependent on agents' choices in the preceding periods, as such, a parameter whose evolution natively embeds the evolution of choice. Also, let a *MAC_p* denote a macroeconomic outcome which is anticipated to have parameterization solely as a macroeconomic price. In Balinski and Baumol[50], Heal[51], Takayama and Woodland[52], or Weitzman[53], seminally every *msv* embeds a quantity problem and a price dual equivalent of the quantity problem. If then the same *msv* is to be robustly parsed as either a quantity or a price, applying the stylized prediction of the enumerated studies, there exist, at the very least, two different macroeconomic factors, κ and ρ , satisfying, $\kappa \neq \rho$, which, respectively span the quantity or price dimensions of the same *msv*. Regardless, qualitatively either κ or ρ span the exact same equilibrium. Imposing a formal theoretical insight that is enumerated in the immediate preceding paragraph, either κ and ρ embeds a kink in the interior of its distribution. Additionally, this study's formal theory infers as follows. The quantity problem that is embedded in a *msv* is parsed, on the one hand, by an increasing linear or an increasing strictly convex relation, or on the other, by an increasing strictly concave relation between the *msv* and κ . If the relation that subsists between a *msv* and κ is increasing linear or increasing strictly convex, rather sub-optimally the evolution of the *msv* has been restricted to the low-growth segment of a continuum (opportunity set) for growth, equivalently for risk; with both the MPA and a macroeconomy's fiscal policies latent in the data, there is arrival at the inference that a macroeconomy's regulatory efforts have, in aggregate, yielded a sub-optimal evolutionary path. If contrarily a *msv* is increasing strictly concave in κ , the *msv* and by extension the macroeconomy has evolved optimally; importantly note that in presence of a kink in the

interior of κ , that the predicted strictly concave relation does not embed any sub-optimality. Turning attention to the price dual, the price dual that is embedded in the selfsame msv is parsed by a decreasing strictly convex relation between the msv and ρ . If the enumerated relation holds in the data, but yet the exogenously inferred relation that subsists between a menu of msv and κ is sub-optimal, there is arrival at the inference that a macroeconomy's price dual is non-aligned with its primal quantity problem. Since the objective of monetary policy has statement as, "*the alignment of the primal quantity problem with the price dual equivalent*", resulting in the management of output, equivalently money growth using prices (e.g. interest rates), there is arrival at the inference that regulation has failed to achieve its objective. If fiscal policy is not failing alongside monetary policy, with the condition unidirectional, such that the non-discovery of any $MACp$ is not contradictory evidence, feasibly there exists a $MACp$, which is increasing strictly concave in ρ and decreasing strictly convex in κ . For a study, which lacks any exogenously inferred benchmark distribution, that spans the same set of macroeconomic outcomes using either a quantity or a price rule, but using a *DSGE* model consisting of about 30 assumed numerical parameters, see Hua et al.^[21].

A composition of the conditions that govern the intertemporal evolution of a msv reveals they endogenously span a *saddle* between the low-growth and the high-growth segments of a macroeconomy's opportunity set for growth. Said trajectory spans an optimal evolutionary path that is *S-shaped*, equivalently quasiconcave - an increasing linear or an increasing strictly convex relation in the lower domain of growth, then a navigation of the saddle, then on to an increasing strictly concave distribution in the upper domain of growth. For evidence that a quasiconcave function can, under conditions which are shown to be satisfied in this study, support rational actions, see Dasgupta and Maskin^[54] or Obrimah^[55]. Attesting to the robustness of the inferred structure, in each of Fama^[40], Mandelbrot^[56], or Press^[41], the return series for every individual firm in their sample are strictly convex in the *low-expected-return* region, span a saddle with approximately zero mass, then are strongly concave in the *high-expected-return* region. The evidence that the evolution of firms' returns is *S-shaped* has yet to be controverted in any subsequent studies. For concreteness, in Chen, Huang, and Lin^[57] the relation between innovation performance and the effort for the appropriation of a firm's innovation is *S-shaped*. Subramanyam^[58] furnishes formal theoretical support. Independent of the effort in Subramanyam^[58], this study's Corollary 2 predicts the stylized *S-shape* to be the outcome of rational investment choice within a macroeconomy. Empirical studies that reiterate the *S-shape* are inclusive of Freeman and Tse^[59] and Das and Lev^[60]. The formal theory shows none of interest rates, inflation, nor the parameters

of *Quantitative Easing* (QE) embed kinks in the interior of their distributions; as such, none has qualification as either κ or ρ . Contrarily Obrimah^[61] infers that the Fama and French^[62] *HML* factor is a kinked proxy for systemic risk. The same study shows that the new construct that it directly derives from *HML*, which is termed the *HMLd* factor, is a kinked proxy for systemic valuation uncertainty risk. This study's formal theory shows *HML* is a proxy for κ and that *HMLd* is a proxy for ρ . Since neither *HML* nor *HMLd* has constitution as a stock return, there is not any violation of the stance, in Bernanke and Gertler^[63] that, ideally a monetary authority does not factor changes to asset prices into assessments as to whether to engage with the *MPA*. For evidence that risk and uncertainty do not coincide, or more specifically that risk is a strictly convex function of uncertainty, see Obrimah^[32].

Switching focus to the choice of an empirical structure, Takayama and Woodland^[52], Lucas and Stokey^[64], and Obrimah^[61] predict or corroborate the robustness of a quadratic structure to the implementation of the empirical tests. Applying Weitzman^[53], all quantity growth equates to money growth, as such, the sufficiency of the primary targets of the *MPA*, namely any of, *Real GDP Per Capita* (*GDPCAP*), *GDP Growth* (*growth*), or the proportion of a country's workforce that are vulnerable to unemployment (*ve*) as the macroeconomic outcomes that trace out the intertemporal path of a macroeconomy. Suppose also the adoption of *gross domestic investments*, *credit to the private sector*, *savings*, and the *foreign exchange rate* as the candidate *msv*. Whereas this study's empirical structure is robust to assessments of the welfare effects of any monetary ideology, the United States of America (USA) is the poster country for a macroeconomy in which the management of the macroeconomy is primarily premised on a monetary policy ideology. If then the efficacy of this study's new instrumental structure is to be most robustly assessed, data for the USA seem most appropriate to the exercise. Using data from 1960 through 2017, evincing the optimality of the evolution of the price dual, the relation between any of *GDPCAP*, *growth*, or each candidate *msv* and $\rho \equiv HMLd$ is shown to be decreasing and strictly convex. Regardless, evincing a sub-optimal path for the evolution of the primal quantity problems, the relation between any of *GDPCAP*, *growth*, *ve*, or each candidate *msv* and $\kappa \equiv HML$ is increasing strictly convex. Consistent, as such, with the largely non-replicable evidence for a structural damage to output and employment that is furnished in Romer and Romer^[65], the evolution of money growth in the USA is indicated to have evolved sub-optimally. In stated respect, the empirical evidence is unambiguously consistent in the assessment that the macroeconomic growth of the USA has been both low-growth and revolving circularly around recurring diminishments to agents' monetary welfare. For concreteness, consistent with the predictions in each of Kydland and Prescott^[66] and Calvo^[67], relative to each other,

the intertemporal evolution of GDPCAP and growth are shown to be parameterized by time inconsistency (for a study which also infers time inconsistency in the context of the *MPA*, see ^[18]). Additionally, rather damningly, with ve increasing strictly concave in $\rho \equiv HMLd$, the empirical evidence implies that, at the margin, the U.S. economy spans growth within its low-growth domain via a gutting of employment, then cuts back on the gutting of employment as the macroeconomy evolves into its high-growth segment. With U.S. firms then attempting to please investors in the course of the low-growth regimes via cost cutting, as opposed to via a concerted effort for new innovations that feasibly facilitate high-growth in the future periods, the resulting '*manipulated-growth*' is unable to support high growth, resulting in a macroeconomy whose growth is constrained to be low-growth. Said evidence is consistent with the formal theoretical inference in several seminal studies, namely Stiglitz^[68], Jensen and Long (1972), Leland^[69], Ekern and Wilson^[70], and Baron^[71], all of which infer that, in the long-run equilibrium, efforts for guaranteeing some forecast return to investors are Pareto dominated by alternate efforts that infer the optimal allocation of firms' resources.

The highlighted empirical outcomes align with the inference in this study's formal theory that the availability of a *rational-risk incentivization instrument (rii)*, which incentivizes *rational risk bearing*, Pareto dominates the availability of any monetary policy instrument. Concurring with the necessity of a *rii*, in Bauer et al.^[29] monetary policy affects agents' risk appetites in a manner that is detrimental to the progression of innovation in an economy. Aggregating study findings, whereas fiscal policy cannot be deemed to have been sub-optimal, and whereas the combination of fiscal policy and the *MPA* has not been detrimental to the maintenance of an optimal price path, regardless the evolution of money growth, equivalently output in the USA has been sub-optimal. Study findings rationalize that the sub-optimality derives from the formal theoretical inference that the *MPA* lack the qualities of a *rii*, namely the capacity for switching from the incentivization of higher rational risk in the lower domains of growth to the incentivization of a preference for rational (safe) risk in the upper domains of growth. The empirical evidence in this study that the monetary authority of the USA, that is, the Federal Reserve has parameterization explicitly as a risk averse agent, as such, as an agent that foregoes additional returns, equivalently additional money growth, in exchange for the safety of the existing returns refers. Also relevant to a rationalization of study findings is the implication, in Weitzman^[53] that, in the contexts in which private sector equilibriums ought to be perturbed by some regulatory intervention, whereas a direct regulation of quantities, that is, the π is more robust to such interventions, the U.S. Federal Reserve tends to employ the *MPA*. Further, whereas it is not optimal to regulate either prices or quantities

whenever firms are both doing relatively poorly and there is heightened uncertainty as to the true factor prices, regardless the U.S. Federal Reserve tends to continue to engage with deployments of the *MPA*. In aggregate, the robustness of the formal theoretical outcomes and the empirical findings recommend the new empirical structure towards empirical tests of the welfare effects of the management of a macroeconomy. The rest of the study is organized as follows. Section 2 derives the structural analytical (qualitative) conditions – the functional yet structural (distributional or evolutionary) shapes – that govern the instrumental empirical structure. The formal theoretical evidence for the existence of a kink in the interior of any robust measure of macroeconomic risk or uncertainty is both generically developed and customized to *HML* and *HMLd* in Section 3; the imposition of the primal quantity problem and the price dual equivalent of the quantity problem on the formal theoretical outcomes is also engaged with in Section 3. Section 4 reports the empirical results from the direct implementation of the predictions of the formal theory. Section 5 concludes.

2. Growth, Risk, HML, HMLd, and the structural (evolutionary) conditions that govern the evolution of HML

The convention in the finance and economics literature posits growth is separable from risk. In stated respect, the Fama and French^[62] high-minus-low book-to-market (HML) arbitrage factor is computed as the return on a (Decile 10) portfolio consisting of firms that have the highest realizations of book-to-market ratios, less the return on a (Decile 1) portfolio consisting of firms that have the lowest realizations of book-to-market ratios. Whereas there exist departures (see ^[61]), the evidence is broadly supportive of the origin hypothesis in Fama and French^[62] that typically a Decile 10 portfolio outperforms a Decile 1 portfolio. By construct, a Decile 1 portfolio consists of the assets that are deemed by the market to have the best growth prospects. Contrarily Fama and French^{[72][73]} and Ogrimah^[61] infer the possibility that a Decile 10 portfolio is made up of assets that have high default risk; however, there exists an alternative characterization of the Decile 10 assets, which is both robust to this study's objective and more robust to general equilibrium than the alternate aforementioned characterization. Suppose a Decile 10 portfolio is made up of low growth assets. The target ratio theory predicts low growth stocks have higher leverage ratios than high growth stocks (see for example, ^{[74][75][76]}). Since, however, the higher leverage ratios feasibly are optimal, the higher leverage ratios do not, in of themselves, portend realizations of default risk that are sub-optimally high. Absent, as such some additional evidence, a higher leverage for a Decile

10 portfolio stock is as optimally realized as a lower leverage ratio for a Decile 1 portfolio stock. Since a low growth asset ought ideally not correspond to a high default risk asset, relative to the default risk characterization, there is arrival at an orthogonal characterization of the assets that make up a Decile 10 portfolio.

The question remains, however, as to how exactly low-growth assets can be anticipated to outperform high-growth assets; but then that is exactly the essence of the *value-growth anomaly* (see for example, [77][78][79]), namely that across business cycles, typically low-growth assets outperform high-growth assets. Yet again imposing rational expectations, a value firm is restricted not to coincide with a firm that has a high default risk. The exposition to follow rationalizes the predicted superior performance of either the low-growth or the high default risk firms in the context of *HML*; with a focus on the *MPA*, the analysis goes on to demonstrate that the Decile 10 firms are restricted to be low-growth firms. In stated respect, as is the case in Obrimah^[30], let the structure of asset risk be parameterized by the distribution of *Certainty Equivalent Cash Flows* (*CEQs*). Further, let the structure of uncertainty be parameterized by the distribution of the systemic discount rates (see for example, [80]) that are applied to the pricing of the *CEQs*. By explicit formal theoretical proof in that study, discount rates are monotone increasing in *CEQs*.

