Research Article

A Trace-Coupled Scalar Clock for the Problem of Time in Canonical Quantum Gravity

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Canonical quantum gravity has no external time parameter, so the choice of a physical clock becomes part of the dynamics rather than a background assumption. Prior relational, dust, geometric and scalar-clock constructions show how internal time can work in special regimes, but they leave a sharper question open: can one specify a common matter-accessible clock sector in four-dimensional covariant gravity and test, within one action, its canonical consistency, dynamical monotonicity and quantum probability interpretation? This paper answers that question for the Tempon, a Lorentz-scalar clock field coupled to matter through the conformal metric $A^2(\Phi)g_{\mu\nu}$. Its first-order expansion yields the trace interaction ΦT, while the full covariant action fixes the matter momentum and Hamiltonian before any reduction is made. For an explicit scalar matter model, the exact Legendre transform gives the corrected kinetic structure and the ADM constraints, and the scalar-sector bracket calculation verifies preservation of the classical first-class hypersurface-deformation algebra. The same model then supplies a clock test rather than a clock assumption. In homogeneous FLRW cosmology, an exact Tempon momentum balance separates open fixed-sign branches from turning-point solutions where monotonic relational time fails. In the reduced quantum theory, a positive-frequency homogeneous or gauge-fixed sector supports Schrödinger evolution relative to the Tempon and a conserved conditional inner product under self-adjointness and adiabaticity conditions. The resulting framework turns the clock-choice problem into a calculable viability program, with direct extensions to clock-quality measures, fifth-force searches, precision-clock comparisons and scalar gravitational-wave response.

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1. Introduction

Quantum theory and general relativity use the word time in structurally different ways. In ordinary wave mechanics, time is the external parameter that orders the unitary motion of a state, while attempts to represent it by a universal self-adjoint operator encounter well-known difficulties for semibounded Hamiltonians [1][2]. Relativity removes the Newtonian background clock: temporal intervals are determined by the spacetime metric and by the worldline of an observer, and simultaneity is not universal [3][4]. These two roles coexist successfully when gravity is classical, but they become incompatible when the geometry itself is quantized.

In the canonical formulation, general covariance appears through constraints rather than through an ordinary Hamiltonian evolution. Dirac's analysis of constrained systems provides the general framework [5], and the gravitational Hamiltonian constraint becomes, after quantization, the Wheeler–DeWitt equation [6]. The wave functional of a closed universe then has no external parameter of the Schrödinger type. Kuchař and Isham emphasized that this frozen formalism is only one face of a broader family of problems involving observables, global clocks, refoliation, Hilbert-space construction and the choice among inequivalent internal times [7][8]. Anderson's modern synthesis organizes the Problem of Time into nine interlinked facets rather than a single frozen-equation problem [9]. Unruh and Wald examined whether proposed internal times support a consistent probabilistic interpretation, showing why formal deparameterization alone does not settle the physical interpretation [10]. The difficulty is therefore not adequately summarized by saying that an equation lacks the symbol t.

Recent work sharpens the scientific target. Mozota Frauca shows that resolutions demonstrated in deparametrizable models need not transfer to full general relativity, which is generally non-deparametrizable [11][12], and asks whether quantum cosmology retains empirically meaningful duration, including the age of the universe [13]. Klinger and Leigh pursue a complementary route in which the temporal problem reflects the quantum-gravitational framework itself [14]. Together, these studies establish a demanding benchmark: a proposed clock sector must be physically specified, canonically consistent, dynamically usable and explicit about the domain in which its quantum evolution is defined.

The present paper meets this benchmark with a concrete clock-sector hypothesis. The Tempon is a dedicated Lorentz scalar physically connected to the matter sector through a conformal matter metric. The name "Tempon" is coined from tempus, the Latin root for time, together with the particle-like suffix

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“-on”; the term is used here to denote a proposed scalar clock sector rather than to assume a fundamental particle interpretation. The first-order expansion of the conformal coupling generates the scalar trace interaction ΦT, while the full nonlinear coupling supplies a self-consistent canonical completion. This construction replaces the schematic procedure of appending ΦT to an already Legendre-transformed Hamiltonian and instead derives the matter momentum and Hamiltonian from the coupled action itself.

Here universal denotes matter-wide operational access: the same conformal factor acts on every matter sector included in the model, so the clock is a common field rather than a selected laboratory subsystem. Massive and nonconformal sectors couple directly through the trace, while conformal sectors can participate through anomaly-induced or higher-order interactions. The relevant clock domain is the open sector in which the Tempon gradient is timelike, its level sets form a regular foliation, its momentum retains one sign and conditional evolution remains well defined.

The paper delivers four concrete results. First, it derives the covariant action and exact combined Legendre transform for an explicit matter field. Second, it establishes classical first-class closure from the generally covariant two-scalar structure and verifies the decisive scalar-sector cancellations directly. Third, it obtains an exact homogeneous momentum balance that separates sign-preserving clock branches from turning-point solutions. Fourth, it identifies the positive-frequency, gauge-reduced and self-adjoint sector in which the Tempon generates relational Schrödinger evolution. Together, these results convert the universal-clock question into a definite viability problem with calculable success and failure boundaries.

This combination gives the proposal a clear identity within the crowded literature on relational time. Dust clocks achieve exact deparameterization by adding a special reference medium; geometric clocks exploit variables already present in the gravitational phase space; scalar clocks often rely on symmetry reduction; and clock-neutral methods prioritize transformations between perspectives. The Tempon follows a different route. It introduces a common scalar sector whose relation to matter is fixed by the same conformal metric that generates the trace interaction. The resulting theory therefore links three questions that are usually studied separately: whether the matter coupling admits a consistent canonical completion, whether the coupled dynamics preserves a usable clock branch, and whether that branch supports a probabilistic quantum evolution. The strength of the proposal lies in treating those questions within one action rather than importing a clock after quantization.

