Research Article

The Uniqueness of the Medieval Persian Art

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I examine the symmetry used in the ornamentation of various medieval civilizations and interpret it as a characteristic feature of each cultural group. I first classify the symmetries according to the 17 wallpaper groups of crystallography, then determine their frequency and apply statistical methods to obtain quantitative and comparative information for each of these groups. The calculations show that the civilizations can be divided into two groups according to their habits: the Islamic group of Seljuks and Arabs and the Greco-Roman group, which includes Armenians, the Eastern Roman Empire and the Kingdom of Andalusia. Remarkably, Persia stands out from this division with its own art movement until the 16th century.

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1. Introduction

It is a nomadic expansion when peoples move around, conquer new lands, seek new adventures and, if possible, settle in a place other than their original homeland. The Ottoman Empire in the past or today's United States of America are just two examples of this practice. Other peoples remain in their birthplace, even if their borders are altered somewhat over time and even their names change over the centuries. China, India, Egypt or Greece are examples for this behavior. And Persia is certainly a prominent member of this type of empire. It is located in the middle of the Silk Road, in the prairies and high mountains of Western Asia, between the Caspian Sea and the Indian Ocean. Figure 1 shows a map of Iran, today's Persia. In the east, it has Indo-European neighbors such as Afghanistan, which speaks a language of the same origin as Persian, or some Turkic-speaking states such as Turkmenistan. In the west, there is a diverse combination of cultures represented by Armenia, Azerbaijan, Iraq and Turkey. The Persians themselves are of Indo-European descent, they have Arian roots [1].

My aim is to study Persian artworks to find ornaments that are periodic in two dimensions and cover the surface. I examine their symmetry properties, Persian preference for symmetries, and compare it with the habits of the neighboring Middle Eastern civilizations. I argue that this comparison will shed light on the cultural interactions of the Persians with their neighbors. If the invading powers had some influence on the artistic output of the Persians, this would be evident in the results.

In Section 2, I outline the historical development of Persia with regard to the invading hordes up to the beginning of the 16th century, after which, due to the mobility of peoples, there is an intense mixing of artistic practice. Cultural characteristics of individual groups gradually disappear into a global artistic culture. I also summarize the crystallographic method used to classify the works of art and to establish connections between the products of different cultural groups. The next Section is devoted to the data collection and the analysis of the results using statistical instruments. In the final Section, I present the results of the analysis and draw conclusions.

2. Methods

I investigate the traces of culture of some medieval societies by examining mosaics or ornaments in general, especially, two-dimensional periodic ornaments found in most of the artworks. I have selected medieval peoples of the Middle East, from the pre-Christian period to the 16th century. My aim is to identify any significant mutual influence between Persia and the invading powers that might be recognizable in the results. I set the upper time limit to avoid a strong mixing of cultures.

Figure 1. A geographical map of present-day Iran (highlighted) with its neighboring countries. In ancient times, it expanded as far as Romania, waging war for years with the Hellenic cities of Athens and Sparta in the west and occupying Turkmenistan and Afghanistan in the east. (Adopted from *Maps*, Apple Inc.)

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Symmetry is used as evidence of the characteristic nature of the artwork. Analogous to the symmetry properties of atoms aligned on a crystal surface, ornaments can be arranged in 17 crystallographic groups, the *wallpaper groups* [2][3]. Analyzing the symmetry of the ornaments in the form of mathematical groups provides an objective quantity of the respective mosaic work. This idea is implemented by Edith Müller in her dissertation in mathematics at the University of Zurich [4]. In her work, Müller proved that the wallpaper group explains all possible symmetries created by the craftsmen in the Alhambra Palace in Granada.

Washburn and Crowe later introduced the idea that the symmetry distribution of all patterns in the 17 wallpaper groups is characteristic of the culture in which the patterns were created [5]. Thus, the pattern in a handcrafted mosaic is associated with a clear quantitative measure of its culture. Much later, I took this idea and applied it to different cultural groups to compare their achievements [6]. I assume that this comparison contains invaluable information about the interactions of the civilizations under consideration. Since the collected information about the symmetry of the mosaics is available in quantitative form, we can apply statistical recipes to draw rigorous conclusions about the cultural status of the peoples. The novel idea lies in the interpretation of artistic creation in numerical form to enable the application of statistical methods to extract information from the symmetry data that is not accessible by other methods.

2.1. Persia

Persia is one of the oldest known civilizations, with urban settlements in the Near East dating back 6000 years [1][7]. Throughout history, Persia flourished several times due to its military strength and was also a leader in art and architecture, literature, medicine, chemistry, etc. Agriculture flourished, arts and crafts, including weaving, were the best among other groups. Trade contributed significantly to Persia's wealth due to its excellent location. The golden age of Persia, the Achaemenid Empire under Cyrus the Great, stretched from Central Asia westwards to present-day Romania (cf. Figure 1). Persepolis was the capital. Achaemenid architecture produced magnificent examples. The empire's forces invaded the Hellenic peninsula several times. In return, the Macedonian troops of Alexander the Great put an end to the empire by burning Persepolis to the ground in 330 BC. In the following centuries, first the Seleucid Empire, which was led by one of Alexander's generals, then the Parthians and from 224 AD on the Sassanid Empire helped Persia to its glory. Nevertheless, the Romans and later the Eastern Roman Empire challenged Persia's western borders. The long-lasting Roman wars weakened Persia to such an extend that the newly-founded Muslim-Arab troops under Khalif Omar were able to invade the country in 632. The highly-developed Persian civilization aroused great admiration not only in the Roman Empire, but also in the Arab world after the Muslim conquest. The Persians were forced to convert to Islam and abandon their Zoroastrian tradition. The Arabs adopted the Persian customs, but brutally suppressed the traditional Zoroastrian community and tried to impose Arabic as the main language.