2.1. Decile 10 assets are high default risk assets: a possibility

Relative to the assets in the Decile 1 portfolio, suppose the assets in the Decile 10 portfolio are parameterized by higher realizations of default risk. Suppose also that the two sets of assets are associated with exactly the same sizes of expected future cash flows, but as is already assumed, differ with respect to the riskiness of the respective cash flows. Let r denote expected returns. Since a high default risk is riskier than a high growth rate, as such the rational expectation that, $HML = [r(10) - r(1)] > 0$, with the expected future cash flows held to be identical, the *CEQ* for a Decile (D) 10 portfolio is predicted to be smaller than that of a Decile 1 portfolio. Applying Obrimah^[30], relative to Decile 1, Decile 10 is associated with a lower discount rate, δ , as such, feasibly the assets in *D10* attain to a higher valuation than the assets in *D1*, but yet an anticipation that is inconsistent with rational expectations. For concreteness, in presence of, $CEQ(D10) < CEQ(D01)$ and $\delta(D10) < \delta(D01)$, any of,

$$\frac{CEQ(D10, DR)}{\delta(D10, DR)} \underset{<}{>} \frac{CEQ(D01, HG)}{\delta(D01, HG)}; \quad (1)$$

are feasible; contrary, as such to rational expectations, there is arrival at the feasibility that a distressed asset attracts a higher valuation than a comparable asset having a relatively safer growth potential. Let $E[CF]$ denote firms' expected future cash flows. Importantly, note that $CEQ \neq E[CF]$; that in fact, always, for risky cash flows, $CEQ < E[CF]$. Let V_{t+1} denote a future valuation and $MV_t = CEQ/\delta$ the current market valuation. Also, let σ denote risk. With g already reflected in the CEQ and incorporated into δ , and with $g(D10) = g(D1)$ by assumption, but yet, with $\sigma[g(D10)] > \sigma[g(D1)]$ - the essence of the dichotomy that subsists between $D1$ and $D10$ - note that the δ and the CEQ are sufficient parameters for g ; since then the exclusion of g does not have any materiality for study inferences, for parsimony, I have excluded g from the specification of the Gordon model. Suppose then that,

$$\begin{aligned} \frac{CEQ(D10, DR)}{\delta(D10, DR)} &> \frac{CEQ(D01, HG)}{\delta(D01, HG)} \\ &\rightarrow MV_t(D10, DR) > MV_t(D01, HG). \end{aligned} \quad (2)$$

With the expected future cash flows identical; each of risk and uncertainty already priced via, respectively the CEQ and δ ; and information unchanging, imposing rational expectations, namely the materialization of the $E[CF]$, as is already established to be the case in Ogrimah^[81] - if the $\{D01, HG\}$ are fairly valued, rather necessarily the $\{D10, DR\}$ are, however slightly, overvalued, yet the expected cash flows remain exactly the same - the return that accrues to investors between times t and $t + 1$ is bounded by,

$$\frac{V_{t+1}(D10, DR)}{MV_t(D10, DR)} < \frac{V_{t+1}(D01, HG)}{MV_t(D01, HG)} \rightarrow HML < 0, \quad (3)$$

which contradicts the HML construct, namely $HML > 0$. If then the assets that have high default risk span HML , necessarily,

$$[MV_t(D10, DR) < MV_t(D01, HG)] \rightarrow \left[\frac{CEQ(D10, DR)}{\delta(D10, DR)} < \frac{CEQ(D01, HG)}{\delta(D01, HG)} \right],$$

which with $CEQ(D10, DR) < CEQ(D01, HG)$, spans the sufficient condition, $\delta(D10, DR) > \delta(D01, HG)$, an outcome that is consistent with rational expectations. Note, however, that the sufficient but non-necessary condition, $\delta(D10, DR) > \delta(D01, HG)$, violates Ogrimah^[30], as such is predicted to be unlikely to be realized. For concreteness, since, imposing the Ogrimah^[30] restriction, $\delta(D10, DR) < \delta(D01, HG)$, always there exist $\delta(D10, DR)$ high enough that facilitate $[MV_t(D10, DR) < MV_t(D01, HG)]$; with the sufficient condition then not doubling as a necessary condition, feasibly the Decile 10 stocks are high default risk stocks, yet simultaneously the prediction in Ogrimah^[30] is satisfied. For additional concreteness, with δ strictly convex in $CEQs$,

δ increases faster than $CEQs$, hence the feasibility of the combination, $\delta(D10, DR) < \delta(D01, HG)$ and $[MV_t(D10, DR) < MV_t(D01, HG)]$. The superior robustness of, $\delta(D10, DR) < \delta(D01, HG)$ is evident in the realization that, whenever $\delta(D10, DR) > \delta(D01, HG)$, the $\{D01, HG\}$ are priced to *First Order Stochastically Dominate (FSD)* the $\{D10, DR\}$; with the demand for the $\{D10, DR\}$ then reduced to null, there is arrival at a binding contradiction.

2.2. Decile 10 assets are low growth assets: an alternate possibility

Since risk increases with growth, relative to the low growth assets, it is the high growth assets that are more likely to underperform expectations. Imposing yet again the assumption that the expected future cash flows are identical, the *low growth* assets (LG) are predicted to have higher $CEQs$ and discount rates than the *high growth* (HG) assets and there is arrival at:

$$\frac{CEQ(D10, LG)}{\delta(D10, LG)} \begin{matrix} > \\ < \end{matrix} \frac{CEQ(D01, HG)}{\delta(D01, HG)}. \quad (4)$$

The phenomenon that is referred to as the *value-growth* anomaly asserts that feasibly,

$$\frac{CEQ(D10, LG)}{\delta(D10, LG)} > \frac{CEQ(D01, HG)}{\delta(D01, HG)}. \quad (5)$$

Whenever markets are rationally populated with assets, if low growth firms are to have positive demand, they are not dominated by high growth firms. Let μ denote rational expectations returns; σ the rational expectations standard deviation of returns; and λ (μ/σ) the rational expectations information ratio. Imposing,

$$\mu(LG) = \mu(HG) \quad (6)$$

$$\sigma(LG) < \sigma(HG), \quad (7)$$

there is arrival at, $\lambda(LG) > \lambda(HG)$, that is, arrival at a satisfaction of the rational expectations condition that is inferred in Obrimah^[82] which ought to subsist between the LG and the HG ; regardless, as is the case with the derivation of equation (3), $MV_t(D10, LG) > MV_t(D01, HG)$ implies,

$$\frac{V_{t+1}(D10, LG)}{MV_t(D10, LG)} < \frac{V_{t+1}(D01, HG)}{MV_t(D01, HG)} \rightarrow HML < 0,$$

which contradicts the HML construct, namely $HML > 0$. If then the Fama and French^[62] HML factor is replicated, it is supported by,

$$MV_t(D10, LG) < MV_t(D01, HG) \quad (8)$$

$$\rightarrow \frac{V_{t+1}(D10, LG)}{MV_t(D10, LG)} > \frac{V_{t+1}(D01, HG)}{MV_t(D01, HG)} \rightarrow HML > 0, \quad (9)$$

such that, yet again the Obrimah^[30] rational expectations condition, $\delta(D10, LG) > \delta(D01, HG)$ is the equilibrium condition. For a generic formal theoretical treatment of the derivation of equations (8) and (9), see Obrimah^[81]. Note that the evidence that *HML* proxies for default risk in Fama and French^[72] or Obrimah^[61] is empirical, not formal theoretical, as such, is not contradictory to the foregoing.

2.3. The formal theoretical conditions and evidence that formalize the foregoing

The study, Obrimah^[61] arrives at a measure for the severity of valuation uncertainty risk that is termed, *HMLd*, a measure that is derived from the *HML* factor. For concreteness, with r denoting returns; 10 or 1 denoting Deciles; and $\bar{r}$ denoting the average return across the 10 Deciles of book-to-market ratios that are formed for any particular sample year, in Obrimah^[61], *HMLd* has construct as,

$$HMLd_{y,s} = \frac{[r_{y,s}(10) - r_{y,s}(1)] \times \left\{ \frac{1}{N} \sum_{i=1}^{10} [r_{y,s}(i) - \bar{r}(i)_{y,s}]^2 \right\}^{0.5}}{r_{y,s}(1)}$$

$$\equiv \frac{HML_{y,s} \times \left\{ \frac{1}{N} \sum_{i=1}^{10} [r_{y,s}(i) - \bar{r}(i)_{y,s}]^2 \right\}^{0.5}}{r_{y,s}(1)}.$$

That study furnishes evidence that *HMLd* is orthogonal to *HML*. Further, each of *HML* or *HMLd* is shown to embed a kink in the interior of it's distribution. The code for computing *HMLd* is available in Obrimah^[83]. For concreteness of their satisfaction of the enumerated condition, Exhibit 1 replicates the distributions of *HML* and *HMLd* that are reported in Table 1 of Obrimah^[61]. The exhibit reports the means, minima, and maxima for the three terciles into which each variable is segmented.

	Tercile Means			Tercile Minimums			Tercile Maximums		
	I	II	III	I	II	III	I	II	III
HMLd	-3.01	-0.38	4.96	-17.6	-0.93	0.02	-0.98	0.00	96.6
HML	3.32	-0.16	2.31	-11.9	-5.38	-2.84	19.53	7.34	13.2

Exhibit 1. Distributional statistics for HML and HMLd - excerpts from Table 1 of Obrimah^[61]

The formal theory in Section 3 to follow shows only measures, such as either *HML* or *HMLd*, which embed a ‘kink in the interior of their continuums’ have characterization as robust measures for growth.

First, however, statements and formal proofs for the general equilibrium conditions – the functional yet structural (distributional) shapes – that govern the intertemporal evolution of *HML*. The predictions that pertain to *HMLd* are specified in the context of a generalization, in Section 3.2, of the Conditions 1 through 3, such that they are robust to any menu of candidate proxies for a risk channel for the transmission of growth.

Condition 1. *By construct, Decile 1 of HML consists of high growth firms. Suppose Decile 10 of HML consists of low growth firms. Let HML be a proxy for macroeconomic risk. If growth is increasing linear or increasing strictly convex in HML, growth resides in the low-growth segments of macroeconomic risk, equivalently macroeconomic growth. If otherwise growth is increasing concave or increasing strictly concave in HML, growth traverses each of the low-growth and high-growth segments of macroeconomic risk.*

Proof. See the Appendix. □

Condition 2. *By construct, Decile 1 of HML consists of high growth firms. Suppose Decile 10 of HML consists of high default risk firms. Growth is solely increasing strictly concave in HML.*

Proof. See the Appendix. □

Condition 3. *The combination of low-growth firms that have higher leverage ratios in Decile 10 and high-growth firms that have relatively lower leverage ratios in Decile 1 can, rather erroneously have mischaracterization as the presence of higher default risk in Decile 10 and lower default risk in Decile 1. Feasibly, as such Condition 1 holds, but Condition 2 is assumed. Conditional on monetary policy actions, rational expectations predicts Decile 10 assets are low-growth assets, not high default risk assets.*

Proof. See the Appendix. □

Whenever there exist conditions that are robust to general equilibrium, such conditions are not tied to some specific variables, rather have parameterization as concepts or principles. Conditions 1 through 3, which specify structures, equivalently distributions that are intertemporally satisfied, pass the test of conditions that are robust to general equilibrium. There are not any prior empirical studies of the efficacy of monetary policy whose governing conditions are natively structurally (distributionally) qualitative, as opposed to consisting in the directionality of regression coefficients. In stated respect, whereas prior studies premise the interpretation of the evolution of a macroeconomy on the priors that

undergird their empirical models, contrarily in this study, the interpretation of the evolution of a macroeconomy is premised on a menu of exogenously derived structural (distributional) governing conditions. For concreteness, note that so long as a measure of macroeconomic risk or uncertainty embeds a kink in the interior of its distribution (the evidence to follow in Section 3) and so long as a dependent variable has characterization explicitly as a macroeconomic outcome, that Conditions 1 through 3 govern the interpretation of any regression that is populated with any such two variables. Such a general equilibrium condition is not typical of the studies that employ the *DSGE* structure. Conditions 4 through 6 generalize the predictions in Conditions 1 through 3.

3. Risk, Growth, HML, and HMLd

Let g , δ , μ , ς , ϑ , and χ denote, respectively, growth rates, discount rates, expected cash flows, assets' idiosyncratic risk, assets' systemic risk, and assets' certainty equivalent cash flows. Let ' Δ ' denote a change operator. Applying Theorem 1 of Obrimah^[30], always,

$$\Delta g \rightarrow \Delta \varsigma. \quad (10)$$

Further, in presence of industry waves,

$$\Delta g \rightarrow \Delta \vartheta. \quad (11)$$

Let g^h denote a high-growth asset and g^l a low-growth asset. Applying, equations (10) and (11) and suppressing functional notation, because it is immaterial (it is ς and ϑ that implement g , not vice versa) and a nuisance to the set theoretic analysis to follow (for a functional exposition, see ^[30]),

$$g^h = \{\varsigma^h \cup \vartheta^l\} \text{ or } g^h = \{\varsigma^h \cup \vartheta^h\} \quad (12)$$

$$g^l = \{\varsigma^l \cup \vartheta^l\}, \quad (13)$$

such that noting that, $\varsigma^h \cup \varsigma^l \equiv \varsigma^h$,

$$[g^l]^c = [\varsigma^h \cup \vartheta^h]; \quad (14)$$

$$[g^h \cap g^l] = \vartheta^l \text{ if } g^h = \{\varsigma^h \cup \vartheta^l\} \rightarrow [g^h \cap g^l]^c = [\vartheta^l]^c \equiv [\varsigma^h \cup \varsigma^l \cup \vartheta^h] \equiv [\varsigma^h \cup \vartheta^h]; \quad (15)$$

$$[g^h \cap g^l] = \emptyset \text{ if } g^h = \{\varsigma^h \cup \vartheta^h\}. \quad (16)$$

3.1. Theorems and Corollary

Theorem 1. Risk is non-separable from Growth. As such, assets are solely separable into 'high growth-high risk (h -type)' assets or 'low growth-low risk (l -type)' assets, are not separable into high growth vis-a-vis high risk assets. For agents who invest in the h -type assets, portfolio diversification that is implemented using risky

assets^[84] is solely facilitated by the l -type assets. A rational agent is, as such, parameterized by preference for the h -type assets and preference for diversification via investments in the l -type assets. If a continuum has characterization as a robustly measurable space for growth, g , it embeds a kink, equivalently an inflexion point or threshold in the interior of the continuum.

Proof. See the Appendix. $\square$

As ought to be the case if a risk channel of monetary policy transmission has materiality independent of monetary policy, Theorem 1 shows any such risk channel spans the stylized facts of portfolio theory. In stated respect, whereas Markowitz^[85] states portfolio theory in relation to a risk free asset, Black^[84], whose structure for portfolio theory is refined in Obrimah^[82], states portfolio theory in relation to the search for a lower risk asset that, asymptotically facilitates the same diversification benefits as a risk free asset, but yet at a higher return potential. Switching attention to the evidence in Theorem 1 for the necessity of an inflexion point in the interior of the continuum of risk, equivalently growth, whereas the prior studies of monetary policy that allow for non-linearities in the evolution of the path of a macroeconomy do not characterize the stated allowance to be a necessity, rather characterize it to be either *intuitively appealing* or *beneficent* to study outcomes, Theorem 1 shows such an allowance is a parameter of the most robust assessments of the welfare effects of the MPA. Ravn and Sola^[86] concur with the prediction in Theorem 1, find they only are able to dichotomize monetary policy regimes into either ‘low mean-low risk’ regimes or ‘high mean-high risk’ regimes.

Corollary 1. *The feasibility of a partition of an economy's risk continuum into two non-intersecting segments is parameterized by the existence of lower and upper bounds for the risk of each of the l or h type assets.*

Proof. See the Appendix. $\square$

Theorem 2. *Let conventional monetary policy (CMP) have definition as the management of economic growth, g , using solely reference interest rates (i) and the effects of current changes to i on future inflation (π) – see for example, Bernanke and Gertler^[63]. The distributions of the tools of CMP, namely, i and π violate Corollary 1, as does the distribution of ‘Quantitative Easing (QE)’. Regardless whether an economy engages with either CMP or QE, a macroeconomy that is robustly managed adopts some ‘rational-risk incentivization instrument (rii)’, $\pi^c \in \left[\left[\zeta^{\hat{h}} \cup \vartheta^{\hat{h}} \right], \left[\zeta^{\check{h}} \cup \vartheta^{\check{h}} \right] \right]$ that incentivizes rational risk taking. A robustly formulated π^c Pareto dominates any and all engagements with either CMP or QE.*

Proof. See the Appendix. $\square$

Corollary 2. *Macroeconomic outcomes are functions of risk and uncertainty; necessarily, as such, all substantive macroeconomic outcomes are non-linearly distributed. If a macroeconomy evolves rationally, its evolutionary path has representation as a quasiconcave function of either risk or uncertainty, equivalently as a somewhat S-shaped function that has a rational saddle sandwiched between the low-growth and the high-growth segments of macroeconomic growth. The ranking of the performance (μ) that is expected from the rational saddle of a measure of macroeconomic risk or uncertainty vis-a-vis the ranking of the performance that is expected from the low-growth or the high-growth segments is,*

$$\mu(\text{rational_saddle}) > \mu(\text{low_growth}) > \mu(\text{high_growth});$$

as such, μ is increasing strictly concave in macroeconomic risk or uncertainty.