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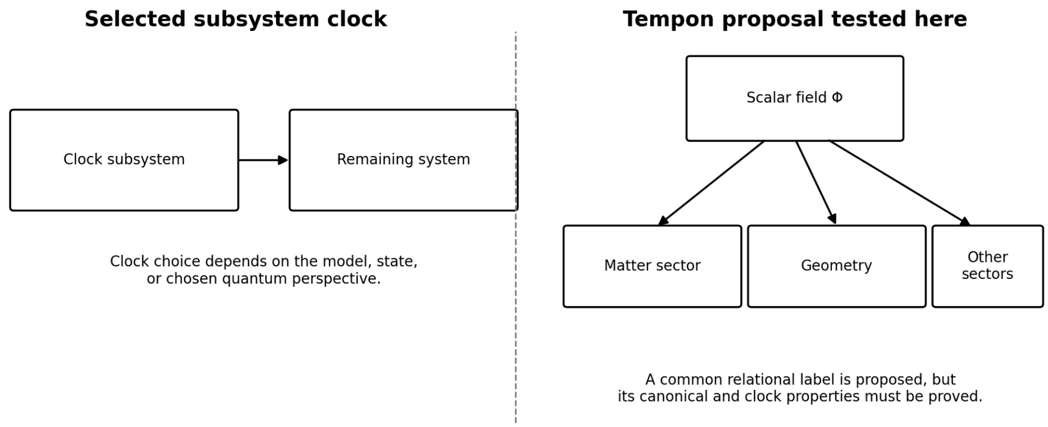

Figure 1. *Conceptual distinction between a selected subsystem clock and a dedicated trace-coupled scalar clock sector.*

2. Literature Positioning and the Specific Research Gap

2.1. Conditional and clock-neutral relational dynamics

The Page–Wootters mechanism showed that a globally stationary state can contain conditional evolution relative to a subsystem used as a clock [15]. The mechanism has been illustrated experimentally in controlled quantum systems [16], and later work argued that apparent ambiguities can disappear when the clock-system decomposition is transformed consistently [17]. Modern reviews place this family of ideas within the broader theory of quantum clocks, including finite resolution, backreaction and nonideal timekeepers [18]. The conceptual achievement is substantial: external time is not logically required for every effective dynamical description. Foundational relational formulations by Rovelli developed timeless quantum models and the hypothesis that time is absent at the fundamental level [19][20]. These works establish an important conceptual lineage for relational evolution, while the physical selection and canonical testing of a common matter-wide clock remain separate questions. The partial-and-complete-observables programme later supplied systematic constructions of relational Dirac observables in canonical general relativity and more general constrained Hamiltonian systems [21][22]. These constructions clarify how observables can be defined relative to chosen clock variables, but they do not by themselves select a unique matter-wide clock.

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Clock-neutral approaches go further by treating the physical state as prior to any clock perspective. A description relative to one internal time can be transformed into a description relative to another [23][24]. Within appropriate relativistic constraint systems, relational observables, Page–Wootters dynamics and quantum deparameterization can be equivalent descriptions of the same reduced physics [25][26]. Quantum-reference-frame methods also show that subsystem structure and entanglement may depend on the selected perspective [27]. These results form an important alternative to the idea that nature must choose a single privileged clock. A Tempon theory must therefore demonstrate an operational advantage or a dynamical selection mechanism; merely adding another variable does not erase clock-neutral equivalences.

2.2. Reference matter, geometric time and the multiple-choice problem

Concrete internal clocks remain indispensable because they permit explicit calculations. York time uses the trace of the extrinsic curvature and is therefore geometrical and foliation-sensitive [28]. Brown–Kuchař dust supplies an exact reference system and a true Hamiltonian, but introduces special pressureless matter with its own physical assumptions [29]. Husain and Pawłowski likewise obtain a physical Hamiltonian by fixing a pressureless dust scalar as time, providing a close canonical comparator that still relies on a dedicated reference medium [30]. Scalar clocks are widely used in quantum cosmology, where monotonicity can hold in symmetry-reduced regimes; the measurement problem for such clocks nevertheless remains nontrivial [31]. These constructions demonstrate that relational evolution is technically possible, but they do not single out a universal physical clock for the full theory. Loop quantum cosmology provides a prominent symmetry-reduced example in which a massless scalar serves as internal time and supports singularity-resolving dynamics in homogeneous isotropic models [32][33].

The multiple-choice problem is not merely semantic. Gielen and Menéndez-Pidal showed that different clock choices in the same cosmological model can change whether a quantum singularity is resolved [34], and can lead to different unitary completions and different global histories [35]. A related de Sitter analysis found that the simultaneous demands of unitarity and a chosen time can generate low-curvature effects tied to the foliation [36]. These examples make a central point for the Tempon program: the physical status of the clock must be defended independently of the mathematical convenience of deparameterization.

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2.3. Field-theoretic scalar clocks, periodic clocks and recent experiments

Scalar relational time has already moved beyond minisuperspace. Earlier GFT matter constructions coupled quantum geometry to matter in three dimensions, and subsequent condensate-cosmology and scalar-matter formulations established the field-theoretic setting from which relational GFT clocks were later developed [37][38][39][40]. These results supply the relevant lineage, but they do not by themselves select a universal matter-wide clock for four-dimensional canonical gravity. A massless scalar produces a relational Hamiltonian in group field theory [41], and Page–Wootters relational dynamics has been developed in the same field-theoretic quantum-gravity setting [42]. A recent stochastic ADM proposal treats scalar-clock momentum as influenced by coarse-grained gravitational fluctuations [43]. The JT model itself originates in the two-dimensional gravity formulations of Teitelboim and Jackiw [44][45]. In two-dimensional de Sitter JT gravity, the dilaton can function as a physical clock only for states satisfying finiteness, conservation, positivity and classical-limit tests [46]. These works are close comparators: they show that a scalar clock is neither new nor automatically satisfactory.