In 750, the Abbasids of Baghdad, replaced the Umayyads by force and took power in Persia. During this time, Persia had an increasingly multi-ethnic structure with a growing proportion of Turkic peoples from Central Asia. The conversion to Islam gave Persian society a certain incentive to progress in the fields of culture and science, including medicine, so that one can speak of the Golden Age of Islam. After several generations of Abbasid rulers, the Persian administration gradually gained power, but this time it was the Seljuks from the east who took over at the beginning of the 11th century. Nevertheless, Persia retained its identity and Persian culture exerted a strong influence on the invaders, including the revision of the new religion. Persian Islam had developed into Shiism and replaced Sunni belief. Through the military strength of the Seljuk dynasty in Persia, which became known as the Great Seljuk Empire, Persian culture spread from Afghanistan across Anatolia, where another sultanate was founded as Seljuks of Rum.

Around 1220, the Mongols led by Genghis Han conquered the Near East, destroying several cities and massacring the people; the invasion was devastating for Persian culture. Under the rule of later Mongol chieftains, Persia was able to recover, but was then ravaged by the Black Death, which killed a third of the population around 1350. Further destruction of the people and their facilities was caused by another Mongol invasion, this time by Timur. His empire is known as the Timurid Empire and lasted until the beginning of the 16th century. Unfortunately, Persia's favorable location was very attractive for the invading tribes who conquered Persia for their own interests. Macedonians, Arabs, Turks and Mongols left deep traces of devastation. Persian culture revived after 1500 with the Safavid dynasty, when the unity of the Persian Empire was restored [7]. I looked at the works of art up to this period. Today, the official name of the country is "The Islamic Republic of Iran".

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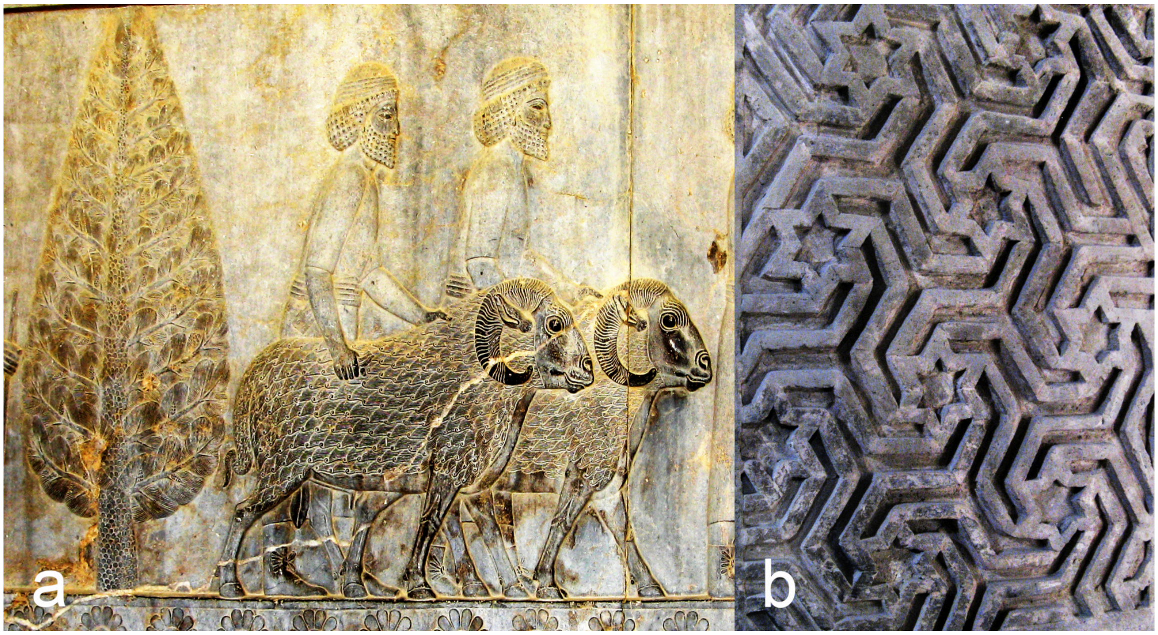

Figure 2. (a) Two shepherds with their rams, created around 500 BC in Persepolis. Note the periodic decorations on the tree trunk, the animals skin and the men's hair. The bas-relief of Sogdians is found in Apadana. (b) A wall decoration in the domed chamber of Tadj al-Mulk in Isfahan from the Islamic period in the 11th century. (Author's photographs)

2.2. Symmetries and the Wallpaper Group

In crystallography [8], an ordered surface phase is uniquely described by the *point-group operations* in which a *unit cell*, an atom or a molecule, is held stationary and rotations and reflections around one or more mirrors are applied. The rotations are expressed in numbers n that divide 2π and leave the unit cell unchanged when applied. Permitted numbers are $n = 1, 2, 3, 4, 6$. Larger numbers lead to overlaps, 5 leads to voids. Together with the reflections, these operations allow 10 possibilities, which are referred to as the *crystal class*.