Proof. See the Appendix. $\square$

Theorem 3. *Regardless whether changes to g are induced by monetary policy or otherwise, the Fama and French^[62] HML factor and the Obrimah^[61] HMLd factor are robust proxies for the risk, respectively, the uncertainty components of changes to g ; equivalently are robust proxies for changes to either ς or ϑ .*

Proof. Follows directly from the formal theoretical evidence in Obrimah^[61] and Exhibit 1, which, culled from Obrimah^[61] shows each of HML or HMLd satisfy Theorem 1 and Corollary 1. $\square$

Whereas the relation,

$$\mu(\text{rational_saddle}) > \mu(\text{low_growth}) > \mu(\text{high_growth})$$

might seem on the face of things counterfactual, because it asserts, $\mu(\text{low_growth}) > \mu(\text{high_growth})$, note that, $\mu(\text{low_growth}) > \mu(\text{high_growth})$, is exactly the essence of the *value-growth anomaly*, namely the finding that typically value stocks outperform growth stocks with respect to return performance (the relevant citations in Section 2). In essence, as in Section 2 and as is already rigorously proofed in Obrimah^[81], Corollary 2 predicts the value-growth anomaly is, in fact, a rational expectation, not an anomaly. For concreteness, in Obrimah^[81], in presence of a conferring of innovation risk premiums on a high-growth firm, such that the market price is higher than the fundamental price, in presence subsequently of the delivery of expected cash flows to investors, as such, the absence of any positive surprise performance, aggregated the enumerated conditions span, $\mu(\text{low_growth}) > \mu(\text{high_growth})$. Whereas then, natively, it ought to be the case that $\mu(\text{low_growth}) < \mu(\text{high_growth})$, in imperfect markets in which there exists a strict preference for high growth firms, with the strict preference having qualification as a rational market friction, the

existence of a systemic price premium for the high growth firms and the reality, to wit, the vast majority of high-growth firms do not outperform expectations, plus the reality that high-growth firms severely underperform low-growth firms during market downturns span, $\mu(low_growth) > \mu(high_growth)$.

With v denoting systemic uncertainty, the rest of the inequalities, namely either $\mu(rational_saddle) > \mu(low_growth)$ or $\mu(rational_saddle) > \mu(high_growth)$ follow directly from $v(rational_saddle) > v(low_growth)$, respectively, $v(rational_saddle) > v(high_growth)$, because rational-saddle firms' expected cash flows are more severely discounted in relation to the expected cash flows to either the low-growth or the high-growth firms.

3.2. Refining the Conditions 1 through 3 using Theorems 1 through 3 and the stylized insight that every 'quantity problem' has respecification analogously as a 'price dual' equivalent

Let a msv be as in the introductory section. Since it is already well accepted that HML is a systemic risk factor, it ought to rationalize the intertemporal evolution of a msv . In similar vein, since $HMLd$ is inferred to be a robust proxy for the systemic uncertainty as to the states of the world to emerge in the future periods, it ought also influence the evolution of a msv . In stated respect, the evidence in studies, such as, Dixit and Pindyck^{[87][88]}, Smales^[89], Moore^[90], or Al-Thaqeb and Algharabali^[91] that agents curtail investments in response to increases to the severity of uncertainty refers. Relatedly, the findings in this study reiterate the evidence in Ogrimah^[61] that HML is orthogonal to $HMLd$ (in that study, the correlation between the two variables amounted to 0.00). Macroeconomic variables that qualify as msv are inclusive of, but not limited to, *domestic investments*; *credit to the private sector*; *savings*; and *exchange rates*. For concreteness, whereas *domestic investments* is a yearly parameter, it is subject to a rationality constraint, namely that whenever investment choice in the current year is rational, it is reference-dependent on investment choice that already transpired in the preceding periods. Whereas then *domestic investments* is a periodic parameter, imposing the assumption that agents act rationally, it's yearly realizations are bounded by some rationality constraints, such as, the expectation that it's evolution is heteroscedastic (see for example, ^[92]). By the same argument, the yearly realizations of *credit to the private sector*, *savings*, or *exchange rates* are also bounded by some rationality constraints. For additional concreteness, it is straightforward that the emergence of a financial crisis in some time period, t , possesses implications for the foreign exchange (fx) rates that obtain in the next time period, $t + 1$. Further, since a fx rate facilitates the exchange of quantities, whereas it is natively a price, simultaneously it connotes quantities, as such, in aggregate has qualification as a msv . Contrarily, since either Real GDP Per Capita

or GDP Growth solely measure money growth, neither can be reference-dependently conditioned on any events that feasibly transpired in some preceding period.

Suppose the evolution of a *msv* is solely a function of agents' engagements with rational choice.

Applying Theorem 2, there exists a policy parameter, π^c that incentivizes agents to act rationally throughout the domain of growth, as such, the connectedness of the low-growth and the high-growth segments of the growth continuum. However, if the saddle that is inferred in Corollary 2 is a parameter of general equilibrium – and the findings in the literature indicate it is (the citations in the introductory section) – regardless the evolution of returns is non-linear. Further, there is arrival at the necessity of measures of either macroeconomic risk or uncertainty that embed a kink in their interior, as such, yet again the relevance of macroeconomic variables, such as, either *HML* or *HMLd*. Let Λ denote the amount of money in circulation, plus assume the conventional inverse relation that subsists between the systemic level of prices and the amount of money in circulation. It is straightforward that the condition,

$$\mu(\text{rational_saddle}) > \mu(\text{low_growth}) > \mu(\text{high_growth})$$

which is inferred in Corollary 2 directly spans,

$$\Lambda(\text{rational_saddle}) > \Lambda(\text{low_growth}) > \Lambda(\text{high_growth}),$$

namely an increasing and strictly concave relation between money growth and macroeconomic growth, equivalently between money growth and either macroeconomic risk or uncertainty. Importantly, yet again, there is arrival at an *S-shaped* relation between the performance of, and the progression of risk or uncertainty in an economy. For a study which infers that the increasing and strictly concave relation that is spanned by Λ endogenously results in a *S-shaped* evolution, see Hashai^[93]. Imposing the inverse relation that natively subsists between the systemic level of demand and the systemic level of prices, with p denoting the price level, the inequalities for Λ directly span,

$$p(\text{rational_saddle}) < p(\text{low_growth}) < p(\text{high_growth}),$$

resulting in a negative strictly decreasing relation between a macroeconomic price and either macroeconomic risk or uncertainty.

In Balinski and Baumol^[50], Heal^[51], Takayama and Woodland^[52], or Weitzman^[53], under very general conditions, which asymptotically have summary as the uniqueness, equivalently the invertibility of the demand and supply functions, an optimal *quantity problem* has specification, alternately as an optimal *price dual* problem. Further, with the saddle value for the quantity problem restricted to be equal to the

saddle value for its price dual, as such, with the existence of a saddle required, as is well known to be true, invertibility is robust to the existence of saddle points, equivalently is robust to the existence of a finite number of redirecting kinks that have zero mass. Combined, Theorems 1 through 3 and Corollary 1 and 2 satisfy the conditions that are inferred in all four enumerated seminal studies. In stated respect, the *S*–*shape* that is inferred in Corollary 2 whose saddle is spanned by two kinks, each having zero mass, satisfies all of the enumerated conditions. Applying the saddle theory, the demand and supply equilibrium that is portended by any *msv*, such as domestic investments has a quantity problem formulation in, say, κ terms, and a price dual formulation in say, ρ terms. Naturally, rather necessarily, $\kappa \neq \rho$. Since *HMLd* has construct, natively as a proxy for a price measure, namely the systemic uncertainty as to the true valuations of the assets that emerge in a macroeconomy^[61], it has construct, natively as a price dual of a quantity optimization problem. Anticipatedly, as such, *HMLd* is a proxy for ρ . By the same token, since *HML* is a proxy for risk, as such, a proxy for returns, it is not a proxy for price, rather is a proxy for growth to quantities, as such, anticipatedly a proxy for κ . Applying the foregoing and assuming optimality, whereas a *msv* is increasing strictly concave in *HML* (equivalently in Λ or κ), contrarily the same *msv* is decreasing strictly convex in *HMLd* (equivalently in p or ρ). Aggregating all of the foregoing, there is arrival at the following Conditions, which refine Conditions 1 through 3.

3.2.1. Conditions 4 through 6

Condition 4. Let a *MACp* denote a macroeconomic outcome which does not natively embed a quantity problem. Suppose the π^c in Theorem 2 exists, such that a macroeconomy evolves optimally. Each of money growth or a *MACp* is an increasing strictly concave function of ρ . Simultaneously, a *MACp* is a decreasing function of ρ . It is solely the *MACp* that are robust to inferences in respect of the welfare effects of a macroeconomy's fiscal policy.

Proof. Follows directly from the foregoing, that is, from,

$$\Lambda(\text{rational_saddle}) > \Lambda(\text{low_growth}) > \Lambda(\text{high_growth}), \text{ or}$$

$$\mu(\text{rational_saddle}) > \mu(\text{low_growth}) > \mu(\text{high_growth}),$$

because in the absence of a quantity problem, a price series spans returns, μ . Since holding incomes constant, quantity decreases with price, simultaneously there is arrival at the necessity of the following restriction, namely that a *MACp* decrease with ρ . Optimality is evident as follows, namely in presence of

a saddle, the low-growth region is non-contiguous with the high-growth region. Imposing the discontinuity the returns in the low-growth or high-growth regions are not directly comparable. $\square$

Condition 5. *The quantity problem that is embedded in a msv is parsed by the relation between the msv and $\varkappa$. An increasing linear or increasing strictly convex relation between the msv and $\varkappa$ restricts the msv to the low-growth segment of a feasible continuum (opportunity set) for growth, equivalently for risk. An increasing strictly concave relation between the msv and $\varkappa$ infers the msv navigates the low-growth and the high-growth segments of a feasible continuum (opportunity set) for growth, equivalently for risk. The price dual that is embedded in the selfsame msv is parsed by a decreasing strictly convex relation between the msv and ρ .*

Proof. Follows directly from the necessity, to wit, the inequalities that govern μ , equivalently Λ (Corollary 2) span an increasing strictly concave function and are, for the accompanying product prices, restricted to span,

$$p(\text{rational_saddle}) < p(\text{low_growth}) < p(\text{high_growth}),$$

that is, a decreasing and strictly convex relation. The monetary policy-specific prediction is directly from Condition 1. $\square$

Condition 6. *In this study, $HML \equiv \varkappa$ and $HMLd \equiv \rho$. A quadratic structure is robust to the specification of either the quantity problem or the price dual of the quantity problem.*

Proof. Follow directly from Takayama and Woodland^[52], Lucas and Stokey^[64], and Orlin^[61]. $\square$

3.2.2. Robustness Tests: Empirical Tests of Conditions 4 through 6

The empirical results that are reported in Table 1 serve primarily to certify whether $HML \equiv \varkappa$ and $HMLd \equiv \rho$ and as preliminary evidence as to whether the path of the U.S. macroeconomy can be deemed to have evolved optimally or sub-optimally between 1960 and 2017. Suppose then the following menu of msv , namely, *domestic investments; credit to the private sector; savings; and exchange rates*. Since all four msv 's are outcomes of quantity problems – e.g. savings is what is left consequent on optimal consumption, and exchange rates are spanned, in part, by a macroeconomy's trade balance – none have characterization as a $MACp$ (a search through a menu of feasible proxies for a $MACp$ did not yield any success, because none satisfied Condition 4).

This table reports the relations that obtain between each of four macroeconomic factors and either HML, predicted to a measure of risk, or HMLd, predicted to be a measure of uncertainty. The four macroeconomic factors are, respectively, investments normalized by Gross Domestic Product (GDP); credit to the private sector normalized by GDP; the US\$ exchange rate; or domestic savings normalized by GDP. The HML measure, which is natively kinked and a proxy for κ , a factor that parameterizes a macroeconomy's 'primal quantity' problem, is the Fama and French^[62] HML factor; HMLd is a natively kinked measure of uncertainty that is constructed in Obrimah^[61], which is a proxy for ρ , a factor that parameterizes a macroeconomy's 'price dual equivalent' of the primal quantity problem. The code that constructs HMLd is available in Obrimah^[94]. The quadratic structure is validated by each of the formal theory, Takayama and Woodland^[52], Lucas and Stokey^[64], and Obrimah^[61]. The regression models have specification as:

$$msv_t = \beta_1 HML_t + \beta_2 HML_t^2 + \hat{\varepsilon}_t$$

$$msv_t = \beta_3 HMLd_t + \beta_4 HMLd_t^2 + \check{\varepsilon}_t$$

$$msv_t = \tilde{\beta}_{10} HML_t + \tilde{\beta}_{20} HML_t^2 + \tilde{\beta}_{30} HMLd_t + \tilde{\beta}_{40} HMLd_t^2 + \tilde{\varepsilon}_t$$

where the msv are, successively, each of the four macroeconomic factors. The measure of macroeconomic uncertainty, HMLd, is derived from HML (for the mathematical construct, see Section 2. The data for HML are from the Kenneth French Data Library. All other data are from the World Bank and cover the period, 1960–2017 (58 observations). The regression models are implemented using Generalized Least Squares (GLS) model and model p-values are derived using Newey–West standard errors, as such are corrected for autocorrelation.

Panel A						
	investments			credit to private sector		
	i	ii	iii	iv	v	vi
HML	.0171		.0069	5.027 ^c		4.365 ^c
	(.24)		(.45)	(.00)		(.00)
HML ²	.0034 ^b		.0036 ^c	.5376 ^c		.5405 ^c
	(.05)		(.00)	(.01)		(.01)
HMLd		-.1911 ^c	-.1761 ^c		-15.92 ^c	-9.346 ^c
		(.01)	(.00)		(.00)	(.00)
HMLd ²		.0011 ^c	.0010 ^c		.002 ^c	.0538 ^c
		(.01)	(.00)		(.00)	(.00)
BIC	-124.62	-125.69	-126.72	7.7e+05	9.7e+05	7.4e+05

Panel B						
	savings			exchange rates		
	i	ii	iii	iv	v	vi
HML	.9359 ^c		.8757 ^c	.0537 ^c		.0503 ^c
	(.00)		(.00)	(.00)		(.00)
HML ²	.0922 ^c		.0930 ^c	.0043 ^c		.0043 ^c
	(.00)		(.00)	(.00)		(.00)
HMLd		-2.115 ^c	-1.378		-.1187 ^c	-.0550 ^b
		(.01)	(.12)		(.00)	(.02)
HMLd ²		.0118 ^c	.0076		.0007 ^c	.0003
		(.00)	(.11)		(.00)	(.01)
BIC	-124.6	1.8e+04	1.1e+04	-191.9	-172.2	-184.4

Table 1. Macroeconomic Outcomes vis-a-vis HML or HMLd: A Robustness Test

a, b & c denote statistical significance at the 10%, 5% and 1% confidence levels

Consistent with Condition 6, the empirical evidence in Table 1 designate *HML* to be a macroeconomic factor, κ , that transmits the quantity effects of monetary policy. Simultaneously, consistent with Condition 6, *HMLd* has characterization as a macroeconomic factor, ρ , that transmits the price effects of monetary policy. With *HMLd* then yet again shown to be orthogonal to *HML*, study findings revalidate the origin insight in Ogrimah^[61]. Applying Conditions 4 and 5, consistent with the notion that the *CMP* restrain rational growth, as such are damaging to agents' welfare, all of the results for *HML* evince an intertemporal evolution that is constrained to be low-growth.