Periodic clocks further weaken the assumption that a useful clock must be globally monotonic. Relational dynamics may be described cycle by cycle or through appropriate covariant constructions even when the clock repeats [47]. The Tempon analysis therefore adopts a narrower statement: a monotonic sector is sufficient for a single-valued Schrödinger reduction, but not a universal definition of clockhood. Turning points are treated as boundaries of that sector rather than as proof that all relational dynamics is impossible.

The matter coupling is guided by effective field theory. Low-dimension Lorentz-invariant operators organize the controlled low-energy expansion [48], and gravity itself can be treated consistently as an effective field theory below its cutoff [49]. The conformal matter metric used here is the nonperturbative completion of the linear ΦT interaction. It is not claimed to be the unique ultraviolet theory, but it provides a precise Lagrangian from which momenta and constraints can actually be derived.

2.4. Semiclassical time, operational clocks and the gap addressed here

The Born–Oppenheimer/WKB route to time remains both productive and contested. Chua and Callender argue that the physical justification of the approximations can reintroduce temporal notions and thereby threaten circularity [50]. A constructive response connects semiclassical branches, decoherence, probability and the arrow of classical time [51]. Other recent work separates the roles of observer and timekeeper in closed-universe gravity [52] and asks how an internal clock can be read by another subsystem [53]. These issues prevent the phrase emergent Schrödinger equation from being treated as the end of the analysis.

Experimental analogues now make the subject less abstract. Barontini constructed an isolated cold-atom system in which an internal entropic variable orders repeated evolution and reproduces an effective Schrödinger description [54][55]. However, the result is an analogue model rather than a universal gravitational clock.

The wider debate includes timeless configuration-space programs [56], evolving-block accounts of becoming [57], thermodynamic arrows [58][59], and thermal time derived from modular flow [60]. Conditional-time models have also been extended through relativistic time operators and generalized coherent states [61][62]. Standard quantum-gravity and quantum-cosmology treatments provide the background for semiclassical expansion and boundary conditions [63][64][65][66]. Proper-time interferometry and optical clocks demonstrate that relativistic timing effects and precision frequency comparisons are experimentally meaningful [67][68][69].

After this literature is included, the gap is specific. No absence-of-clocks claim remains. The open task is to test, in one four-dimensional generally covariant model, a dedicated scalar sector whose trace-type matter coupling is derived from a complete action, whose matter momentum and constraints are obtained by a combined Legendre transform, whose clock branches are separated from its turning-point failures, and whose quantum reduction states its branch, gauge and inner-product assumptions. The value of the Tempon proposal lies in this integrated viability test, including the possibility of a no-go result.

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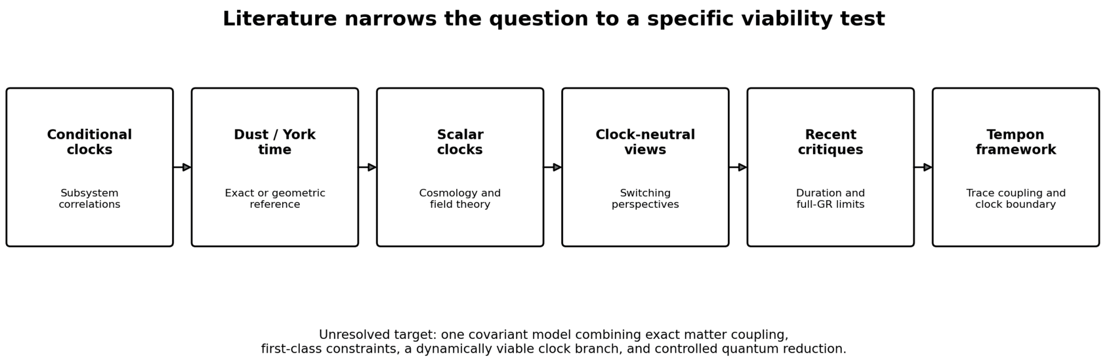

Figure 2. *Literature map: existing approaches narrow the problem to a specific four-dimensional viability test.*

Framework	Principal achievement	Open question for the Tempon program
Page–Wootters / clock-neutral	Conditional dynamics and transformations between clock perspectives	Operational selection of a matter-wide clock
York time	Geometric global-time candidate in selected foliations	Foliation dependence and global existence
Brown–Kuchař dust	Exact deparameterization and true Hamiltonian	Physical assumptions of the reference medium
Scalar cosmological clock	Simple relational evolution in reduced models	Turning points and extension beyond symmetry reduction
GFT scalar clock	Field-theoretic relational Hamiltonian	Relation to a common matter-wide coupling
JT dilaton clock	Explicit norm, probability and classical-limit tests	Extension from two-dimensional to four-dimensional gravity
Periodic clock	Relational evolution through cycles	Connection between cyclic and single-valued clock sectors
Tempon test	Exact trace-type coupling, 4D constraints, clock-sector boundary	Full inhomogeneous and nonperturbative extension

Table 1. Comparison of representative clock frameworks and the Tempon viability test

3. Exact Covariant Tempon–Matter Model

3.1. Conformal completion of the trace interaction

Let $g_{\mu\nu}$ be the Einstein-frame metric and Φ the Tempon scalar. Matter is assumed to couple to the conformally related metric

$$\tilde{g}_{\mu\nu} = A^2(\Phi) g_{\mu\nu}, \qquad A(\Phi) > 0.$$

The complete action is

$$S = S_{GR} + S_\Phi + S_m \left[A^2(\Phi) g_{\mu\nu}, \Psi_m \right].$$

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This is a standard generally covariant scalar–matter construction. The Tempon interpretation is new at the level of the clock-sector question; the conformal coupling belongs to familiar scalar-tensor and effective-field-theory mathematics [70]. The positivity condition A>0 preserves the metric signature, and regularity requires A and $A'(\Phi)$ to remain finite on the field domain. The construction extends to several matter sectors by assigning the same conformal factor to each one.