To obtain an extended pattern in two dimensions, the stationary point group must be embedded in a two-dimensional translation, a lattice, with the appropriate symmetry. This is the *translational group*, which consists of five crystal systems: the oblique, the rectangular, the square and two hexagonal systems with threefold and sixfold symmetry. The combination of the point-group and the translation symmetry results in 17 groups that can be used to describe any two-dimensional pattern. They are referred to as the *wallpaper group* and consist of *p1, p1m1, p1g1, c1m1, p211, p2mm, p2mg, p2gg, c2mm, p3, p3m1, p31m, p4, p4mm, p4gm, p6, p6mm*. The letter c stands for a centered cell with a motive in the center of the grid. Otherwise it is a primitive cell with the designation p. The letter m stands for a mirror reflection, and g for a glide reflection, which results from the combination of the point-group and the translational symmetry [9][10]. A comprehensive study of the geometry

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of stucco ornamentation from the medieval Sassanid period was carried out by Mahsa Kharazmi, Reza Afhami and Mahmood Tavoosi, who outlined the seven one-dimensional freeze and the seventeen wallpaper groups and analyzed the artworks within this framework [11].

Figure 2 shows two examples of bas-relief ornaments that were created in Persia at different times. I have chosen these two artworks as representative of the period before (Fig. 2a) and after (Fig. 2b) the Islamic rule. The process of determining the wallpaper group for each specimen has already been described in my work on the marble floor of San Marco Basilica in Venice [12]. Figure 3 shows four excerpts from the photos shown in the previous figure. On the left-hand side (Fig. 3a) we see the simplest member of the symmetry group *p1*, which is found on the animal's fur. The blue arrows are the translation vectors that define the unit cell of the symmetry group and the periodicity of the unit cell in two dimensions. There are no symmetry features within the unit cell. The next selection (Fig. 3b) is the tree trunk. In addition to the blue translation vectors of equal length, we recognize mirror reflections *m* in red and glide lines *g* in green. The ornament belongs to the group *c1m1*. Moving to the right, we see in Fig. 3c a section from the shepherd's beard who is standing on the right-hand side in Figure 2a. Here we have two translation vectors in blue of the same length that are perpendicular to each other. We see mirror reflections in two principal directions and two others that are rotated by 45°. This pattern is a typical member of the *p4mm* symmetry group. For the sake of simplicity, neither the glide reflection lines nor the axes of twofold and fourfold rotations are included. On the far right we see in Fig. 3d the ornament of the bas-relief on the wall of Tadj al-Mulk. We have two translation vectors of equal length in blue at 30° to each other. Also indicated are the axes of rotation perpendicular to the plane of the figure. Sixfold symmetry axes are shown in red, threefold in green and twofold in yellow. There are no mirror and glide reflections, so the symmetry group is *p6*.

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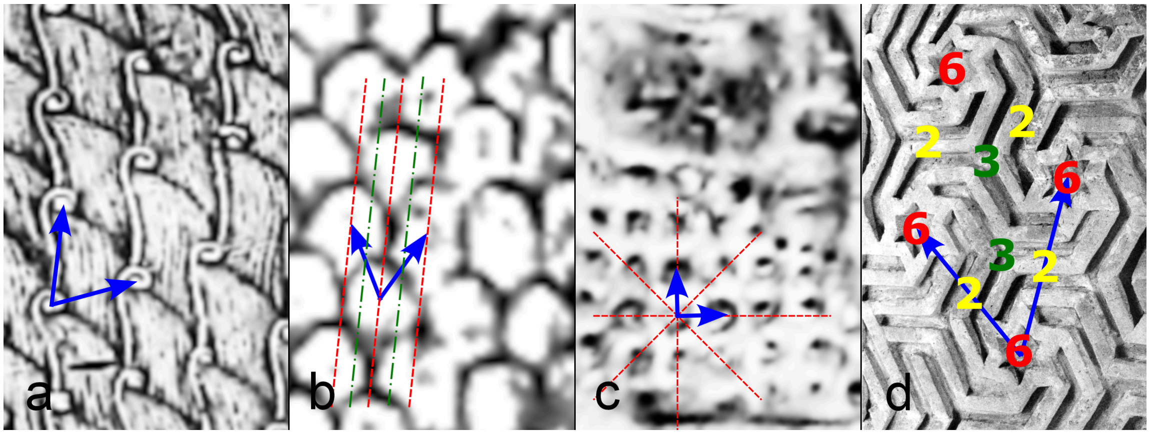

Figure 3. Assignment of symmetry properties to the ornaments shown in Figure 2. Four excerpt are shown, from left to right, (a) of the animal skin in *p1*, (b) tree trunk in *c1m1*, (c) shepherd's beard in *p4mm*, and (d) wall decoration in *p6* symmetry group.