4. The data and the main empirical results

The relevance of this study's empirical structure is evident as follows, namely whereas Weale and Wieladek^[16] infer the quantitative materiality of monetary policy, that is, infer "an asset purchase

announcement of 1% of GDP leads to a statistically significant rise of 0.58% (0.25%) and 0.62% (0.32%) rise in real GDP and CPI for the US (UK)", in the absence of estimates of the optimum (desirable) outcome, the study is unable to arrive at any inferences in respect of the welfare effects of the quantitative outcomes. Regardless, with any effort for the estimation of a quantitative optimum also fraught with 'errors', because they require priors that themselves embed either imprecision or abstractions from reality, such efforts are not anticipated to be more robust than estimates of the quantitative effects of monetary policy (see for example, the critique of studies which attempt to infer optimums in [95]).

Contrarily, with the governing conditions exogenously derived and shown to be robust to general equilibrium, evidence that a macroeconomy has evolved either sub-optimally or optimally facilitates robust inferences as to the welfare effects of a monetary authority's *MPA*. The availability of governing conditions that facilitate a filtering out of the effects of fiscal policy further enhances the robustness of inferences. Much as it is preposterous to assert that investors' behavior can be robustly modeled only if the exact timing of their actions are simulated, as such, inclusive of the Fama and French^[62] three factor model, the preposterousness of any asset pricing model, so also it is preposterous to assert that the behavior of a monetary policy maker is solely robustly modeled in the context of a simulation of the exact timing of the said authority's actions. To see this, since necessarily there exists a lag between the undertaking of a *MPA* and the impact of the said *MPA* on an economy, the probability of a *MPA* and the evidence for the appropriateness or otherwise of a *MPA* do not coincide in time. Applying the insight, either *HML* or *HMLd* is, simultaneously, but at different points in time a measure of the relevance of a *MPA* and a measure of the appropriateness (outcome) of a *MPA*. For concreteness, suppose risk, equivalently *HML* is deemed to be too high at some time, t . If the monetary authority's inference is correct and the correcting action appropriate, conditional on the location of *HML* a priori, there ought to exist some time period, $t + \tau$, satisfying, $\tau > 1$, at which either $HML_{t+\tau} < HML_t$ or $HML_{t+\tau} > HML_t$. The same variable, *HML*, embeds, as such, both the triggers and the outcomes of the *MPA*. Since it is impossible for a trigger time, t , to coincide with an outcome time, $t + \tau$, there is arrival at the robustness of the dichotomy of the times, t , from the times, $t + \tau$. Regardless, in presence of a satisfaction of the principle of *equilibrium over time* (the *eot*; see for example, Hicks 1939), in the limit, if, $MPA_t | [HML \sim \text{too low}]$ spans, $[Optimal_HML_{t+3}]$, with the *Generalized Least Squares (GLS)* model in tow, the assumed lagged relation is predicted to be evident in the relation that subsists between the contemporaneous realizations for each of a *msv* and the contemporaneous realization for *HML*. The

empirical results that are presented in Table 2, which report each of lagged and contemporaneous relations demonstrate the robustness of the *eot* to the governance of the empirical results.

Since the new empirical structure that is developed in this study facilitates inferences that are inherently qualitative and that are most easily interpreted in the context of a quadratic structure, the quadratic structure suffices for the implementation of the empirical tests of the formal theory. Additionally, since the results in this study serve, simultaneously as robustness tests of the formal theory, with panel data introducing complications in respect of the interpretation of study findings, because the mix of monetary and fiscal policy is heterogeneous across countries, panel data are not as robust to an origin engagement with the implementation of the regression models as are data for a particular macroeconomy. Since, arguably the USA is the poster country for conventional monetary policy, data for the USA are applied to the generation of study inferences. With the data obtained from the World Bank, yearly interval data for 1960 through 2017 are applied to the implementation of the empirical tests. With 1987 about midway between 1960 and 2017, but moreso the fact that 1987 harbors ‘Black October’, that is, October 1987, the month in which on a single day (the 19th) the stock market lost about 21% of it’s market capitalization (see for example, ^[96]), the results for two subperiods, 1960–1987 and 1988–2017 are also reported.

Whereas it is already well established that neither Real GDP Per Capita (*gc*) nor GDP Growth Rates (*gg*) are robust proxies for economic development (see for example, ^{[97][98][99]}), evidence that either or both parameters evolve sub-optimally suffices for study purposes. In stated respect, the evidence in Ogrimah^[99] that an increase to *gc* is a necessary, but insufficient condition for economic development refers. Plus, with either *gc* or *gg* as a primary target of a monetary authority’s interventions, they are proxies for the intentions of the U.S. Federal Reserve. Since the management of unemployment is also a primary objective of a monetary authority, *ve* is relevant to study objectives. In presence of the trade-off that can subsist between inflation and unemployment in the context of a deployment of the *MPA*, *ve* has characterization as an implicit price of growth; as such, simultaneously it facilitates a test of Condition 4. Additionally, since layoffs of labor ought not be premised on any quantity considerations (firms that are struggling to generate demand are excluded from any model of equilibrium; see for example, ^[52]), rather ought to be premised on considerations of efficiency or productivity, ideally *ve* does not embed any quantity problem. As such, in addition to *gc* and *gg*, the proportion of the work force that is vulnerable to unemployment (*ve*), a parameter of the *International Labor Organization (ILO)* which is reported by the World Bank is incorporated as a macroeconomic proxy for ρ . Since, however, in reality *ve* is perhaps

more often than not influenced by quantity considerations – the effort for the delivery of higher returns, that is, μ , equivalently Λ to investors, either of which models as a quantity problem – feasibly *ve* also turns out to be a proxy for $\varkappa$. Also incorporated as a proxy for $\varkappa$ is the macroeconomic outcome, *au* (the proportion of all labor that is vulnerable to unemployment who hold advanced degrees). Conditions 4 through 6 facilitate the formulation of the ‘instrumental’ empirical structure. Let *MAC* denote any of *gc*, *gg*, *ve*, or *au*. In presence of all of the foregoing, with the model intercepts explicitly suppressed, the regression models have specification as:

$$MAC_t = \beta_1 HML_t + \beta_2 HML_t^2 + \hat{\varepsilon}_t \quad (17)$$

$$MAC_t = \beta_3 HMLd_t + \beta_4 HMLd_t^2 + \check{\varepsilon}_t \quad (18)$$

$$MAC_t = \tilde{\beta}_{10} HML_t + \tilde{\beta}_{20} HML_t^2 + \tilde{\beta}_{30} HMLd_t + \tilde{\beta}_{40} HMLd_t^2 + \tilde{\varepsilon}_t \quad (19)$$

$$MAC_t = \tilde{\beta}_{11} HML_{t-3} + \tilde{\beta}_{21} HML_{t-3}^2 + \tilde{\beta}_{31} HMLd_{t-3} + \tilde{\beta}_{41} HMLd_{t-3}^2 + \tilde{\varepsilon}_{t3} \quad (20)$$

$$MAC_t = \tilde{\beta}_{12} HML_{t-5} + \tilde{\beta}_{22} HML_{t-5}^2 + \tilde{\beta}_{32} HMLd_{t-5} + \tilde{\beta}_{42} HMLd_{t-5}^2 + \tilde{\varepsilon}_{t5}. \quad (21)$$

4.1. Hypotheses 1 through 5

The regression models are implemented using a *Generalized Least Squares (GLS)* model that corrects for autocorrelation. Applying the discussions in Sections 2 and 3 and the resulting formal theoretical predictions, in total there is arrival at four complementary Hypotheses. Hypotheses 1 and 2 formalize the signs of the partial effects that are supportive, or otherwise of the objectives of a monetary authority. In stated respect, note that the signage are non-parametric, that is, parameterize a structure, equivalent an intertemporal evolution that spans a distribution. If truly the claim that *HML* and *HMLd* are uncorrelated and orthogonal to each other is robust, the model in equation (19) does not span partial effects that are contradictory to the partial effects that are spanned by equations (17) or (18). For the same dependent variable, the *Bayesian Information Criterion (BIC)* juxtaposes the qualitative fit of the different specifications of the empirical model. Hypothesis 3 formalizes study expectations as they relate to a juxtaposition of equation (19) with either equation (17) or (18). Lastly, functioning as robustness tests, equations (20) or (21) examine whether allowing time lags for the effects of a monetary authority’s actions to diffuse throughout an economy alters the inferences that are garnered from any of equations (17) through (19). The evidence in Romer and Romer^[65] that it takes four to five years for the effects of monetary policy actions to diffuse throughout an economy refers. Hypothesis 4 formalizes study expectations in so far as they relate to a substitution of lagged values for the contemporaneous realizations of *HML* or *HMLd*. Hypothesis 5 highlights the following importance, namely if study inferences are robust, a substitution of the terciles of either *HML* or *HMLd* for the quadratic structure ought to maintain the inferences that are derived in the context of Hypothesis 3.

Hypothesis 1. Suppose equations (17), (19), (20), or (21). If a monetary authority's policy actions restrict growth to be low, $\{\beta_1, \tilde{\beta}_1\} \geq 0$ and $\{\beta_2, \tilde{\beta}_2\} \geq 0$; if, contrarily, a monetary authority's policy actions restrict growth from becoming too high, $\{\beta_1, \tilde{\beta}_1\} \geq 0$ and $\{\beta_2, \tilde{\beta}_2\} < 0$.

Hypothesis 2. Suppose equations (18), (19), (20), or (21). If a macroeconomy's price path evolves optimally, $\{\beta_3, \tilde{\beta}_3\} < 0$ and $\{\beta_4, \tilde{\beta}_4\} \geq 0$; if, however, a *MAC* is, simultaneously a *MAC_p*, contrarily $\{\beta_3, \tilde{\beta}_3\} \geq 0$ and $\{\beta_4, \tilde{\beta}_4\} < 0$.

Hypothesis 3. The qualitative fit of the model in equation (19) is comparable to the qualitative fit of the models in either equation (17) or (18).

Hypothesis 4. Suppose equations (19) through (21). If the inferences in equations (17) through (19) are robust, an allowance for time lags of three or five years between the necessity of a policy action and the realization of the target macroeconomic objectives does not result in dichotomous qualitative inferences.

Hypothesis 5. If the inferences from the implementation of equation (19) are robust, a segmentation of each of *HML* or *HML_d* into terciles does not result in inferences that are contradictory to the inferences that are garnered from the implementation of equation (19).

4.2. Empirical Results

Table 2 reports the empirical results from the implementation of equations (17) through (21) for each of *gc*, *gg*, *ve*, or *au*. The results for the entire sample in each of Panels A and B show the U.S. economy has, in the sample period, been restricted to the low-growth segment of the growth continuum. The results in columns (ii) and (v) for the 1988-2017 subsample are qualitatively identical to the results for the entire sample. The results in columns (iii) and (vi) for the 1960-1987 sub-sample do not yield any inferences in respect of either beneficence or harm. However, since the absence of a strictly concave structure implies a non-traversing of a high-growth segment, regardless the evidence is not positive. Applying the evidence, rather unequivocally the beneficence of monetary policy to welfare, if any, has deteriorated post-1987.

This table reports the relations that obtain between each of four macroeconomic outcomes (*msv*) and two macroeconomic measures, one for risk, the other for uncertainty, the properties of which are formally and theoretically established in Sections 2 and 3 of the manuscript. The four *msv* are, respectively, *GDP Per Capita* (*gc*); *GDP Growth* (*gg*); the proportion of the U.S. working population that are deemed vulnerable to unemployment (*ve*); and the proportion of the unemployed that have advanced degrees (*au*). The natively kinked measure for macroeconomic risk, which is a proxy for κ , a factor that parameterizes a macroeconomy's 'primal quantity' problem, is the Fama and French^[62] *HML* factor; the natively kinked measure for macroeconomic uncertainty, which is a proxy for ρ , a factor that parameterizes a macroeconomy's 'price dual equivalent' of the primal quantity problem, is the Obrimah^[61] *HMLd* factor (the code for *HMLd* is available in Obrimah^[94]). The quadratic structure is validated by each of the formal theory, Takayama and Woodland^[52], Lucas and Stokey^[64], and Obrimah^[61]. In Panels A and B, the data in columns (i) and (iv) cover, 1960-2017 (58 observations); in columns (ii), (v), (vii), or (viii), (iii) they cover, 1988-2017; in columns (iii) and (vi), they cover 1960-1987. Note that the data for *ve* or *au* are only available for the 1988-2017 time window. The results in Panel C are from regressions of the contemporaneous realizations of the macroeconomic outcomes on the contemporaneous, 3-year or 5-year lags of *HML* and *HMLd*. The regression models are implemented using the Generalized Least Squares (GLS) model and model p-values are derived using Newey-West standard errors, as such are corrected for autocorrelation. Table 1 lists the data sources.

Panel A: HML												
	gc			gg			ve					
	i	ii	iii	iv	v	vi	vii					
HML	1418 ^c (.00)	2523 ^c (.00)	1480 (.23)	.1420 ^c (.00)	.1272 ^c (.00)	.2090 (.29)	.2660 ^c (.00)					
HML ²	148.5 ^c (.00)	286.6 ^c (.00)	65.27 (.54)	.0041 (.25)	.0067 ^c (.00)	.0067 (.91)	.0305 ^c (.00)					
BIC	5.8e+10	4.2e+10	1.1e+10	152.1	20.46	173.4	256.6					
Panel B: HMLd												
	gc			gg			ve			au		
	i	ii	iii	iv	v	vi	vii			viii		
HMLd	-4393 ^c (.00)	-4022 (.31)	2329 (.27)	-3876 ^c (.00)	.0067 (.97)	-.0526 (.87)	-.2061 (.68)			-.6298 ^c (.01)		

HMLd ²	24.89 ^c (.00)	22.91 (.28)	2914 ^c (.00)	.0021 ^c (.00)	-.0000 (.99)	.2814 ^c (.01)	.0012 (.65)			.0035 ^c (.01)		
BIC	7.4e+10	5.7e+10	1.4e+10	211.0	4.54	170.0	445.4			152.5		
Panel C (As in Columns (i) & (iv), the Entire Sample)												
	gc			gg			ve			au		
	cm	3-yr lag	5-yr lag	cm	3-yr lag	5-yr lag	cm	3-yr lag	5-yr lag	cm	3-yr lag	5-yr lag
HML	1236 ^c (.00)	1334 ^c (.00)	1466 ^c (.00)	.1335 ^c (.00)	.0979 ^c (.01)	.1566 ^b (.03)	.2641 ^c (.00)	.2934 ^c (.00)	.2846 ^c (.00)	.2276 ^c (.00)	.1514 ^c (.00)	.1821 ^c (.00)
HML ²	149.3 ^c (.00)	160.1 ^c (.00)	161.2 ^c (.00)	.0040 (.21)	.0137 ^c (.00)	.0048 (.42)	.0312 ^c (.00)	.0308 ^c (.00)	.0294 ^c (.21)	.0251 ^c (.00)	.0194 ^c (.00)	.0164 ^c (.00)
HMLd	-2554 ^c (.00)	-1631 ^b (.05)	-1817 ^b (.03)	-2696 ^b (.05)	.0831 (.35)	.2115 ^c (.00)	.2395 (.21)	.2911 ^a (.06)	.2429 (.17)	-.0300 (.86)	-.0204 (.85)	-.0044 (.95)
HMLd ²	14.73 ^c (.00)	9.822 ^b (.03)	10.84 ^c (.01)	.0014 ^b (.05)	-.0004 (.37)	-.0011 ^c (.00)	-.0012 (.23)	-.0015 ^a (.07)	-.0013 (.18)	.0002 (.79)	.0002 (.76)	.0001 (.87)
BIC	5.6e+10	5.6e+10	5.3e+10	151.3	89.98	132.3	257.3	263.6	280.9	103.1	120.2	132.3

Table 2. Macroeconomic Outcomes vis-a-vis Macroeconomic Risk and Uncertainty

a, b & c denote statistical significance at the 10%, 5% and 1% confidence levels.