Varying the matter action with respect to Φ gives the exact identity

$$\frac{\delta S_m}{\delta \Phi} = \sqrt{-g}\,\frac{A_{,\Phi}(\Phi)}{A(\Phi)}\,T,$$

where T is the Einstein-frame matter trace defined consistently from the conformal action. For $A(\Phi)=e^{\alpha\Phi/M}$, the small-field expansion yields

$$S_m\left[A^2 g, \Psi_m\right] = S_m\left[g, \Psi_m\right] + \frac{\alpha}{M}\int d^4x\sqrt{-g}\,\Phi T + \mathscr{O}\left(\alpha^2\Phi^2/M^2\right).$$

The linear trace interaction is therefore an expansion of a complete covariant matter action, not an independent Hamiltonian term that may be appended after the matter Legendre transform. This distinction is the technical starting point of the revised framework.

3.2. Explicit scalar matter model

To make the canonical calculation unambiguous, take a real scalar matter field χ with potential $U(\chi)$. Its action in the conformal metric is

$$S_\chi = -\frac{1}{2}\int d^4x\sqrt{-g}\,\left[A^2(\Phi)g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + 2A^4(\Phi)U(\chi)\right].$$

The scalar sector is a two-field nonlinear sigma model with field-space metric $\mathrm{diag}(1,A^2)$. It is nondegenerate whenever A is finite and nonzero. This fact is important: the coupling changes the matter kinetic coefficient but does not introduce higher time derivatives or a degenerate primary constraint. Fermions, gauge fields and quantum trace anomalies require separate treatment; the scalar model is used because it exposes the canonical issue in a transparent form without pretending to represent every matter sector.

The equations of motion are

$$\nabla_\mu\nabla^\mu\Phi - V'(\Phi) - A(\Phi)A'(\Phi)(\nabla\chi)^2 - 4A^3(\Phi)A'(\Phi)U(\chi) = 0,$$

$$\nabla\mu[A^2(\Phi)\nabla^\mu\chi] - A^4(\Phi)U'(\chi) = 0.$$

The first equation provides a decisive diagnostic for the clock sector. The matter source contains the trace combination and can take different signs for different equations of state, while the potential force can either support or reverse the Tempon motion. Clock viability is therefore determined by the coupled dynamics rather than by covariance alone.

3.3. ADM split and combined Legendre transform

Write the spacetime metric in ADM form with lapse N, shift N^i and spatial metric h_{ij}. Define the normal velocities

$$v_\Phi = \frac{\dot{\Phi} - N^i D_i \Phi}{N}, \qquad v_\chi = \frac{\dot{\chi} - N^i D_i \chi}{N}.$$

The scalar Lagrangian density is

$$\mathscr{L}_{\Phi\chi} = N\sqrt{h}\left[\frac{1}{2}v_\Phi^2 - \frac{1}{2}D_i\Phi D^i\Phi - V + \frac{1}{2}A^2 v_\chi^2 - \frac{1}{2}A^2 D_i\chi D^i\chi - A^4 U\right].$$

Because the full Φ–χ Lagrangian is transformed at once, the momenta are exact:

$$\pi_\Phi = \sqrt{h}\, v_\Phi, \qquad \pi_\chi = \sqrt{h}\, A^2(\Phi) v_\chi.$$

The velocity inversion is

$$\dot{\Phi} = \frac{N\pi_\Phi}{\sqrt{h}} + N^i D_i \Phi, \qquad \dot{\chi} = \frac{N\pi_\chi}{\sqrt{h}\, A^2(\Phi)} + N^i D_i \chi.$$

The A^{-2} factor in the second equation is the key modification lost when the uncoupled matter momentum is retained. Performing $\pi_\Phi\dot{\Phi}+\pi_\chi\dot{\chi}-\mathscr{L}$ gives the exact scalar Hamiltonian density

$$\mathscr{H}_{\Phi\chi} = \frac{\pi_\Phi^2}{2\sqrt{h}} + \frac{\sqrt{h}}{2}D_i\Phi D^i\Phi + \sqrt{h}V + \frac{\pi_\chi^2}{2\sqrt{h}A^2} + \frac{\sqrt{h}A^2}{2}D_i\chi D^i\chi + \sqrt{h}A^4 U.$$

and the scalar contribution to the momentum constraint

$$\mathscr{H}_i^{\Phi\chi} = \pi_\Phi D_i\Phi + \pi_\chi D_i\chi.$$

The standard form of the momentum constraint confirms that the fields transform correctly under spatial diffeomorphisms, but it is not by itself a proof of the Hamiltonian–Hamiltonian bracket. The $A(\Phi)$-dependent kinetic and gradient coefficients must still cancel in that bracket, or closure must be inferred from the generally covariant nondegenerate action. Both arguments are given below and in Appendix B.

3.4. Universality, conformal sectors, and operational coupling

The conformal metric makes the coupling universal at the level of the matter action: every included sector responds to the same scalar function A(Φ). This is stronger than selecting one material degree of freedom as a clock, but weaker than saying that every field has a nonzero classical trace. Massive particles, bound systems and nonconformal fields respond directly. Radiation and other classically conformal sectors may have T=0 on shell, while quantum anomalies, masses generated after symmetry breaking, or higher-dimensional operators can restore a response. The model therefore distinguishes universality of the metric prescription from universality of a nonzero signal.