3. Data Collection

I examine the practice of medieval cultures with regard to their preference for the use of different symmetries in the creation of works of art. These artworks are mainly found in architectural constructions, such as wall decorations, in the form of paintings, frescoes, mosaics or bas-reliefs. I determine the wallpaper groups of the artwork and note the frequency or occurrence for each wallpaper group. I assume that the choice of the symmetries is characteristic of the civilization under study. This technique translates the occurrence of symmetry into numbers, and further work with such numbers leads to rigorous numerical results. It is therefore an evaluation of art in terms of mathematical methods.

There are several books on Persian art, and I have consulted some of them to collect data. For the pre-Islamic artworks, I have "The Splendour of Iran" [13] and "L'Art de L'Iran" [14] and for Islamic Iran up to the 16th century "L'Art de L'Iran" [14] and "Persian Painting" [15]. The details of the sources are listed in the Appendix. Works of art with clear ornamentation that pass through at least 3 periods in both crystallographic directions are evaluated and assigned to a wallpaper group. This process is repeated for all the ornaments found in the literature cited. The percentage frequency of the individual wallpaper groups is listed in Table 1.

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	PreIslam	IslamicP	Persia			PreIslam	IslamicP	Persia
p1	8.3	2.1	4.2		*p3*	0.0	1.4	0.9
p1m1	6.9	2.1	3.7		*p3m1*	0.0	0.7	0.5
p1g1	0.0	0.0	0.0		*p31m*	0.0	0.7	0.5
c1m1	15.3	7.8	10.3		*p4*	5.6	4.9	5.1
p211	5.6	0.0	1.9		*p4mm*	16.6	14.1	15.0
p2mm	0.0	2.1	1.4		*p4gm*	1.4	0.7	0.9
p2mg	16.6	1.4	6.5		*p6*	2.8	4.2	3.8
p2gg	2.8	4.9	4.2		*p6mm*	5.6	14.8	11.7
c2mm	12.5	38.0	29.4					

Table 1. Occurrence of the individual symmetry groups (in percent) in the artifacts of the Persian civilization, displayed in two sets of columns. The first set contains the wallpaper groups *p1* to *c2mm* and the second set *p3* to *p6mm*. "PreIslam" stands for Persia before the Muslim conquest (72 objects) and "IslamicP" for Moslem Persia (142 objects). "Persia" is the sum of the two periods with 214 objects.

In Table 1, we see that the wallpaper group *c2mm* is the most frequently used, followed by *p4mm*, *p6mm*, *c1m1* and *p2mg*. The first three use double-mirror reflections and the other two use one mirror reflection, which shows that the mirror symmetry is a favorite tradition of Persian artists. This seems to be a general behavior of Persian art that has remained more or less the same with the advent of Islam. It is surprising, however, that the use of *p2mm* is almost negligible.

In an earlier paper, I calculated the *correlation function* for the observations based on the 17 variables, the wallpaper groups, and drew conclusions about the possible mutual influences of the civilizations [6]. The correlation function is calculated pairwise for all considered cultural groups. If there are g such groups, one obtains $g \times (g-1)/2$ correlations. There are therefore 21 such correlation numbers for seven groups. It is then not easy to recognize the similarities between groups at first glance. For larger values of g, statistical techniques such as *multidimensional scaling* (MDS), *principal components analysis* (PCA) and *hierarchical cluster analysis* (HCA) are used to find the similarity groups [16]. In my recent study with seven civilizations, I also employed such statistical techniques to find similarity groups [10]. There are two distinct clusters: On the one hand, the Hellenistic, Eastern Roman, Armenian and Andalusian groups, and on the other hand, the Seljuks and the Arabs. In the following, I will examine the situation of Persian art in relation to these two groups of Middle East civilizations.

4. Comparison with the Neighbors

Since Persian art production did not change significantly with the advent of Islam, as indicated by the frequency of wallpapers in Table 1, I will use the sum of Persian artifacts in this comparison. In this way, the total sum of the artifacts used in the analysis increases to 214, which raises statistical accuracy. Table 2 lists the number of Persian specimens ("Persia") identified in 17 wallpaper groups in the far right column along with the values of the other seven civilizations. Some Persian values are similar to those of other civilizations, but it is almost impossible to make a reliable comparison about the general trends. Therefore, statistical tools are used for a healthy comparison.

	Hellen	E Roman	Armen	Arabs	Andalus	Seljuk E	R Seljuk	Persia
p1	6/9.8	16/14.0	1/0.8	0/0	1/1.0	1/0.8	1/0.3	9/4.2
p1m1	1/1.6	11/9.7	6/4.9	0/0	0/0	0/0	0/0	8/3.7
p1g1	0/0	1/0.9	0/0	0/0	0/0	1/0.8	0/0	0/0
c1m1	1/1.6	5/4.4	3/2.4	0/0	0/0	0/0	2/0.6	22/10.3
p211	0/0	3/2.6	0/0	8/3.6	1/1.0	3/2.4	4/1.1	4/1.9
p2mm	4/6.6	11/9.7	7/5.7	13/5.8	2/1.9	8/6.4	32/8.8	3/1.4
p2mg	1/1.6	6/5.3	1/0.8	1/0.4	0/0	1/0.8	3/0.8	14/6.5
p2gg	0/0	1/0.9	1/0.8	0/0	0/0	4/3.2	2/0.6	9/4.2
c2mm	4/6.6	4/3.5	2/1.6	29/12.9	2/1.9	14/11.2	31/8.5	63/29.4
p3	0/0	0/0	1/0.8	0/0	1/1	0/0	3/0.8	2/0.9
p3m1	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/0.5
p31m	0/0	0/0	3/2.4	1/0.4	0/0	2/1.6	7/1.9	1/0.5
p4	1/1.6	2/1.8	15/12.2	1/4.4	28/27.2	12/9.6	30/8.2	11/5.1
p4mm	38/62.4	40/35.1	67/54.5	65/28.9	53/51.5	32/25.6	101/27.8	32/15.0
p4gm	4/6.6	13/11.4	6/4.9	12/5.3	6/5.8	16/12.8	19/5.2	2/0.9
p6	0/0	1/0.9	2/1.6	10/4.4	3/2.9	10/8.0	29/8.0	8/3.8
p6mm	1/1.6	0/0	8/6.6	76/33.8	6/5.8	21/16.8	100/27.5	25/11.7
# / %	61/100	114/100	123/100	225/100	103/100	125/100	364/100	214/100