In stated respect, rather than a moderation of high growth, such that it does not become irrationally high, monetary policy seems to be ‘policing’ ς^* (see the proof of Theorem 2 of Section 3), the rationality parameter that connects the low and high growth segments of the growth continuum, such that growth is sub-optimally capped to remain in the low-growth region. Combined, the finding that the proportion of jobs that are vulnerable to unemployment increased at an increasing rate (Panel A) and the finding that the proportion of the unemployed that hold advanced degrees decreased at an increasing rate in the 1988-2017 subsample in Panel B are consistent with the notion that the *MPA* do not mediate harm to agents’ welfare. For concreteness, whereas an increase to risk induced a strictly convex increase to the

risk of unemployment, as such, an adverse state, which subsequently was not reversed as uncertainty increased, regardless the proportion of the unemployed with advanced degrees decreased at an increasing rate, an inference that is consistent with the gutting of the manufacturing sector of the USA (see for example, [\[100\]](#)). Consistent with a gutting of the manufacturing sector (the results in Panel C), whereas ve is insensitive to $HMLd$ in the context of a predictive 5-yr lag and in the context of a contemporaneous relation, it is increasing and strictly concave in $HMLd$ in the context of a predictive 3-yr lag. However, with ve not simultaneously decreasing with HML , it fails the test of a $MACp$ that is specified in Condition 4. The economic intuition for, in aggregate, the inference that the trends in the evolution of ve in relation to $HMLd$ are sub-optimal is straightforward, to wit, the implication that an increase to risk in the low-growth segment of the continuum of growth spans initially an increase to unemployment is inconsistent with rational expectations. In stated respect, the increasing strictly concave relation then only can be spanned by a gutting of labor for the generation of growth in the low-growth segment, then a cutting back on the said gutting in the high-growth segment of the growth continuum. In aggregate, with firms attempting to please investors in the course of low-growth regimes via cost cutting, as opposed to innovation that spans future growth, the ‘*manipulated-growth*’ or ‘*corrupt-growth*’ is unable to support any higher realizations of growth, resulting in a macroeconomy whose growth is stunted.

Panel C of Table 2 reports the empirical results from the implementation of equation (19). As is the case in each of Panels A and B, all of the partial effects in the columns labelled cm continue to evince growth that is restricted to the low-growth segment of the growth continuum (it is the results in the columns cm that are directly comparable to the results in Panels A or B). Switching attention to the time trends (Panel C), with focus on gc , there is arrival at a similar qualitative inference. In stated respect, the progressiveness with which the partial effects for each of HML or HML^2 are attenuated, first between the 5-yr and 3-yr lags, then between the 3-yr lag and the contemporaneous effect, evince the attenuation of growth that is already low, as such, evince progressiveness of harm to agents’ welfare. Turning attention to the relation that subsists between gg and HML , growth is either increasing linear or increasing strictly convex in HML , resulting yet again in the inference that growth is restricted to be low-growth. The time trends concur, namely whereas growth starts off linear in HML at the 5-yr lag, it transforms to be strictly convex in HML at the 3-yr lag, then reverts to be linear in HML in the contemporaneous window, transitions which evince, sequentially a spurring and constraining of growth within the low-growth segment of the growth continuum. Additionally, however, with reference to

economic growth, there is arrival at the insight that the U.S. economy has description as a circularly evolving economy that, in the limit, was deteriorating. The partial effects of each of $HMLd$ or $HMLd^2$ for the contemporaneous period remain consistent with the prediction in Condition 6 that $HMLd$ is a proxy for the price path of a macroeconomy. Combined, the results for gc and gg corroborate the finding, in Fitoussi, Sen, and Stiglitz^[98] that the management of economy-wide growth using inflation and interest rates mismeasures agents' welfare. In aggregate, all of the macroeconomic variables, namely gc , gg , ve , and au yield exactly the same qualitative inference in respect of the efficacy, or otherwise of monetary policy actions.

The totality of the results in Panel C of Table 2 do not support the notion that the monetary policy actions of the U.S. Federal Reserve have been beneficent to agents' welfare. Consistent with study findings, Svensson^[48] and Kockerols and Kok^[101] find evidence that non-accommodative policies, equivalently, the policies that implement a *Leaning Against the Wind* (*LAW*) by a monetary authority induce a net cost to an economy. In stated respect, as in this study, unemployment increases with a monetary authority's incentive to engage with *LAW*. Consistent with this study's insights, namely that monetary policy redirects investments to low growth assets, in Laseen, Pescatori, and Turunen^[45], using financial variables that are accommodative of non-linear behavior, rather counterfactually, "an excessive stabilization of output leads to a compression of risk premia, higher asset prices, and higher investment levels". Tatom^[102] also infers counterfactual outcomes from the interventions of a monetary authority.

Switching focus to asset allocations, whilst also allowing explicitly for non-linearities, Jin and De Simone^[46] find evidence that investors herd into real estate funds (relatively safe assets) in response to monetary policy interventions and that subsequently the real estate funds experience vulnerability. Consistent with this study's findings, in Benchimol, Saadon, and Segev^[47], explicitness of allowance for non-linear dynamics is key to inferences in respect of the effects of uncertainty on the monetary policy actions of a monetary authority. The empirical evidence in Bekaert, Hoerova, and Duka^[103] also concur with study inferences. With focus on either the evolution of returns in markets or investment behavior, study inferences rationalize relevant findings in Taylor^[104], respectively Baele et al. (2020). In aggregate, study inferences accord with the insight in Romer and Romer^[65] that U.S. monetary policy has been harmful to each of employment and output.

4.3. Robustness Tests

Suppose a monetary authority's monetary policy actions are guided by risk aversion, that is a willingness to sacrifice growth, equivalently higher returns for some perception of safety - the Pratt^[105] or Arrow^[106] parameterization of risk aversion. Since the Arrow-Pratt risk aversion trades-off additional gains for a mitigation of risk, applying the outcomes of the formal theory, namely that it is solely *HML* that proxies for quantities (π), equivalently for money growth, the monetary authority's risk aversion is solely evinced by *HML*, not by *HMLd*. Each of Sharpe^[107] or Ross^[108] imply that the returns to risk aversion, equivalently the distribution of the resulting macroeconomic outcomes will turn out *J*-shaped. Relative to either *HML* or *HMLd*, Fig. 1 graphs the predicted outcomes in Table 2 for each of *gc*, *gg*, *ve*, and *au*. Consistent with the highlighted prediction, all of the relevant graphs for *HML* - Figs. 1a, 1c, 1e, and 1g - evince *J*-shapes, as such, characterize the representative U.S. agent, equivalently the U.S. Federal Reserve to be risk averse. With the evidence from stock markets evincing *U*-shaped distributions for return outcomes (see for example, ^[109]^[110]; and all of the studies that are cited in Hong and Wang^[110], pg. 299) as such evincing markets that are populated by both risk averse and risk seeking agents (the formal theoretical evidence in Obrimah^[82]), the evidence in Fig. 1 indicates the U.S. Federal Reserve is, rather inadvertently picking sides in markets, as such, more likely than not, is distorting market outcomes.

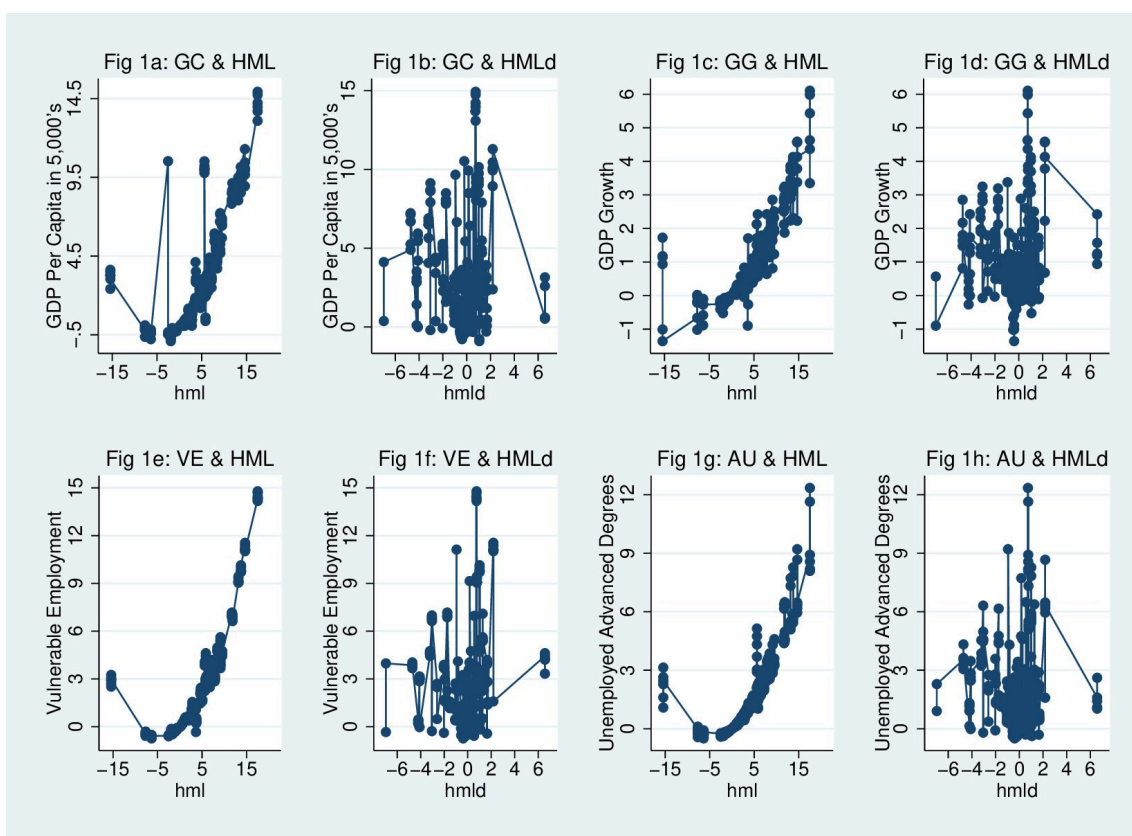


Figure 1. Graphs the predicted macroeconomic outcomes in Panel C of Table 1 vis-a-vis either *HML* or *HMLd*. The variables, *HML* and *HMLd* are as constructed in Section 2. *HML* is original to Fama and French^[62]; *HMLd* is original to Obrimah^[61].

Applying the formal theoretical outcomes, *HMLd* is solely a proxy for ρ , the price path of a macroeconomy. Consistent with the predicted dichotomy, none of Figs. 1b, 1d, 1f, or 1h evince *J-shaped* graphs. Further, the graphs evince that the macroeconomic prices that obtained in the U.S. are most clustered in the interior of the distribution of *HMLd*, which in Exhibit 1 is bounded by the closed set, $[-0.93, 0.00]$; the graphs then evince that prices are second-most clustered in the third tercile of *HMLd*, a continuum bounded by the closed set, $[0.02, 96.6]$; it of course is straightforward that the prices then are least clustered in the first tercile of *HMLd*, namely in the continuum bounded by the closed set, $[-17.6, -0.98]$. In increasing order of severity, the ranking of the severity of uncertainty in Obrimah^[61] is, the third tercile, the first tercile, then the second tercile. Applying the evidence, the uniqueness of the prices that obtain in the U.S. economy deteriorated with the severity of uncertainty, an outcome that is dichotomous to the following rational expectation, namely that efforts that are robust to

the management of uncertainty maintain, asymptotically the uniqueness of a macroeconomy's price structure. In stated respect, with the supporting citations in Section 2, prices that become more uncertain with uncertainty enhance investment stasis, that is, induce agents not to be able to formulate rational expectations, resulting in a decision to postpone investment decisions. Since such postponements solely occur in the context of high realizations of uncertainty, equivalently high realizations of risk, there is arrival at a capping of growth via postponements of engagements with investment choice. In aggregate, clearly the results in Fig. 1 are supportive of the inferences in each of Theorem 2 and Table 2. In stated respect, rather than a strategic response to uncertainty, such that prices are restrained to be uniquely specified, contrarily the monetary authority, namely the U.S. Federal Reserve is indicated to be fatalistically accepting of the uncertainty regime that emerges within a macroeconomy.

4.3.1. Substituting Terciles of HML or HMLd for the quadratic structure

Table 3 and Fig. 2 report the results from the empirical tests of Hypothesis 5. Consider the results for the terciles of *HML*. The results for *gg* show growth decreases in response to a transition from the lowest realization of risk (Tercile 1, the reference tercile) to the median realization of risk (Tercile 2). However, relative to Tercile 1, growth is not any different in Tercile 3, the highest realization of risk. As such, as in Benhabib et al.^[111] and as is the case in Table 2, the evidence evinces a macroeconomy that is cycling around an adverse outcome then returning to some previous state, as opposed to progressing. Clearly, there is arrival at evidence of harm, not evidence of beneficence. The results for *gc* show GDPCAP in the second tercile remains the same as in the first tercile, then decreases significantly in the third tercile. Yet again, there is arrival at an inference of harm, as opposed to an inference of beneficence. The assumption of a one-period lag in the transmission of the growth effects into real GDP effects reconciles the findings for *gc* with those for *gg*. With the results for either *ve* or *au* qualitatively identical in the sense of indicating harm, as opposed to beneficence, there is not any evidence that the *MPA* are beneficent to agents' welfare.