The same distinction matters for the equivalence principle. A single A(Φ) for all matter species preserves universal free fall at the level of the idealized matter metric, whereas species-dependent functions $A_a(\Phi)$ would produce composition dependence. Quantum corrections can nevertheless generate different effective scalar charges for composite objects because electromagnetic, nuclear and binding contributions enter their masses differently [71]. A complete phenomenology must therefore match the conformal action to the Standard Model and calculate the scalar charge of each clock or test body rather than assuming exact species independence.

Operational access creates a characteristic balance. Increasing the conformal coupling strengthens correlations between matter and Φ and therefore improves readability, while simultaneously increasing backreaction, entanglement and fifth-force sensitivity. Weak coupling preserves the autonomy of the clock sector but reduces the signal available to matter. This accuracy–backreaction trade-off is a quantitative feature of the model and supplies a natural criterion for comparison with dust and geometric clocks. The Damour–Polyakov least-coupling mechanism shows that cosmological evolution can suppress a dilaton's matter coupling, but applying it to the Tempon would require additional coupling functions and background dynamics beyond the minimal ansatz [72].

4. Classical Constraint Structure

4.1. Exact constraints

Using the ADM gravitational variables and standard conventions [73], the total Hamiltonian is

$$H_{tot} = \int_{\Sigma} d^3x \, \left[N\mathscr{H} + N^i\mathscr{H}_i + \lambda_N \pi_N + \lambda^i \pi_i\right].$$

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The primary constraints $\pi_N \approx 0$ and $\pi_i \approx 0$ generate the secondary Hamiltonian and momentum constraints. With $M_p^{-2} = 8\pi G$,

$$\mathscr{H} = 2/(M_p^2\sqrt{h})[\pi_{ij}\pi^{ij} - \pi^2/2] - (M_p^2\sqrt{h}/2)\,{}^3R + \mathscr{H}_{\Phi\chi} \approx 0,$$

$$\mathscr{H}_i = -2D_j\pi_i^j + \pi\Phi D_i\Phi + \pi_\chi D_i\chi \approx 0.$$

The differentiability of the smeared generators depends on boundary conditions and, where required, boundary charges. Regge–Teitelboim boundary control is assumed throughout [74]. For compact slices without boundary, the surface terms vanish. For asymptotically flat or cosmological slices, the falloff conditions must be chosen so that the Hamiltonian generators are finite and functionally differentiable. Closure statements made below refer to the bulk first-class algebra with the appropriate boundary completion.

4.2. First-class closure

The cleanest argument begins from covariance. The action contains the metric and two scalar fields, is invariant under spacetime diffeomorphisms, and has a nondegenerate velocity Hessian in the scalar sector. Therefore lapse and shift remain Lagrange multipliers and the secondary constraints generate hypersurface deformations. The Hojman–Kuchař–Teitelboim structure identifies the corresponding algebra [75], while the Dirac–Bergmann interpretation and possible second-class complications are described systematically by Henneaux and Teitelboim [76]. The resulting smeared brackets are

$$\{D\left[\overrightarrow{N}\right], D\left[\overrightarrow{M}\right]\} = D\left[\mathscr{L}_{\overrightarrow{N}}\overrightarrow{M}\right],$$

$$\{D\left[\overrightarrow{N}\right], H[M]\} = H\left[\mathscr{L}_{\overrightarrow{N}}M\right],$$

$$\{H[N], H[M]\} = D\left[h^{ij}\left(N\partial_j M - M\partial_j N\right)\right].$$

Appendix B provides the explicit scalar-sector check. The $A'(\Phi)$-dependent pieces generated when the Hamiltonian acts on A^{-2}, A^2 and A^4 cancel between the kinetic and gradient sectors or combine into the standard momentum density. The cancellation follows from the covariant sigma-model structure. At the classical level, no additional scalar primary constraint arises while A remains finite and nonzero.

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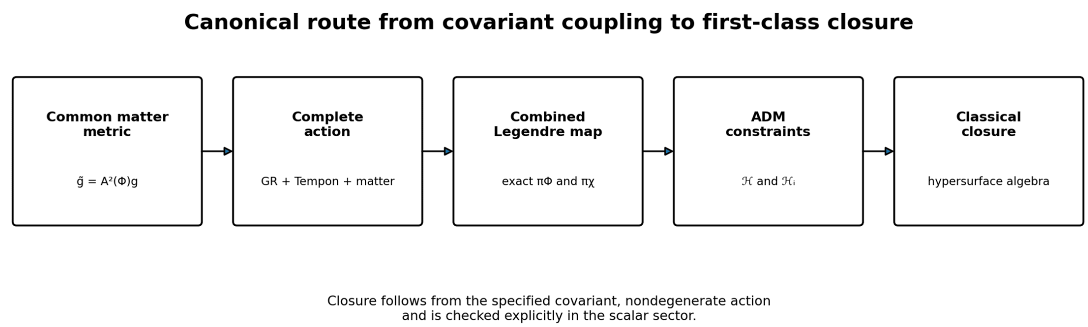

Figure 3. *Canonical route from the covariant matter metric to exact constraints and classical first-class closure.*

4.3. Scope and extensions of the closure result

The closure result establishes the classical canonical consistency of the complete trace-coupled theory. Its domain is the Poisson algebra of the covariant, nondegenerate action with the stated boundary conditions. Quantum anomaly control forms the next layer of the construction, where operator ordering, regularization and boundary domains determine whether the commutator algebra realizes the same structure.

The theorem covers the minimal derivative-free conformal coupling with a common matter metric and $A(\Phi)$ finite and nonzero. Derivative interactions, species-dependent conformal factors and degenerate surfaces $A(\Phi)=0$ define well-posed extensions of the research program because each changes the kinetic matrix or the symmetry realization and therefore requires a dedicated Dirac analysis.