Table 2. Occurrence of the individual symmetry groups in the artifacts of eight cultures, in numbers (#) and percentages (%). "Hellen" stands for Hellenistic, "E Roman" for the Eastern Roman Empire, also known as Byzantium, "Armen" for the Armenians, "Arabs" for the group from the Arabian Peninsula, "Andalus" for the Moorish kingdom in Andalusia, "Seljuk E" for the Seljuk Empire in Persia and "R Seljuk" for the Rum Seljuks in Asia Minor. "Persia" is a list of 214 artifacts from antiquity to about 1600. The rotational symmetries are summarized in groups. The data for the first seven groups have already been reported in a previous paper [10]

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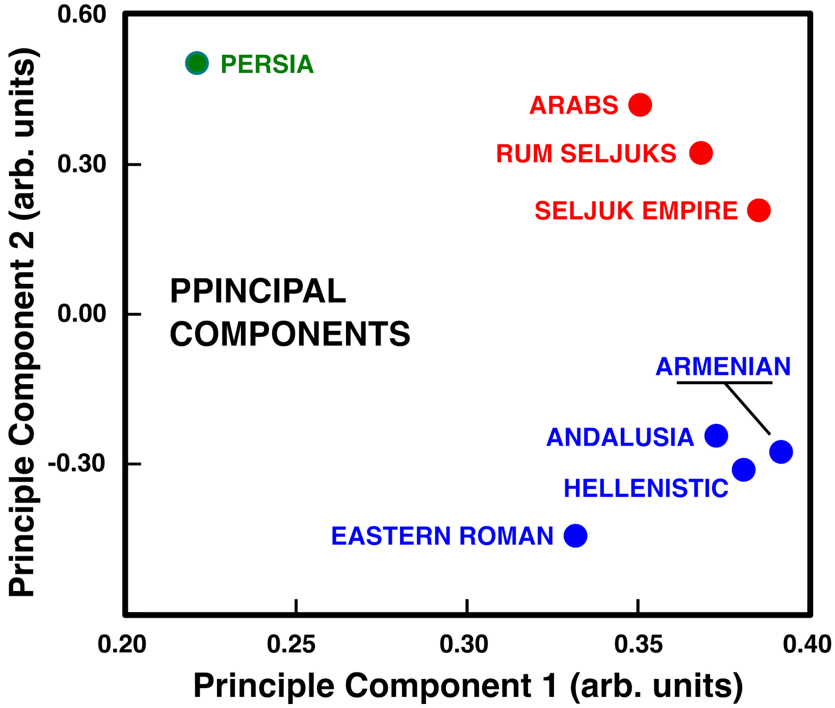

Figure 4. Illustration of the first two principal components of the data from Table 2, obtained using PCA. We see two different groupings of civilizations, colored red and blue. Persia, in green, stands alone.

4.1. Principal Components Analysis

PCA is a technique for reducing the dimensions with the aim of still preserving the essential information. As a rule, an attempt is made to reproduce the data in two dimensions in order to visualize the results and to identify the trends inherent in the observations. The new dimensions are the principal components that represent the greatest variation in the data [17][18]. I used the data in Table 2 and calculated the behavior of art creation in eight civilizations. The results are shown in Figure 4.

In my previous analysis, I have already established that there are two different groups of civilizations [10]. Arabs, Seljuks of Rum and the Seljuk Empire, which are colored red, form one group, the Islamic group. The Hellenistic, Eastern Roman, Andalusia and Armenian civilizations, which are colored blue, form another group, the Greco-Roman group. I have also come to the conclusion that the red group is inspired by the Arabs, and the blue group by the Hellenic civilization that established its colonies in western Asia Minor around 650 BC. I use the term "Hellenistic" after the "Hellenic" colonies were invaded by the Roman intruders after 150 BC. The results of the PCA in Figure 4 show this division in a similar way. I have included Persia in our analysis. The results show that Persia, colored green, is far from both clusters and does not belong to either group.

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PCA is applied on the original data shown in Table~2, projecting them into their principal components, with the aim that most of the information of the data is contained in the first few orthogonal components. This way, the dimensions are reduced while preserving the covariance of the data, i.e., the information contained in the original data.

