Peer Review

Review of: "Generalized Inverse Preservers of Hadamard Circulant Majorization"

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1. Independent researcher

I doubt that Lemma 1.2 (which is taken from Lemma 2 and Lemma 3 of [1]) is correct.

Let $E_{i,j}$ be the matrix whose i, j-th entry is 1 and 0 elsewhere. Denote by E the matrix in which every entry is 1, and by $\circ$ the Hadamard product.

For each fixed pair i, j, consider the following linear operator:

$$T_{i,j} : A \mapsto a_{i,j} E,$$

where $a_{i,j}$ is the i, j-th entry in A.

Clearly, if there is a circulant matrix C such that

$$A \circ C = B,$$

then

$$(T_{i,j} A) \circ (c_{i,j} E) = (a_{i,j} E) \circ (c_{i,j} E) = b_{i,j} E = T_{i,j} B,$$

where $b_{i,j}, c_{i,j}$ are the i, j-th entries in B, C respectively.

Each $c_{i,j} E$ is, of course, circulant, hence each $T_{i,j}$ is a Hadamard circulant majorization preserver, but none of $T_{i,j} E_{i,j}$ can be dominated by any circulant permutation matrix. I think one can only conclude that they are dominated by some circulant matrix (instead of some circulant permutation matrix; there is no reason for that to hold).

The conclusion of the paper (as well as [1]) might actually still be true, but an alternative and correct proof should be provided.

Besides, I have the following three suggestions:

(1) Please include a brief introduction as to why this problem is important or how it can be applied in applications.

(2) Please include an example to show that such preservers and their different types of inverses exist a priori, i.e., something like example 1 in [1], in which we can easily see that the preserver has an inverse and is likely still a preserver (which I did not check).

(3) Terms like "... is dominated by ..." as in the statement of Lemma 2 should be defined before use. Also, typos should be fixed (e.g., at the end of theorem 1.1, parentheses are missing).

Declarations

Potential competing interests: No potential competing interests to declare.