General relativity also constrains the admissible spacetime domain. Global hyperbolicity, regular Cauchy slices and causal conditions are standard requirements for evolving initial data [77][78]. A valid clock additionally needs its level sets to be spacelike. The next section addresses that extra dynamical question.

5. Clock Viability and the Turning-Point Boundary

5.1. Geometric conditions for a scalar clock

A scalar becomes a useful single-valued clock on a region where three conditions hold. The norm of its gradient must be negative, so that Φ=constant defines spacelike hypersurfaces. The level sets must

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remain regular and nonintersecting, and the normal derivative of Φ must keep one sign, equivalently $\pi\Phi$ must stay away from zero in the chosen foliation. These are dynamical properties of a solution sector.

Clock viability imposes stronger conditions than canonical consistency. Covariant scalar dynamics may contain oscillatory solutions, spatial-gradient domination or turning points, whereas periodic-clock constructions show that relational descriptions can also be organized cycle by cycle [47]. The Tempon analysis focuses on the monotonic sector because it supplies a single-valued parameter for conventional Schrödinger evolution and makes the viability boundary mathematically transparent.

5.2. Exact homogeneous momentum balance

Consider a spatially flat FLRW metric $ds^2=-dt^2+a^2(t)d\boxtimes x^2$ with homogeneous $\Phi(t)$ and $\chi(t)$. The exact equations are

$$\ddot{\Phi}+3H\dot{\Phi}+V_{,\Phi}+AA_{,\Phi}\dot{\chi}^2-4A^3A_{,\Phi}U=0,$$

$$\ddot{\chi}+\left[3H+2\frac{A_{,\Phi}}{A}\dot{\Phi}\right]\dot{\chi}+A^2U_{,\chi}=0.$$

Define the canonical homogeneous momentum $P\Phi=a^3\Phi$. Then

$$\dot{P}_{\Phi}=-a^3\left[V_{,\Phi}-\frac{A_{,\Phi}}{A}T_{\chi}\right],$$

with the sign of $T\chi$ fixed by the metric convention and the trace definition used in the action. Integrating between t_0 and t gives

$$P_{\Phi}(t)=P_{\Phi}\left(t_0\right)-\int_{t_0}^{t}dt\,a^3(t)\left[V_{,\Phi}-\frac{A_{,\Phi}}{A}T_{\chi}\right].$$

This identity is the central clock diagnostic. Multiplication by a^3 removes the explicit Hubble-friction term, leaving the potential slope and the matter-trace source to control the accumulated impulse. Dust, radiation and scalar matter generate different trace contributions, so the sign of P_Φ follows from the exact dynamical balance rather than from an energy condition alone.

5.3. Sufficient positive sector

A simple sufficient condition follows directly from the integral balance. On a time interval I, suppose $P_\Phi(t_0)>0$ and

$$\int_{t_0}^{t}dt\,a^3(t)\left[V_{,\Phi}-\frac{A_{,\Phi}}{A}T_{\chi}\right]<P_{\Phi}\left(t_0\right),\qquad t\in I.$$

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Then P_Φ remains positive on I. The condition becomes predictive whenever the source integral is bounded by the field equations or by a controlled background solution. A free massless Tempon with weak conformal coupling admits an open set of positive initial momenta, and exponential or plateau sectors can support the same behavior for suitable initial data and matter histories.

A local comparison form is also useful. If the projected equation yields $\dot{P}_\Phi \geq -C(t) P_\Phi$ with locally integrable C, Gronwall's inequality gives $P_\Phi(t) \geq P_\Phi(t_0) \exp[-\int C dt] > 0$. The homogeneous dependence on P_Φ produces the lower bound that protects the sign branch.

5.4. Turning points and no-go boundary

The same balance yields a sharp turning-point boundary. When the accumulated force equals the initial momentum, the branch reaches $P_\Phi=0$. A massive quadratic potential provides a transparent example: in the weakly coupled or vacuum regime the Tempon oscillates, $\Phi \approx \Phi_0 \cos(mt+\delta)$, and repeated sign changes occur. Trace coupling shifts the location of the boundary through the matter source and can enlarge or shrink the viable region.

This boundary is a substantive prediction of the framework. It partitions the space of potentials, matter traces and initial data into clock-admissible and turning-point sectors. A cosmological or midisuperspace calculation can therefore determine whether the viable domain is broad, finely tuned or absent, giving the Tempon proposal a clear mathematical test.

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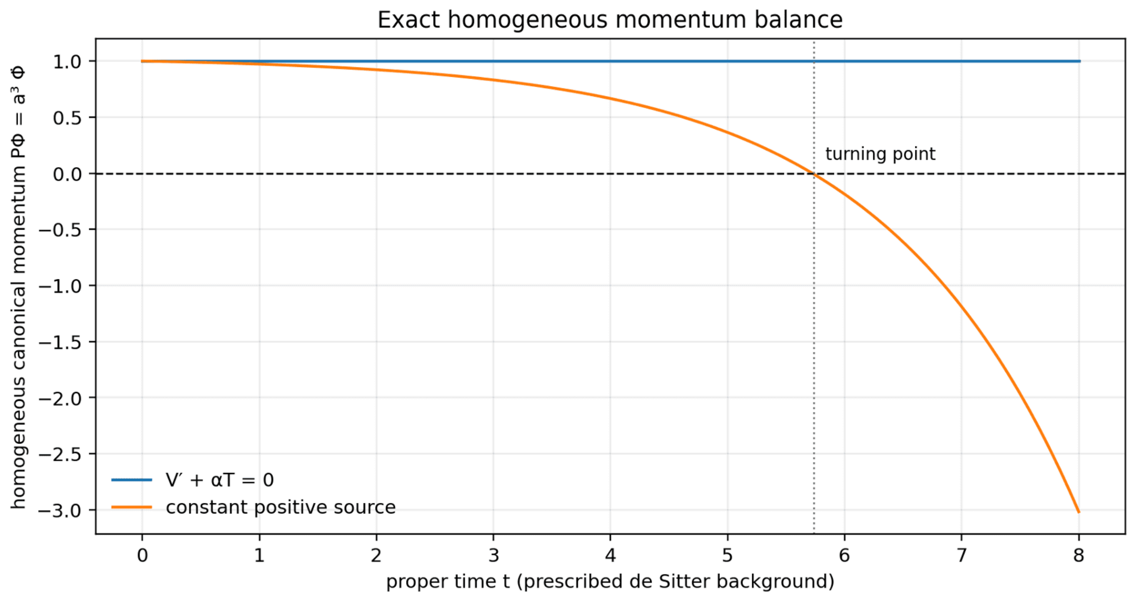

