## Qeios

# Peer Review Review of: "Kronecker-Pauli Operators"

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Review of the Paper: Kronecker-Pauli Operators

## **Mathematical Review**

## Strengths

- **Clarity of Definitions:** The paper defines Kronecker-Pauli matrices (KPMs) with precise properties (orthogonality, hermiticity, etc.) and uses Dirac notation effectively to connect operator form with matrix representations.

Systematic Construction: The step-by-step construction of KPMs for prime dimensions (especially 5×5) is well-detailed and aligned with known generalizations (like Pauli and Gell-Mann matrices).

- Use of Permutations and Roots of Unity: The use of symmetric permutation matrices and substitution of entries with Nth roots of unity is mathematically valid and shows insightful generalization.

## Weaknesses and Suggestions

#### 1. Rigor in Proofs:

- Proposition 3 claims that the constructed operators form a KPM set, but the proof relies on vague phrases like "it is straightforward" or "let us suppose the contrary," without formal steps or lemmas.

- Clarify the logic around the symmetric permutation  $\sigma$  (e.g., the use of  $\sigma(k) = -k + 2l \mod N$ ). Why this specific form? Is it unique or canonical?

#### 2. Notation Consistency:

- Notation like



for operators is unconventional. A clearer definition of the indexing and its meaning (product or sum?) is necessary.

- Dirac notation should be carefully distinguished between bras, kets, and outer products.

### 3. Mathematical Gaps:

- When introducing the matrices  $\chi_i$ , it would be helpful to label which operators or constructions each matrix corresponds to.

- The reason behind selecting certain permutations (e.g.,  $\sigma(0) = 4$ ,  $\sigma(1) = 3$ , ...) needs justification.

### 4. Further Generalization:

- The conclusion speculates on "other sets" of KPMs using non-standard bases. This idea is promising but not explored mathematically.

## Language and Style Review

## Strengths

- The overall structure (introduction, development, conclusion) is appropriate for a theoretical paper.
- The use of Dirac notation shows familiarity with the conventions of quantum physics literature.

## **Issues and Recommendations**

#### 1. Grammar and Clarity:

- Use more formal academic language.

#### 2. Abstract/Introduction:

- Needs refinement. For example:

Original: "In quantum mechanics, there are some classical bases which are generalized to higherdimensional matrices."

Improved: "In quantum mechanics, classical matrix bases such as the Pauli matrices are often generalized to higher dimensions."

#### 3. Logical Flow:

- Transitions between sections are abrupt. Use linking sentences to explain why a new section or concept is introduced.

#### 4. Formatting:

- The references are inconsistently formatted. Ensure consistency.
- Long matrix expressions could be summarized or illustrated with LaTeX formatting or figures.

## **Overall Recommendation**

With moderate revisions, this paper could contribute effectively to the literature on operator generalizations in quantum information. The mathematical content is promising, especially for finitedimensional quantum systems and basis constructions. However, it needs sharper proofs and notations, and thorough language editing for grammar, clarity, and academic tone.

## Declarations

Potential competing interests: No potential competing interests to declare.