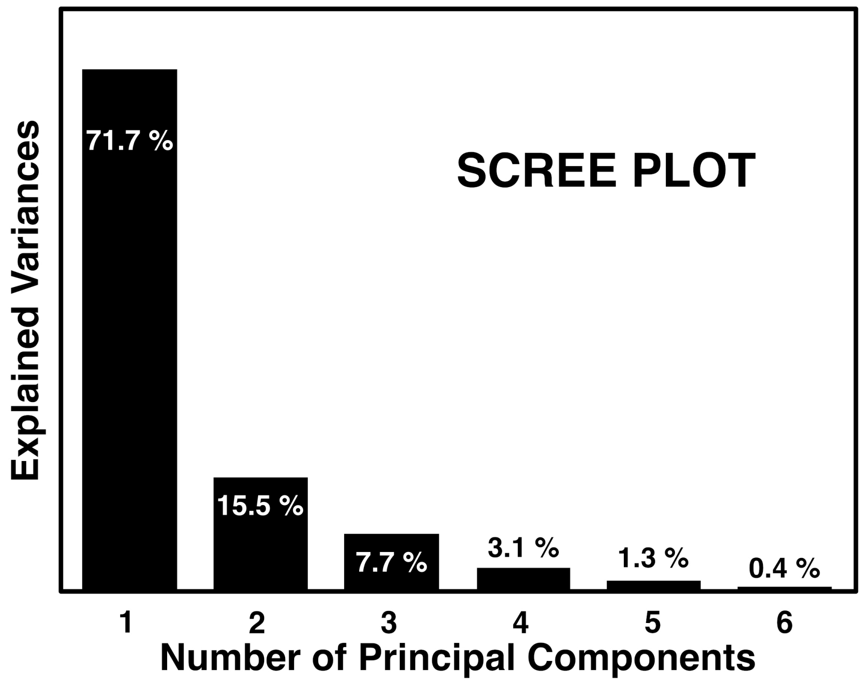

Figure 5. A plot of the first six principal components of the data listed in Table~2. It shows the contribution of each component (as a percentage) to the total variance. The sum of the first two is approximately 87 %, and can therefore be considered representative of the data.

A *scree plot* calculates the contribution of each component, and if the sum of the first two exceeds about 80 %, it ensures that the new representation actually captures the essential part of the information. The scree plot is a test of the success of the PCA operation. The scree plot in Figure 5 shows the contribution of the first six components to the total variance of the data. The sum of the first two components accounts for about 87 % of the total information. This value therefore justifies the representation of the entire data by only two components, as shown in Figure 4.

4.2. Multidimensional Scaling

In addition to PCA, MDS is another method for reducing the dimensions of the data [19][20]. They are closely related, but use different inputs. MDS works with a distance matrix and preserves the distances between data

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points. The input does not necessarily have to be a distance matrix. Similarities or dissimilarities can be used just as well. In practice, the MDS includes an additional calculation step, namely the calculation of distances or pairwise correlations between individual data points, which are then used as input for the MDS. This fact in turn allows the choice of the type of correlation or distance matrix. If the covariance in the data is equal to the *Euclidean distances* between the data points, PCA and MDS provide the same result [16].

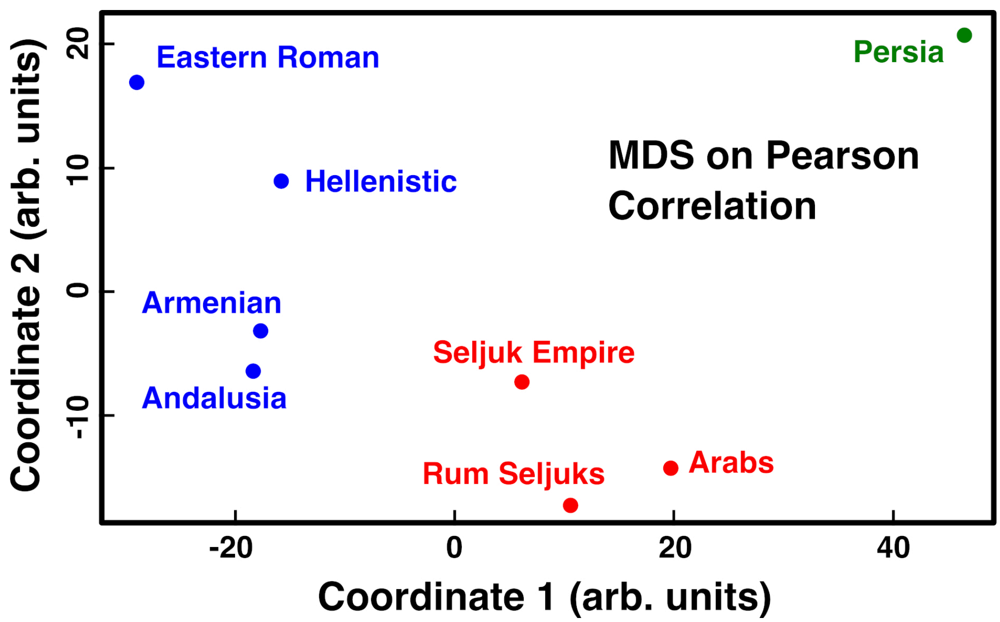

Figure 6. Illustration of the first two principal components of the data from Table 2, obtained with MDS using Pearson correlation. We see two distinct groupings of civilizations in red and blue. Persia, in green, stands alone.