Figure 4. *Clock-sector boundary: sign-preserving branches are separated from turning-point solutions.*

5.5. Inhomogeneous field limitation

For the full field $\Phi(x)$, local clock viability requires the gradient to remain timelike and the foliation to remain regular. Spatial Laplacians and fluxes can create local zeros even when the spatial average remains positive. The present analysis establishes the exact covariant system and its homogeneous viability boundary; the corresponding inhomogeneous extension is an energy-estimate problem for the coupled hyperbolic equations and provides the natural next theorem or numerical test.

6. Quantum and Semiclassical Relational Time

6.1. Full field clock versus a single time parameter

The canonical quantum constraint is a functional equation $\Psi[h_{ij}(x),\Phi(x),\chi(x)]$. If Φ is retained as a local field, its conjugate operator is $-i\delta/\delta\Phi(x)$, and the natural relational structure is multi-fingered. A single derivative $\partial/\partial\Phi$ appears only after an additional reduction: a homogeneous zero mode, a gauge choice $\Phi(x)=\tau$ on each slice, or a projection onto a collective clock coordinate.

The Schrödinger reduction is formulated in the homogeneous sector, where the Tempon zero mode supplies a single collective time coordinate. The full theory carries a multi-fingered functional evolution. Extending the construction to local deparameterization requires solving the Hamiltonian constraint for

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$\pi_\Phi(x)$ while preserving the diffeomorphism constraints and the clock-branch conditions, directly addressing the challenges emphasized for non-deparametrizable gravity [11][12].

6.2. Homogeneous Wheeler–DeWitt reduction

In a minisuperspace with scale factor a, homogeneous Φ and matter variables q, the Hamiltonian constraint can be written schematically as

$$[-\partial^2/\partial\Phi^2 + \Theta(a, q; \Phi)]\Psi(a, q, \Phi) = 0,$$

after an appropriate lapse choice and factor ordering. This has the structure of a Klein–Gordon equation in the clock variable only when the kinetic term is nondegenerate and cross derivatives have been removed. Selecting a positive-frequency branch gives

$$i\, \partial\Psi_+/\partial\Phi = \sqrt{\Theta}\Psi_+ \equiv \hat{\mathrm{H}}_{\mathrm{phys}}\Psi_+,$$

The square-root Hamiltonian is defined when Θ admits a positive self-adjoint realization on the chosen domain. For Φ-dependent Θ, the adiabatic expansion controls branch mixing. The JT analysis provides a useful benchmark because its finiteness, positivity and classicality conditions select the physically meaningful sector from the formal solution space [46].

6.3. Conditional inner product and self-adjointness

Within a positive-frequency sector, a conditional inner product at fixed Φ may be defined as

$$\langle\psi_1|\psi_2\rangle\Phi = \int d\mu_{\mathrm{red}}(a, q|\Phi)\psi_1^*(a, q; \Phi)\psi_2(a, q; \Phi),$$

The measure is not chosen arbitrarily. Gauge degrees of freedom must be removed or fixed, the reduced kinetic operator must be symmetric, and boundary conditions must select a self-adjoint extension. The indefinite DeWitt supermetric is not converted into a positive metric by notation alone. Positivity belongs to the reduced branch and its Hilbert-space construction. These requirements parallel the careful relational-inner-product analyses in the clock-neutral literature [25][26].

Self-adjointness of the effective Hamiltonian gives $d\langle\psi|\psi\rangle\Phi/d\Phi=0$. A Φ-independent domain yields exact conservation within the reduced positive-frequency sector, while an adiabatically varying domain produces controlled corrections. The resulting conditional inner product supplies the probabilistic structure required for relational evolution.

6.4. Born–Oppenheimer/WKB limit

A semiclassical expansion provides a second route. Write the homogeneous state as $\Psi(\Phi,Q)=A(\Phi)\exp[iMS_0(\Phi)]\psi(Q;\Phi)$, where Q denotes the remaining geometry and matter variables and M is a large action scale. At leading order one obtains a Hamilton–Jacobi equation for S_0. At the next order, the derivative along the classical Tempon trajectory yields

$$i\,\partial\psi/\partial\tau = \hat{\mathrm{H}}_{eff}(\tau)\psi + \Delta_{BO}\psi, \partial/\partial\tau = (\partial S_0/\partial\Phi)\partial/\partial\Phi.$$

The expansion is controlled by a semiclassical clock branch with slowly varying amplitude, limited nonadiabatic mixing and a small backreaction operator Δ_{BO}. A large WKB action supplies the relevant scale; it need not coincide with a large particle mass. This distinction preserves compatibility between semiclassicality and monotonicity, since an excessively massive quadratic field would tend to oscillate.