This table reports the relations that obtain between each of four macroeconomic (monetary) outcomes and either HML, predicted to a measure of macroeconomic risk, or HMLd, predicted to be a measure of uncertainty. The four macroeconomic outcomes are, respectively, GDP Per Capita (gc); GDP Growth (gg); the proportion of the U.S. working population that are vulnerable to unemployment (ve); as well as the proportion of the unemployed that have advanced degrees (au). The HML measure, which is natively kinked and a proxy for κ , a factor that parameterizes a macroeconomy's 'primal quantity' problem, is the Fama and French^[62] HML factor; HMLd is a natively kinked measure for uncertainty that is modeled in Ogrimah^[61], which is a proxy for ρ , a factor that parameterizes a macroeconomy's 'price dual equivalent' of a primal quantity problem. The programming code for HMLd is available in Ogrimah^[94]. Each of HML or HMLd is tiered into three terciles; the regression partial effects that are reported for HML_2, HML_3, HMLd_2, or HMLd_3 are, as such, with reference to the first terciles (HML_1 or HMLd_1) of HML, respectively HMLd. By design, the intercept ($\tilde{\beta}_0$ below) is the effect that is associated with either HML_1 or HMLd_1. The regression model has specification as:

$$MAC_t = \tilde{\beta}_0 + \tilde{\beta}_{10}HML_2_t + \tilde{\beta}_{20}HML_3_t + \tilde{\beta}_{30}HMLd_2_t + \tilde{\beta}_{40}HMLd_3_t + \tilde{\varepsilon}_t$$

where the MAC are, successively each of the four macroeconomic (monetary) outcomes. The regression models are implemented using the Generalized Least Squares (GLS) model and model p-values are derived using Newey-West standard errors, as such have been corrected for autocorrelation. The data consists of 58 (1960-2017) yearly observations. The data sources are listed in Table I.

Panel A		
	gc	gg
HML_2	-679.4	-.7571 ^b
	(.53)	(.03)
HML_3	-10240 ^c	.1312
	(.67)	
HMLd_2	-.1449	-.4153
	(.40)	(.16)
HMLd_3	-638.7	.3390
	(.55)	(.18)
$\tilde{\beta}_0$	39131 ^c	2.28 ^c
	(.00)	(.00)

Panel B		
	ve	au
HML_2	.1821 ^b	.1099
	(.04)	(.31)
HML_3	.4886 ^c	1.409 ^c
	(.00)	(.00)
HMLd_2	.3640 ^c	.0011
	(.01)	(1.00)
HMLd_3	.3502 ^c	.3080 ^a
	(.01)	(.09)
$\tilde{\beta}_0$	4.103 ^c	2.992 ^c
	(0.00)	(0.00)

Table 3. Macroeconomic (Monetary) Outcomes and HML or HMLd

a, b & c denote statistical significance at the 10%, 5% and 1% confidence levels

Fig. 2 graphs the relations that are obtained from Table 3. Consider Figs. 2c and 2d. Clearly, if a growth rate of about 2.6% (Fig. 2c) or 2.5% (Fig. 2d) in the third tercile is rational, the dip in the growth rate from 2.1% (Fig. 2c) or 2.0% (Fig. 2d) to about 1.6% (Fig. 2c) or 1.75% (Fig. 2d) between the first and second terciles connotes harm, not beneficence. Figs. 2c and 2d furnish graphical evidence of an economy that circles around adverse outcomes back towards some prior origin location.

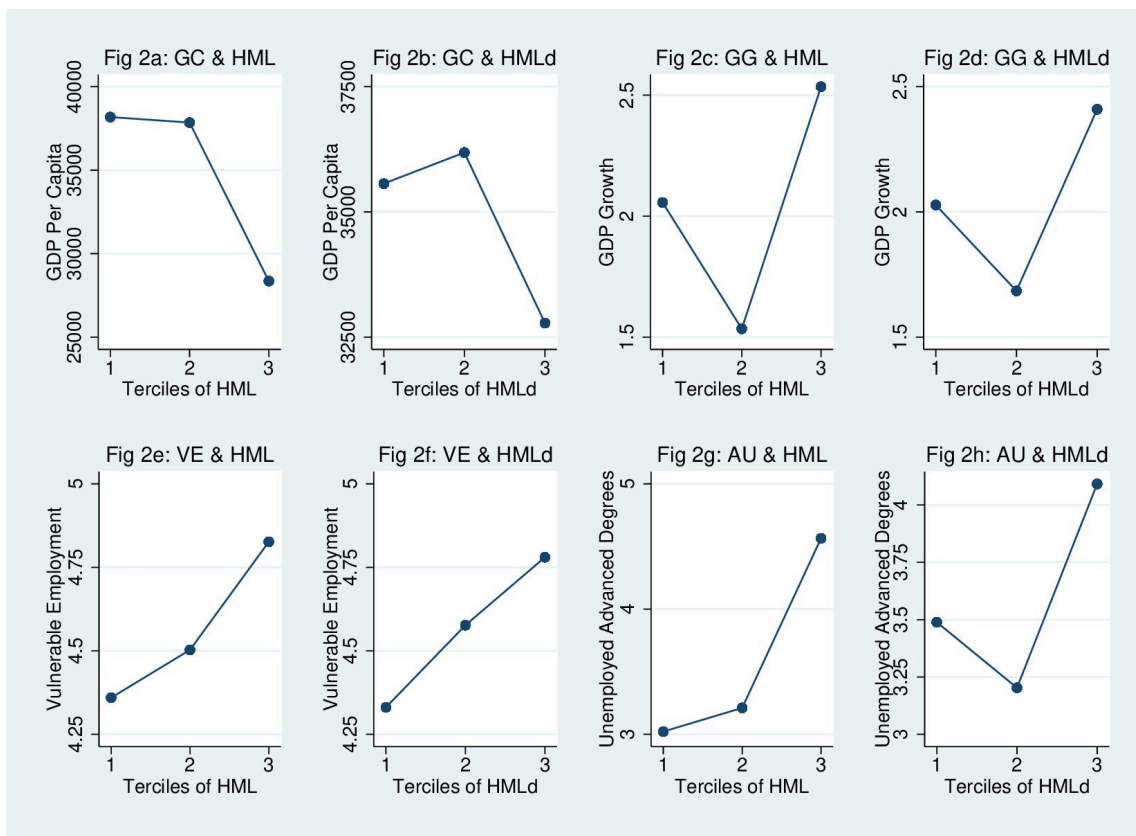


Figure 2. graphs the predicted values of the macroeconomic outcomes in Table 3 against either HML or HMLd.

Invoking Corollary 2, which is established in Section 3, the ordering of performance that evinces an optimal intertemporal evolution of a macroeconomic quantity, $\varkappa$, is for each of gc , gg , ve , or au ,

$$\begin{aligned} \Lambda(\text{rational_saddle}) &> \Lambda(\text{low_growth}) > \Lambda(\text{high_growth}) \\ &\equiv MAC(\text{tercile}_2) > MAC(\text{tercile}_1) > MAC(\text{tercile}_3). \end{aligned}$$

A perusal of Fig. 2 shows it is solely Fig. 2b that satisfies the enumerated condition. However, since $HMLd$ proxies for the price path, ρ , yet again it is solely the evolution of the price path that is shown to be optimal. Further, with the evolution of gc in Fig. 2a, respectively, Fig. 2b directly dichotomous to the evolution of gg in Fig. 2c, respectively, Fig. 2d, regardless there is arrival at time inconsistency in the evolution of gg vis-a-vis gc . In aggregate, the empirical evidence corroborates the predictions in each of Lucas^[112], Kydland and Prescott^[66], and Calvo^[67] that monetary policy is inimical to a sustainable progression to agents' welfare.

4.3.2. Testing for Forecast Power

If the empirical results in Panel C of Table 2 are robust, policy actions that are taken 5 years prior (5-yr lag); 4 years prior (4-yr lag); 3 years prior (3-yr lag); 2 years prior (2-yr lag); 1 year prior (1-yr lag); or contemporaneously (*cm*) are not, for each sample year, homogeneously specified, else the forecasting technology turns out naive, that is, non-dynamic. For concreteness, if the 5-yr lag for either *HML* or *HMLd* created concern within the Federal Reserve, necessarily it engaged with a monetary policy action, the full effect of which feasibly materializes five years subsequently. Importantly, note that the regressions embed a combination of anterior distributions - the historical realizations for *HML* or *HMLd* - and posterior distributions, namely the relevant future realizations of the macroeconomic outcomes, as such are formulated purely for inferences in respect of the robustness of the new empirical structure. Absent any prejudice to the foregoing, the robustness of this study's endogenous forecasting algorithm boils down to evidence. In stated respect, if the forecasting algorithm is robust, it ought to rationalize some of the notable events that have transpired in stock markets, such as *any* of the stock market crash of October 1987; the stock market crash of 2000; the recovery of the stock market by 2003; or the stock market crash of 2008.

Fig. 3 furnishes the graphs for each of the five different forecast horizons and the contemporaneous outcome for each of the years, 1981; 1987-1989; 1993; 1999-2001; 2004; 2007-2009; and 2016. The graphs do not reveal any linear paths. All of the forecast paths are, as such, non-linearly distributed. Further, the graphs show the forecasts and the shapes of the curves do not repeat for any two years that are drawn at random. In aggregate, Fig. 3 shows the forecast algorithm passes the first test of robustness, namely that it's forecasts are dynamically evolving. Suppose then the stock market crash of October 1987. If the predictive algorithm is robust, it is robust to the evidence in studies, such as, Siegel^[113] that the crash of the stock market on October 19, 1987 is not reflected in the macroeconomic outcomes. Further, it is robust to the evidence that the crash did not have any significant impact on spending, equivalently GDP in 1987, and that the macroeconomy, that is, GDP did not sputter in 1988. In stated respect, the historical series of *gc* and *gg* for 1987-1988 are, respectively \$33,761, \$34,853 and 2.54%, 3.24 percent. Consistent with the evidence in the literature and the historical outcomes, all of the forecasts for 1987 increase with a decrease to the forecast window, such that the 2-yr or 1-yr ahead forecasts come out at about \$30,000, which closely approximates the realized *gc* of \$33,761. Further, all of the lagged and current forecasts predict increase to each of *gc* or *gg* between 1987 and 1988, namely: with *gc* in 5,000's, a range of 4 to 8 vis-a-vis a unilaterally higher range of 8 to 14 for *gc*; and a range of 2% to 3% vis-a-vis a range of 2% to

6% for *gg*. Evincing the robustness of each of *HML* or *HMLd*, in the 1986-1989 time period, each is solely negative in 1987 – a realization of $-.61$ for *HML* and a realization of -0.05 for *HMLd* (the distributions of *HML* and *HMLd* are provided in Exhibit 1 of Section 2).

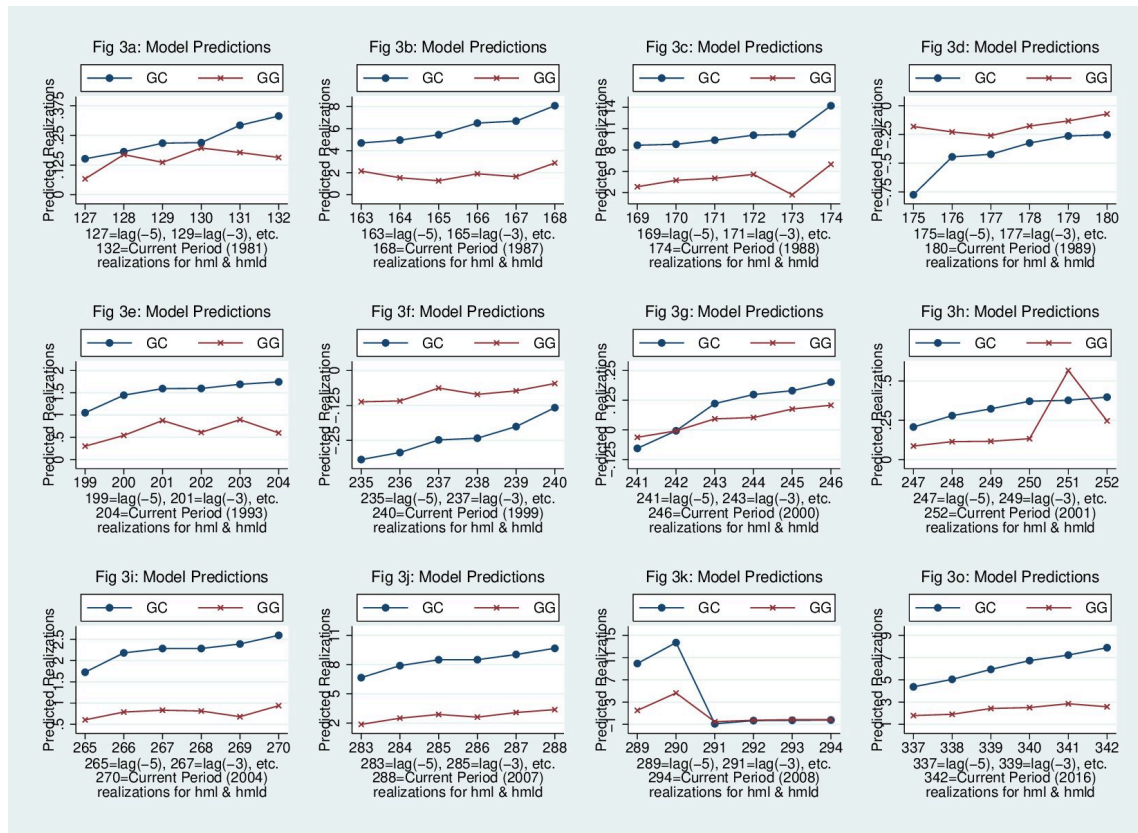


Figure 3. Alongside the predictions that are obtained from equation (19), Fig. 3 graphs forecast structures in which the contemporaneous realizations for each of *HML* and *HMLd* in equation (19) are replaced with their 5-year; 4-year; 3-year; 2-year; and 1-year lags for specific sample years.

Consider then the event years, 1999 and 2000. Whereas 1999 was a better year for stock markets than 2000, in terms of *gc*, the year 2000 outperformed the year 1999 – *gc* increased from \$43,435 to \$44,727 – a feature that is captured by the predictions in Fig. 3. To see this, whereas the forecasted range for *gc* in Fig. 3f is, $-.375$ to $-.125$, the forecast range in Fig. 3g is, $-.10$ to $.20$, a unilaterally higher range. Consistent with the forecasts, whereas *HML* and *HMLd* equal, respectively, $-.11$ and $-.23$ in 1999, in 2000 they equal $.85$ and 5.83 , respectively. In line with the evidence that the stock market did not recover until about 2003, whereas the forecasts of *gg* for 2001 reside between $.25$ and $.40$, for 2004, the same forecasts reside between 1.50 and 2.50 . Lastly, consider the years 2007 and 2008. Per the 3-yr ahead forecast, Fig.

3k shows the model forecasts gc to be lower in 2008 in relation to 2007. Importantly, with 2008 as the sole year with the pattern, the 3-yr ahead forecast embeds a huge correction to the forecast of a higher gc in 2008 by the preceding 5-yr and 4-yr ahead forecasts. Consistent with the 3-yr ahead forecast, gc decreased slightly from \$49,856 in 2007 to \$49,319 in 2008. In line with the evidence that the financial crisis was induced by overinvestment in the economy, that is, by risk that induced an increase to the severity of uncertainty^[114] (see for example, Fostel and Geanakoplos^[115]), whereas HML tatonnes from 0.58 in 2007 to 3.62 in 2008, $HMLd$ decreases from 0.15 to $-.55$. In aggregate, the evidence in Fig. 3 evince the robustness of the forecast structure that is endogenously embedded in this study's newly formulated empirical structure.