I have already presented MDS results for the first seven civilizations using *cosine correlation* between the data points [10]. We have also used MDS with Euclidean distances, as is very often done [21]. To investigate the dependence of the MDS on the type of input matrix and, on the other hand, to corroborate the results obtained with the PCA technique, I applied the MDS to our data. Alternatively, I used the *Pearson correlation* between the data points. Figure 6 presents the results. They compare very favorably with the cosine correlation results, i.e., the civilizations studied are found in two distinct clusters. Most importantly, analogous to the PCA results, Persia also stands alone here, well separated from the two groups.

4.3. Heatmap

Heatmap is the name of a function that contains a clustering algorithm applied to the data [22]. The algorithm calculates the pairwise Euclidean distances and rearranges both the variables and the observations according to their similarity. We have implemented the heatmap function on the numbers listed in Table 2 of eight civilizations for the habits. Figure 7 illustrates the results. The degree of similarity, given in numbers, is shown in colors, with red representing the highest (hottest), i.e., the self-similarity, and blue representing the lowest (coolest) similarity, as the name of the algorithm implies. The algorithm additionally provides a *dendrogram* for the civilizations under consideration. A dendrogram is a direct way to display the results of HCA, which is an agglomerative technique for analyzing the data, i.e., it shows the grouping of the variables. The figure clearly shows the grouping of Islamic (red) and Greco-Roman (blue) civilizations. This dendrogram compares well with earlier results on HCA [10]. Persia's contribution stands alone (green) and joins the dendrogram as the outermost branch.

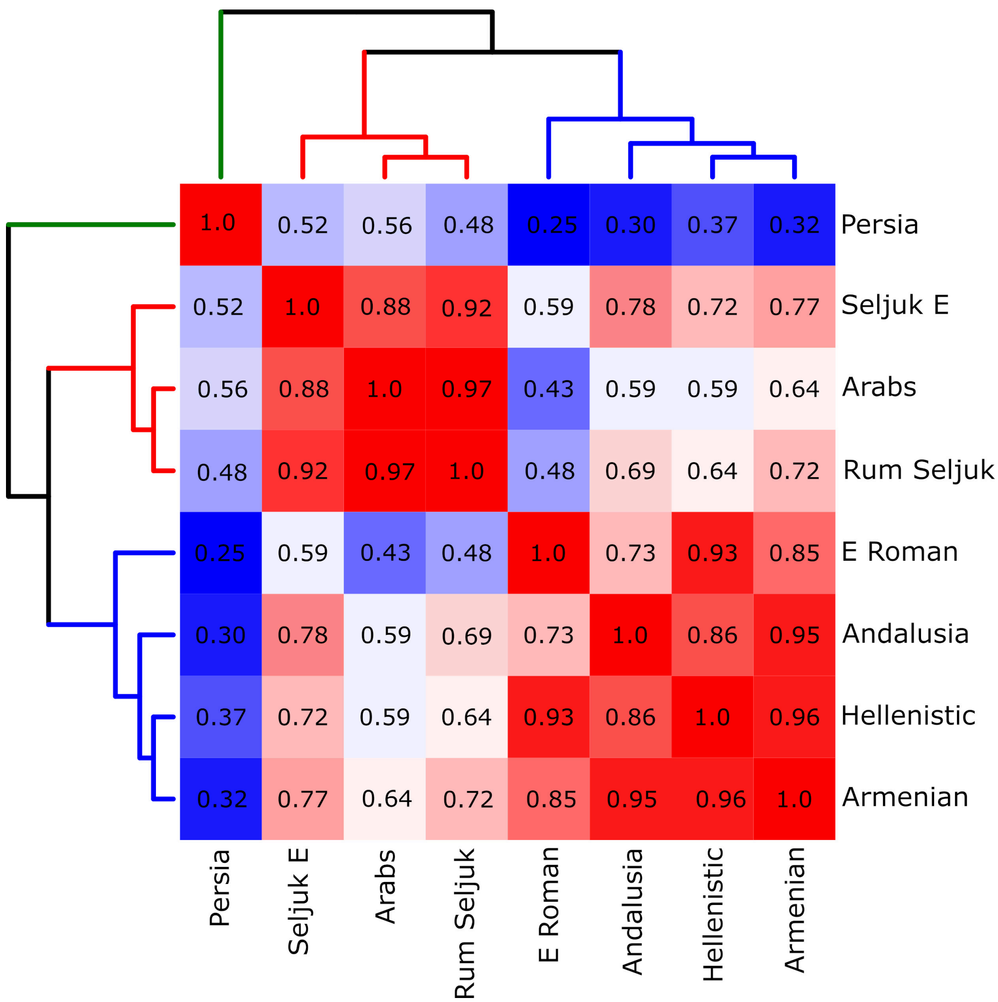

Figure 7. A heatmap based on the data in Table~2. "E Roman" stands for Eastern Roman and "Seljuk E" for the Seljuk Empire. The color code ranges from red for the highest similarity to blue for the lowest, with white representing the mean. The dendrograms show the development of two different groups of civilizations (red and blue). Persia in green, on far left or the top branch, stands out.