5. Conclusions

Whereas a notion that monetary policy can be sub-optimal to the management of a macroeconomy might, rather mistakenly be deemed a pessimist's sentiment, the literature asserts otherwise. In stated respect, rather seminally each of Lucas^[112] and Kydland and Prescott^[66] furnish formal theoretical evidence that it is fiscal policy that ought to be the primary tool for the management of a macroeconomy.

Concurring, Fischer^{[116][117]} arrive at the inference that monetary policy ought to be no more than a policy that, whenever necessary, is applied towards the stabilization of fiscal policy. Also concurring, in Calvo^[67] under perfect conditions, monetary policy induces the evolution of a macroeconomy to embed some time inconsistency (this study explicitly infers time inconsistency in the evolution of the macroeconomic quantities of the U.S. economy). Imposing the enumerated evidence, whenever it is adopted as a primary, as opposed to a secondary policy, monetary policy is predicted to be non-robust to the enhancement of agents' welfare. The finding in Romer and Romer^[65] that monetary policy has had a permanent damaging impact on employment and output in the USA refers. However, with that study premised on proprietary textual data from the Federal Reserve, the study does not facilitate any accessible general equilibrium structure for assessing the impact of monetary policy on agents' welfare.

With this study's empirical structure having characterization as a direct implementation of the predictions of a facilitating formal theoretical structure, this study implements a new 'instrumental' empirical structure which facilitates assessments as to whether a monetary authority's policy actions can be asserted to have been beneficent or harmful to agents' welfare. The empirical structure is 'instrumental' in the sense that it does not require any parameterization of the timing of a monetary authority's policy actions, rather consists of a menu of governing conditions and a property that any

robust measure of either macroeconomic risk or uncertainty is restricted to possess. Whereas the governing conditions and the alluded property are explicitly inferred, they are rendered most effective by the imposition of a canon that is already well established in the literature, namely the insight that every *primal quantity problem* has reformulation, analogously as a *price dual equivalent*. The studies that establish the canon are inclusive of, Balinski and Baumol^[50], Heal^[51], Takayama and Woodland^[52], or Weitzman^[53]. Applying the canon, every macroeconomic outcome having formulation as a macroeconomic quantity (a *msv*) is parameterized by either a macroeconomic factor, $\varkappa$, which is a solution to the primal quantity problem, or by a macroeconomic factor, ρ , that is a solution to the price dual reformulation of the macroeconomic quantity. This study's formal theory infers that either $\varkappa$ or ρ is robust to the workout of a primal quantity problem, respectively a price dual equivalent if it embeds a kink, or more robustly a saddle in the interior of the distribution. The robustness of the enumerated inference is evident in the realization that it is key to the results in each of the four seminal enumerated studies. Simultaneously, the robustness of the requirement of a saddle to monetary policy is evident as follows. Since conventional monetary policy (aka '*leaning against the wind*') attempts to stimulate growth, equivalently risk in the lower segments of growth, yet also attempts to dampen growth, equivalently risk or uncertainty in the upper segments of growth, the said combination is rational *if and only if* there exists a kink, or more robustly a saddle between the low and high growth segments of a macroeconomy's growth continuum.

This study's formal theory shows a macroeconomy whose macroeconomic quantities evolve optimally is parameterized by an increasing strictly concave relation between a *msv* and $\varkappa$. If, simultaneously a macroeconomy evolves optimally with respect to prices, a *msv* is decreasing strictly convex in ρ . If contrarily a macroeconomy does not evolve optimally, a *msv* is either increasing linear or increasing strictly convex in $\varkappa$; the inferred sub-optimality is explicitly linked to monetary policy. Let a *MACp* be a macroeconomic price that, simultaneously is not natively a quantity. A macroeconomic outcome that is increasing strictly concave in ρ and decreasing strictly convex relation in $\varkappa$ has characterization as a *MACp*; the existence of a *MACp* furnishes concrete evidence that a macroeconomy has optimally engaged with the formulation of fiscal policy. Importantly, since every domain of either risk or uncertainty is, via the formal theoretical evidence, restricted to embed a saddle in it's interior, such that the low-growth and high-growth regions are non-contiguous, as such, non-directly comparable, note that the inferred strictly concave optimality condition does not embed any binding contradiction. Absent this study, there is not any prior empirical study of the efficacy of monetary policy whose governing

conditions are natively structurally (distributionally or intertemporally) qualitative, as opposed to consisting in the directionality or statistical insignificance of regression coefficients; in stated respect, note that simulations whose path are spanned by some parametric algorithm lack characterization as structural qualitative outcomes. Applying the enumerated structure, whereas the evolution of prices in the USA has been optimal, contrarily the evolution of the macroeconomic quantities is inferred to have been sub-optimal. Noting that it is prices that are adopted as the tools of monetary policy, as is implied in the enumerated seminal studies (see for example, ^[53]), the adoption of a price, ρ , towards the management of the quantities of which a *msv* is representative is sub-optimal, because in the contexts in which monetary policy attempts to influence quantities, it is the macroeconomic factor, π , that is robust to the management of the quantity dimensions of a *msv*. Evincing the robustness of the enumerated insight, this study's formal theory infers that the management of economic growth is most effective *whenever* a government adopts a *rational-risk incentivization instrument (rii)*, which incentivizes agents to optimally and rationally engage with risk. The availability of a robustly formulated *rii* is shown to Pareto dominate the availability of any monetary policy instrument, because with the *rii* moderating the *price of risk*, such that it is optimally realized, agents are able, via investment choice, to engage with the formulation of optimal quantities. With any robustly formulated *rii* then having rather necessarily formulation as some sort of *fiscal price*, as is seminally recommended in Lucas^[112] and buttressed in each of Kydland and Prescott^[66] or Fischer^{[116][117]}, the management of economic growth using fiscal policy Pareto dominates the alternative, namely the management of growth using monetary policy. Concurring with the insight, Cesa-Bianchi and Rebucci^[118] and Cao and Chollete^[119] arrive at the inference that, rather ineffectively, monetary policy attempts to resolve two dichotomous risks – the need to stimulate growth in the lower segments of risk vis-a-vis the need to dampen uncertainty, equivalently risk in the higher segments of risk – with a monetary policy instrument. This study's formal theory explicitly demonstrates that there is not any monetary policy instrument that possesses the defining property of a *rii*.

Consistent with the existence of a wedge between the risk channel and the real sector channel for the transmission of monetary policy, prior studies find that neither the stock market crash of 'Black October (1987)' nor the stock market crash that transpired in 2000, both of which emanated strictly within the stock market (see for example, Siegel^[113] for the 1987 crash; Hand^[120] for the crash in 2000) had any adverse impact on *Real GDP Per Capita* (GDPCAP). The forecast structure that is endogenously embedded in this study's empirical structure correctly predicts absence of any adverse impact on GDPCAP in either

1987 or 2000. Rather presciently, however, the forecast structure anticipates the adverse impact of the 2008 financial crisis on GDPCAP, a crisis which contrary to the stock market meltdowns of 1987 or 2000, did not emanate from within the stock market, rather originated within the mortgage sector. In stated respect, using the endogenously embedded forecasting structure, the measures for either macroeconomic risk or uncertainty have forecast power for directly predicting qualitative changes (up or down moves) to either GDPCAP or GDP Growth for the adjacent years, 1987/1988; 1999/2000; or 2007/2008. Since all three sets of adjacent periods experienced some adverse perturbations, such that it is unlikely that a smooth process possesses predictive accuracy, the choice of the three sets of years biases the forecast evidence against predictive power. Regardless, all 3-year-ahead forecasts (e.g. a forecast of the GDP Growth for 1987 that is obtained using the κ and ρ that obtained only up until 1984) of the qualitative changes to either GDPCAP or growth are shown to be spot-on; further, switching to the 5-year ahead forecasts, it is solely the 5-year-ahead forecast for the 2007/2008 period that is not spot-on. In aggregate, in presence of the empirical evidence and the emergence of a robustly and pragmatically formulated fiscal policy structure in Obrimah^[121] that conceptually embeds a *rii* – a secondary *rii* which enhances that conceptually formulated *rii* is teased out in a working paper, Obrimah^[122] – it perhaps is time for the monetary policy authorities to engage with the recommendation that has earned Nobel Prizes in Economics for each of Robert Lucas (in 1995), Finn Kydland (in 2004), and Edward Prescott (also in 2004), namely that monetary policy be seconded to fiscal policy, that is, to the adoption of a *rii*.

Footnotes

¹ In Bansal and Yaron^[123], an assumption that growth spans risk rationalizes the evolution of risk and returns in markets; however, the study does not establish the insight, rather finds the *assumption* to be robust to some stylized features of markets.

Appendix

Proof of Condition 1. With risk initially low (due to the availability of low growth assets), feasibly growth is a linear or strictly convex function of risk. However, if an economy is solely parameterized by low realizations of risk, risk is restricted to be sub-optimally low. Suppose otherwise; then with σ_t deemed to be low, that is, relatively safe, $\nexists$ any $\varepsilon > 0$, $\varepsilon \xrightarrow{\text{from_above}} 0$ satisfying, $\sigma_{t+1} = \sigma_t + \varepsilon$, that parameterizes rational risk. Iterating the inference, the probability, ϕ , of a high risk asset satisfies, $\phi = \emptyset$; since there

ought to be some high risk assets in any rationally parameterized economy, there is arrival at the inference that risk is sub-optimally low.

Alternately, holding risk to be initially high, the necessity of a supremum of rational risk^{[61][32]} restricts growth to be strictly concave in risk. Suppose then that an economy attains to high realizations of risk. Since monetary policy actions serve to rein in risk, such that it does not exceed its rational expectations supremum, either growth is concave in *HML*, as such, evidence for adherence to rationality that feasibly is aided by monetary policy, or growth is strictly concave in *HML*, evincing a macroeconomy that exceeds the rational realizations of risk.

With the formal theoretical evidence for the existence of a kink in the interior of *HML* furnished in Section 3, the existence of such a kink – as such, the non-feasibility of a smooth function – rationalizes the feasibility of growth that is either linear or strictly convex in risk at the low realizations of risk and growth that is either concave or strictly concave in risk at the high realizations of risk. Importantly, absent such a kink, a segmentation of *low risk* from *high risk* is lacking in robustness. $\square$

Proof of Condition 2. Follows directly from the proof of Condition 1 and the following argument. Since default risk has characterization as a high risk and simultaneously growth assets have characterization as high risk assets, all of the continuums of risk have characterization as *high risk* continuums. Imposing rational expectations, the feasibility of either a linear or a strictly convex relation between growth and *HML* is excluded. $\square$

Proof of Condition 3. Applying the trade-off theory of capital structure and imposing rational expectations, necessarily leverage (v) satisfies, $v(D10, LG) > v(D01, HG)$. If each of $v(D10, LG)$ or $v(D01, HG)$ are optimal, default risk (d) satisfies, $d(D10, LG) \equiv d(D01, HG)$; as such, Decile 10 does not consist of assets that are parameterized by relatively high default risk. The condition, $v(D10, LG) > v(D01, HG)$ is, as such not a sufficient condition for ascertaining that the assets in Decile 10 are parameterized by high default risk.

Suppose a monetary authority seeks to rein in risk, as such increases a reference interest rate. Imposing rational expectations, the monetary authority does not anticipate an increase to investments in assets that have high default risk. Suppose, contrarily that a monetary authority seeks to stimulate growth. Clearly, growth that is accompanied by high default risk is inimical to the stabilization of an economy. Necessarily, as such, a monetary authority does not anticipate an increase to investments in assets that

have high default risk. Imposing then Conditions 1 and 2 and rational expectations, Decile 10 is made up of low growth firms. $\square$

Proof of Theorem 1. By equation (15), g^h and g^l have a common property only whenever, $[g^h \cap g^l] = \vartheta^l$. The common property is, as such, a risk parameter. By equations (14) through (16), g^h and g^l do not have any property in common whenever, $g^h = \{\varsigma^h, \vartheta^h\}$. Presence of dichotomy between g^h and g^l is, as such, parameterized by risk parameters. Regardless then whether g^h and g^l have a common property or are dichotomously realized, they are described by risk parameters. Applying equations (14) through (16),

$$[g^l]^c = [g^h \cap g^l]^c = [\vartheta^l]^c = [\varsigma^h \cup \vartheta^h],$$

such that, $g^h = \{\varsigma^h \cup \vartheta^l\}$ is solely commensurable with g^l , not $g^h = \{\varsigma^h \cup \vartheta^h\}$, an outcome that is consistent with the stylized fact that it is systemic risk that parameterizes asset prices. Imposing the insight, the systemic risk of the h assets only can be hedged via diversification into assets that incorporate ϑ^l . Since then investments in ϑ^h lack qualification as efforts for diversification, the directionality of diversification is non-invertible.

Suppose then that an agent holds the h type assets that are parameterized by, $[\varsigma^h \cup \vartheta^h]$ and diversifies into the l type assets. In the interior of the continuum, ϑ^l spans a segmentation of the continuum of growth into two incommensurable segments, namely the lower segment parameterized by, ϑ^l , and the upper segment parameterized by, ϑ^h . For concreteness of the incommensurability, suppose a movement, $\tau \in \mathfrak{R}^+$ to either the left or right of ϑ^l for arrival at, respectively, $[\varsigma_\tau^l \cup \vartheta^l]$ to the left or $[\varsigma_\tau^h \cup \vartheta^l]$ to the right. Noting that, $\varsigma_\tau^h > \varsigma_\tau^l$, whereas $[\varsigma_\tau^h - \varsigma_\tau^l] \rightarrow 0$, as $\tau \rightarrow 0$, such that, $g \rightarrow \vartheta^l$, contrarily, $[\varsigma_\tau^h - \varsigma_\tau^l] \rightarrow 0$ spans, $g \rightarrow \vartheta^h$; imposing the foregoing, there is arrival at,

$$([\varsigma_\tau^h - \varsigma_\tau^l] \rightarrow 0) \cap ([\varsigma_\tau^h - \varsigma_\tau^l] \rightarrow 0) = \emptyset,$$

as such, applying equations (14) through (16), the existence of some threshold point, $[\varsigma^h \cup \vartheta^h] = [\varsigma^h \cup \vartheta^h] = \inf(g^h)$, that segments the lower continuum of g that is parameterized by, ϑ^l , from the upper continuum of g that is parameterized by, ϑ^h . $\square$

Proof of Corollary 1. Follows directly from Theorem 1 as follows. Applying Theorem 1, $\exists$ some ς^h which is commensurable with all ϑ^l that is parameterized by, $[\varsigma^h \cup \vartheta^l] > \vartheta^l$ satisfying, $\sup[l] = [\varsigma^h \cup \vartheta^l]$. Further, $\inf[l] = \vartheta^l$, because

$$\vartheta^l < [\varsigma^l \cup \vartheta^l] < [\varsigma^h \cup \vartheta^l] \forall \{\varsigma^l, \varsigma^h\}.$$

For h type assets, applying Theorem 1, $\exists$ some asset $\hat{h}$ satisfying, $\inf [\varsigma^h \cup \vartheta^h] = [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}] > \vartheta^h > \vartheta^l$; further, applying Obrimah^{[82][32]}, $\exists$ some asset $\check{h}$ satisfying, $\sup [\varsigma^h \cup \vartheta^h] = [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]$. $\square$

Proof of Theorem 2.

Scenario I: A decrease to i , equivalently the expectation of a decrease to δ , seeks to induce a decrease to π , as such seeks to stimulate g . With g then presumed to be *too low*, applying Theorem 1 and Corollary 1, necessarily g resides in the low growth segment of g 's kinked continuum. Applying the stated insight, Obrimah^{[61][30]}, Condition 1 of Section 2, and Corollary 1, said risk is lower than or equal to, $[\varsigma^{\acute{h}} \cup \vartheta^l]$.

Since it is unequivocal that, $i \neq \vartheta^h$, and $i \neq \vartheta^l$, the *CMP* parameter i , is unable to facilitate, $[\varsigma^{\acute{h}} \cup \vartheta^l] \rightarrow [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$. Corollary 1 infers the infimum and supremum for each of the low or high growth segments of the growth continuum. Imposing the foregoing and Corollary 1, $[\varsigma^{\acute{h}} \cup \vartheta^l] \rightarrow [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$ is solely facilitated by some $\varsigma^* \in [[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}], [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]]$, satisfying (' $\circ$ ' equates to a *mathematical composition*),

$$[\varsigma^{\acute{h}} \cup \vartheta^l] \circ (\varsigma^*) \equiv [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}];$$

since ς^* is a transforming parameter, which works in time, there is arrival at the necessity of a 'saddle' between $[\varsigma^{\acute{h}} \cup \vartheta^l]$ and $[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$. Further, ς^* connects $[\varsigma^{\acute{h}} \cup \vartheta^l]$ with $[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, *if and only if* the following rationality condition,

$$\varsigma^* \in \mathfrak{R}^+ \equiv [\vartheta^l, [\varsigma^{\acute{h}} \cup \vartheta^l]] \cup [[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}], [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]],$$

is satisfied; but then can the condition be shown to be satisfied, such that ς^* exists?

Suppose that always the investment opportunity set supports all asset types, that is, both low-growth and high-growth assets. Let ς^* be a rationality constraint that facilitates a choice between ϑ^l and ϑ^h using,

$$\begin{aligned} \varsigma^*(\vartheta^h) &\succ \varsigma^*(\vartheta^l) \xrightarrow{\text{choose}} \vartheta^h \\ \varsigma^*(\vartheta^h) &\prec \varsigma^*(\vartheta^l) \xrightarrow{\text{choose}} \vartheta^l. \end{aligned}$$

Clearly, post engagement with the asset that is parameterized by, $[\varsigma^{\acute{h}} \cup \vartheta^l]$, imposing rationality, namely non-satiation with respect to wealth, there is arrival at, $\varsigma^*(\vartheta^h) \succ \varsigma^*(\vartheta^l)$; as such, a rational agent engages with,

$$[\varsigma^{\acute{h}} \cup \vartheta^l] \circ [\varsigma^*(\vartheta^h) \succ \varsigma^*(\vartheta^l)] \equiv [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]. \quad (22)$$

Similarly, since a rational agent who starts off in the high-growth segments chooses $\sup [\varsigma^h \cup \vartheta^h] = [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]$ first – an implementation of the NPV rule – then proceeds to choose high risk projects until arrival at, $\inf [\varsigma^h \cup \vartheta^h] = [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, whenever assets are chosen without replacement, with $\vartheta^h = \emptyset$, post $[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, there is arrival at,

$$[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}] \circ [\varsigma^*(\vartheta^h) \prec \varsigma^*(\vartheta^l)] \equiv [\varsigma^{\hat{h}} \cup \vartheta^l]. \quad (23)$$

Since assets are without replacement, the agent goes on to navigate through all low-growth assets. The foregoing demonstrates that ς^* exists; in stated respect, the assumption that a rational agent jumps from $[\varsigma^{\hat{h}} \cup \vartheta^l]$ to $[\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]$, then navigates through all of the lower return assets is supported by the foregoing. Importantly, given i does not have the property of a rationality constraint, because preferences are predicated on objects, not prices, $i \neq \varsigma^*$.

Applying the foregoing, there is arrival at the necessity of a *rii*, $\pi^c(\varsigma^*)$, which incentivizes agents to engage with the effort, $[\varsigma^{\hat{h}} \cup \vartheta^l] \circ (\varsigma^*)$ or the effort, $[\varsigma^{\hat{h}} \cup \vartheta^h] \circ \varsigma^*$. Since $i \neq \vartheta^h$, and $i \neq \vartheta^l$, $\pi^c(\varsigma^*) \neq i$. Suppose otherwise, namely that $\pi^c(\varsigma^*) = i$. Let the symbol, ‘ $\parallel$ ’ denote ‘commensurable with’, that is, ‘having the same property as’. Note that applying Corollary 1, and the property that, $\varsigma^* \in \mathfrak{R}^+$, there is arrival at the rationality condition,

$$i \in \mathfrak{R}^+ \equiv [\vartheta^l, [\varsigma^{\hat{h}} \cup \vartheta^l]] \cup [[\varsigma^{\hat{h}} \cup \vartheta^h], [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]].$$

The proof to follow shows, using the principle of *Reductio Ad Absurdum* that $i(\varsigma^*) \notin \mathfrak{R}^+$. An i satisfying, $i \parallel [\vartheta^l, [\varsigma^{\hat{h}} \cup \vartheta^l]]$ is robust to $[\varsigma^{\hat{h}} \cup \vartheta^l]$, but is restricted to satisfy,

$$(i \parallel [\vartheta^l, [\varsigma^{\hat{h}} \cup \vartheta^l]]) \rightarrow (i \not\parallel [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]),$$

and vice versa, because in $[\vartheta^l, [\varsigma^{\hat{h}} \cup \vartheta^l]]$, $\Delta i < 0$, but in $[[\varsigma^{\hat{h}} \cup \vartheta^h], [\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]]$, $\Delta i > 0$; plus, $i(\varsigma^{\hat{h}} \cup \vartheta^l) < i(\varsigma^{\check{h}} \cup \vartheta^{\check{h}})$, resulting in combined, the property, “to progress rationality, decrease i if i is already low, or increase i if i is already high”. It is straightforward that the enumerated property of i is incommensurable with, “at $[\varsigma^{\hat{h}} \cup \vartheta^l]$, which is parameterized by, $\varsigma^*(\vartheta^h) \prec \varsigma^*(\vartheta^l)$, to progress, switch to, $\varsigma^*(\vartheta^h) \succ \varsigma^*(\vartheta^l)$; and at $[\varsigma^{\check{h}} \cup \vartheta^{\check{h}}]$, which is parameterized by, $\varsigma^*(\vartheta^h) \succ \varsigma^*(\vartheta^l)$, to progress, switch to, $\varsigma^*(\vartheta^h) \prec \varsigma^*(\vartheta^l)$ ”. The incommensurability is further evidenced as follows. Suppose $i(\varsigma^*, \vartheta^l) \parallel [\vartheta^l, [\varsigma^{\hat{h}} \cup \vartheta^l]]$. It is unequivocal that,

$$[i(\varsigma^*, \vartheta^l) \rightarrow 0] \rightarrow [i(\varsigma^*, \vartheta^l) \rightarrow i(\varsigma^*, \vartheta^h)],$$

because always within the same yield curve, $i(\varsigma^*, \vartheta^h) > i(\varsigma^*, \vartheta^l)$. By the same argument,

$$[i(\varsigma^*, \vartheta^h) \rightarrow \infty] \rightarrow [i(\varsigma^*, \vartheta^h) \rightarrow i(\varsigma^*, \vartheta^l)],$$

because always within the same yield curve, $i(\varsigma^*, \vartheta^h) > i(\varsigma^*, \vartheta^l)$. In aggregate, whereas $\varsigma^*(\vartheta^h) \parallel \varsigma^*(\vartheta^l)$, because equations (22) and (23) are simultaneously true, contrarily $i(\varsigma^*, \vartheta^h) \nparallel i(\varsigma^*, \vartheta^l)$, as such, via *Reductio Ad Absurdum*, there is arrival at,

$$\begin{aligned} [\varsigma^* \nparallel i] &\rightarrow [\varsigma^* \neq i] \\ &\rightarrow [(\pi^c(\varsigma^*) \nparallel i) \rightarrow (\pi^c(\varsigma^*) \neq i)]; \end{aligned}$$

the monetary policy tool, i , is, as such, non-robust to the facilitation of, $[\varsigma^{\dot{h}} \cup \vartheta^l] \rightarrow [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$.

Imposing Theorem 1 and Corollary 1, $\nexists$ any drift (smooth) process that facilitates, $[\varsigma^{\dot{h}} \cup \vartheta^l] \rightarrow [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, because g is kinked at each of, $[\varsigma^{\dot{h}} \cup \vartheta^l]$ and $[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, resulting in the existence of a ‘saddle’ between the two points; in stated respect, since Theorem 1 and Corollary 1 impose, $[\varsigma^{\dot{h}} \cup \vartheta^l] \neq [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$, the existence of a saddle, equivalently the existence of a drift process that is not parameterized by either ϑ^l or ϑ^h is unavoidable. Let ξ denote a function. Since the set of growth points, g^s , such that,

$$g[\varsigma^{\dot{h}} \cup \vartheta^l] < g^s < g[\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}],$$

satisfies, $g^s \neq \{g^l, g^h\}$, and is parameterized by, $g^s = \xi(\varsigma^*)$, it is made up of only one type of growth, namely $g^s \neq \{g^l, g^h\}$, that is by, $g^s = \xi(\varsigma^*)$, as such, consists of a single type of asset, equivalently a single point, resulting in a zero mass, that is the satisfaction of the requirement in Dasgupta and Maskin^[54] that a rational saddle have zero mass.

Let QE denote, ‘Quantitative Easing’, a strategy that is available solely in the context of a low-growth regime. Suppose a substitution of QE for i , such that QE is the monetary policy variable. In the context of QE , to stimulate investments, a monetary authority buys back some government bonds, such that agents have new funds to deploy towards investments in the private sector. Let M denote the monies that are received by agents from the implementation of QE . The rationality of QE implies $\vartheta^l \leq g^M < [\varsigma^{\dot{h}} \cup \vartheta^l]$. If then QE is effective, it facilitates the transition, $g^M \rightarrow [\varsigma^{\dot{h}} \cup \vartheta^l]$. However, since all risk in $[\vartheta^l, [\varsigma^{\dot{h}} \cup \vartheta^l]]$ has commensurably a low systemic risk, imposing rationality, an exhaustion of g^M by rational agents induces exactly, $g^M \rightarrow [\varsigma^{\dot{h}} \cup \vartheta^l]$; as such, rational agents do not require any incentive from government for the tatonnement, $g^M \rightarrow [\varsigma^{\dot{h}} \cup \vartheta^l]$. Suppose then that the agents arrive at, $[\varsigma^{\dot{h}} \cup \vartheta^l]$. Since $M(QE)$ lacks characterization as a rationality condition, it is parameterized by, $M(QE) \neq \varsigma^*$. As such, there is not any guarantee that, $[\varsigma^{\dot{h}} \cup \vartheta^l] \rightarrow [\varsigma^{\hat{h}} \cup \vartheta^{\hat{h}}]$; all that is guaranteed, as such, is an exhaustion of the low-growth assets in an economy. In aggregate, absent ς^* , there is not any guarantee that agents invest in high-growth assets. As then is empirically inferred in Weale and

Wieladek^[17], all else equal, QE is predicted to be no more effective at stimulating investments in high-growth firms than a decrease to interest rates.

Scenario II: An increase to i , equivalently an increase to δ , seeks to induce an increase to π , as such seeks to dampen g . Applying Obrimah^[30], Condition 1 of Section 2, Theorem 1 and Corollary 1, necessarily such a monetary policy action is solely undertaken in an economy that is located in the high-growth segment of g 's kinked continuum. The evidence in the preceding that,

$$\left(i \parallel \left[\vartheta^l, \left[\varsigma^{\acute{h}} \cup \vartheta^l\right]\right]\right) \rightarrow \left(i \nparallel \left[\varsigma^{\check{h}} \cup \vartheta^{\check{h}}\right]\right),$$

and vice versa completes the proof. In aggregate, imposing rational expectations, the availability of $\pi^c(\varsigma^*)$ Pareto dominates any and all engagements with *CMP*. $\square$

Proof of Corollary 2. Parts I and II follow directly from Theorems 1 and 2, Corollary 1, and the empirical evidence to follow in Table 1. Part III is established as follows. Let each of *HML* or *HMLd* be segmented into terciles, such that the highest growth firms are in $T = 1$ (*T1*) and the lowest growth firms are in $T = 3$ (*T3*). The formal theory in Section 2 shows $HML > 0$ is predicated on the following adaptation of equation (9),

$$MV_t[T3] < MV_t[T1] \tag{24}$$

$$\rightarrow \frac{V_{t+1}[T3]}{MV_t[T3]} > \frac{V_{t+1}[T1]}{MV_t[T1]} \rightarrow HML > 0, \tag{25}$$

which is directly spanned by, each of,

$$CEQ_t[T3] < CEQ_t[T1] \text{ and} \tag{26}$$

$$\delta_t[T3] < \delta_t[T1]. \tag{27}$$

Imposing equations (24) through (27), Tercile 3 of each of *HML* or *HMLd* outperforms Tercile 1. Noting that Tercile 2 is not equivalent to Tercile 1, equations (24) through (27) do not parameterize Tercile 2, because it is not an extreme tercile. Additionally, applying Obrimah^[61], for each of *HML* or *HMLd*, Tercile 2 (*T2*) embeds the highest severity of uncertainty, such that applying the proof of Theorem 2, the assets, $g^s \in T2$ satisfy, $g^s \neq \{g^l, g^h\}$, and are parameterized by, $g^s = \xi(\varsigma^*)$. With the defining property of the g^s then more ambiguous than that of the $g^h \in T3$ - the direct implication of a higher severity of uncertainty as to their true value - there is arrival at,

$$CEQ_t[T2] < CEQ_t[T3] \tag{28}$$

$$\rightarrow \delta_t[T3] < \delta_t[T3], \tag{29}$$

which combined, as in Section 2, facilitate a positive probability for,

$$MV_t [T2] < MV_t [T3] \quad (30)$$

$$\rightarrow \frac{V_{t+1} [T2]}{MV_t [T2]} > \frac{V_{t+1} [T3]}{MV_t [T3]} \rightarrow [\mu (T2) - \mu (T3)] > 0. \quad (31)$$

Combined, equations (25) and (31) span,

$$\mu (T2) > \mu (T3) > \mu (T1) . \quad (32)$$

□

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Author's Contribution

The conceptualization, formal theoretical modeling, implementation of the empirical tests, and writing of this study all are attributable to the sole author, Oghenovo Obrimah, PhD.

Data Availability

The data and the Stata program that was applied to the data towards the generation of the empirical results are available in the ScienceDB data repository, specifically at: <https://doi.org/10.57760/sciencedb.11354>.

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Declarations

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Potential competing interests: No potential competing interests to declare.