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5. Results

We have seen that different correlation functions, cosine correlation, Pearson correlation and Euclidean distance, lead to comparable values for the interaction of eight medieval civilizations. Further analyses using PCA, MDS or HCA also lead to equivalent conclusions about these civilizations. We observe a strong tendency towards grouping between the Seljuk Empire, Seljuks of Rum and the Arabs. Similarly, Armenia, the Eastern Roman Empire, the Hellenistic culture and the Moorish kingdom of Andalusia seem to have developed similar artistic habits expressed in the symmetry of the artifacts. However, the ancient Persian culture of artistic creation differs significantly from the artworks of its neighbors, Armenia, the Eastern Roman Empire, the Hellenistic or Arab culture and the Seljuks. We come to this conclusion regardless of the method we use when looking at works of art dating up to the beginning of the 16th century.

6. Conclusions

The algorithms I have applied to the data have clearly shown that Persia stands alone with its art-making culture. This simply means that Persia has not exerted or received any significant influence on the neighboring countries we have looked at so far.

The monumental Persian art was created by the Iranian people, who have many highly civilized ancestors, such as the Sumerians, the Babylonians, the Assyrians, the Medes and the Arians. From this rich combination emerged the Achaemenian civilization, which dominated vast areas, from present-day Afghanistan and Pakistan in the east to ancient Greece and Romania in the west. Persia's wealth and geographic location at the crossroads of nomadic peoples often attracted invaders who destroyed Persian lands every time they crossed or occupied them. Macedonians, Arabs, Turks and Mongols were among these invader groups. Nevertheless, Persia was able to recover each time and did not disappear into history. On the contrary, the glorious Persian culture manifested itself in architecture, medicine, philosophy and poetry, in science and theology. This is a report on the Persian art, which was able to survive for several millennia until the last Mongol occupation in around 1500.

The ancient Persian culture of artistic creation emerged and flourished during the hectic and turbulent Middle Ages. The high cultural level of Persian art influenced the Arab and Turkish peoples, especially in literature and poetry, with numerous heroic and romantic works, thanks to the Middle Eastern sense of philosophy of life and ease of reading the texts. They all used the same Arabic alphabet with a few minor adaptations. However, the symmetry used in the ornamentation is a different level of art, and this analysis shows that these groups did not share much inspiration with Persia. On the contrary, each group developed its own characteristic ornamentation.

I also argue that after the 16th century, a new concept of artistic creation emerged in the Middle East, Central Asia, North Africa and in the areas of Ottoman rule. This new era, known as *Islamic art*, replaced the old Persian art as well as the art of all other Islamic groups. Our next endeavor will be to study of Afghan and Mongolian art to possibly find evidence of mutual influence.

Appendix

Sources consulted for the ornaments

For the pre-islamic period, I have used two books as sources for Persian ornaments. The first is Godard's comprehensive work [14]. Several monuments of the Apadana, served as a source of inspiration, as well as the Bulls on the Gate of All Countries, the Tachara of Darius I, the Terrace, Verandah of the Hall of a Hundred Columns, all in Persepolis. I also found ornaments at Naqsh-e Rostam and Naqsh-e Rajab, both near Persepolis. I looked at some gold and silver coins on display at the Bibliothèque National in Paris. I found total of 19 objects. The other source is "The Splendour of Iran" [1], the first volume on the Elamite Heritage, the Arian Legacy and the Imperial Period. Several works of art are found in the Iran National Museum in Tahran (from Arjan, Chal Tarkan, Esmailabad in Qazvin, Hamadan, Hesar, Marlik, Persepolis, Susa, Tall-e Bakun, Tepe Sialk, Ziwiyeh Castle) in Persepolis (The Gate of All Nations, Tripylon, Xerxes' Thron Hall), in Behistun, in Chogha, in Zanbil, at Takht-e Soleiman and in Taq-e Bustan. I found here 53 ornaments.

I have used two sources for the ornamental art in Islamic Persia. One is Godard's work [14], in which I found 48 ornaments inside and outside the following buildings: The Red Tomb, the Blue Tomb and Circular Tomb at Maragha; Friday Mosque in Nayin; Friday Mosque at Isfahan; the Tomb of Sheikh Safi Al-Din at Ardabil; the Friday Mosque and the Funerary Tower of Ala'Al-Din at Waramin. The second source is the book by Grey [15]. I have 94 ornaments from prominent examples of Persian painting from classical works, such as Shah-name of Firdowsi, Kalila wa Dimna, Diwan of Khwaju Kirmani, Diwan of Sultan Ahmad, Khusrau and Shirin by Nizami, Khamsa by Nizami, Anthology of Iskandar Sultan, Anthology of Baysungur, Gulistan of Sa'di, Zafar-nama of Sharaf Al-Din Ali Yazdi or Shah-nama of Sultan Ibrahim.

Computational details

I have tested the tendency of clustering civilizations with different correlation functions (Euclidean distance, cosine and Pearson). Subsequently, I used different algorithms (PCA, MDS, HCA) to display the results. We also used different environments, Python, MATLAB and R, to run required algorithms. R seems to be the most practical language for the calculations at hand by a non-specialist in the field of data science.

I have used open-source programs:

Image Manipulation Program GIMP 2.10.26,

Vector Graphics Software Inkscape 1.3.2,

Editor and Previewer TeXShop 5.25, and the

Programing Language R 4.3.1 in combination with RStudio 023.06.1+524

to implement the necessary functions.

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