Chua and Callender identify a genuine circularity risk in semiclassical accounts of time [50]. In the Tempon construction, that issue is isolated in the branch-selection and adiabaticity assumptions rather than hidden in the formal expansion. Decoherence suppresses interference between semiclassical branches [51][79][80], after which a sign-preserving Tempon trajectory provides an ordinary Schrödinger parameter for the remaining degrees of freedom. The result is a precise conditional recovery mechanism with explicit control parameters.

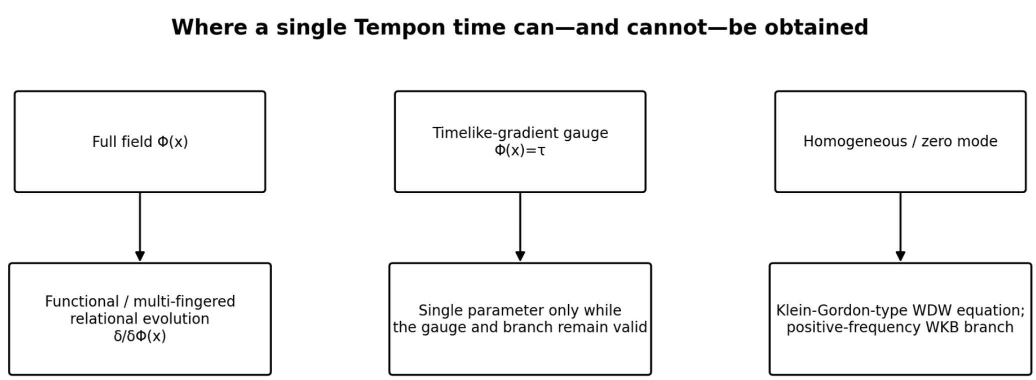

Figure 5. *Quantum reduction: full functional constraint, homogeneous clock sector, branch selection, and conditional Schrödinger evolution.*

7. Analytic Benchmarks and Phenomenological Scope

7.1. Quantitative viability benchmarks

The first quantitative benchmark is a solvable cosmological model. Specifying $A(\Phi)$, $V(\Phi)$, $U(\chi)$, an equation of state and initial data allows direct integration of the exact homogeneous equations and construction of a viability diagram in which the timelike-gradient condition and the sign of P_Φ are tracked across parameter space. A spherically symmetric or other midisuperspace reduction then tests the onset of spatial-gradient failure and the persistence of the clock foliation beyond homogeneity.

A second benchmark compares the Tempon with dust, scalar and geometric clocks inside the same constrained system. The multiple-choice literature predicts that different clock choices can lead to distinct quantum theories [34][35]. The trace-coupled sector has a distinctive potential advantage: every included matter subsystem accesses the same conformal clock field. This advantage becomes quantitative when one compares relational probabilities, clock resolution, branch lifetime and backreaction across the competing descriptions.

The comparison with Brown–Kuchař dust is especially informative. Dust gives a true Hamiltonian through a reference fluid whose momentum enters the constraint in a favorable form, whereas the Tempon retains a quadratic scalar kinetic term and reaches Schrödinger evolution through branch reduction. Dust is therefore structurally efficient for deparameterization, while the Tempon offers a different potential asset: the same field participates in the matter metric and can, in principle, be read by heterogeneous subsystems without introducing a separate comoving medium. York time provides a second contrast. It is built from extrinsic curvature and is directly geometric, while the Tempon is a material scalar whose reading can be correlated with local matter. These distinctions generate measurable comparison criteria rather than a purely verbal hierarchy.

A fair comparison should evaluate at least four quantities on the same background: the interval over which each clock remains single-valued, the conditional uncertainty of common observables, the energy or constraint backreaction carried by the clock, and the map between the corresponding reduced Hilbert spaces. The Tempon becomes scientifically preferable only where matter-wide accessibility compensates for the square-root and branch structure of the scalar clock. This formulation turns “better clock” into a model-dependent statement that can be demonstrated or falsified by one calculation.

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7.2. Phenomenological outlook from the trace coupling

The conformal matter metric turns the clock hypothesis into an experimentally constrained scalar theory. For a small background perturbation $\delta\Phi$, a matter parameter $m_A(\Phi)$ responds as $\delta \ln m_A = q_A \delta\Phi$, where $q_A \equiv d\ln m_A/d\Phi$. An atomic transition I therefore acquires a fractional shift $\delta\nu_I/\nu_I = K_I q_A \delta\Phi$, with K_I determined by the transition sensitivity. A coherent Tempon background would produce correlated signals across a clock network, while differences among sensitivity coefficients would help identify the coupling structure. Atomic-clock searches for coherently oscillating dilaton dark matter provide the closest established phenomenological analogue [81]. Related network proposals target transient topological scalar backgrounds [82], an experimentally relevant but distinct signal class from the coherent narrow-band background considered here. Optical-clock comparisons and relativistic interferometry provide the relevant experimental architecture [67][68][69][83].

The same scalar charge produces a Yukawa-type interaction proportional to $-q_A q_B \exp(-m_\Phi r)/(4\pi r)$, so inverse-square-law and equivalence-principle tests constrain the unscreened region of parameter space [71][84]. In the matter-frame metric, a scalar perturbation contributes a conformal strain proportional to $2[A'(\Phi)/A]\delta\Phi$ and can therefore generate a breathing-like detector response. This connects the framework to network searches for non-tensorial gravitational-wave polarization [85]. Anomaly-induced traces govern access to nominally conformal sectors [86], while portal operators, radiative stability and ultraviolet matching provide complementary particle-physics constraints [87][88][89].

These channels provide a disciplined phenomenological program. The canonical theory fixes the coupling structure, while a chosen Tempon mass, potential, background abundance and screening mechanism determine the actual signal amplitude and spectrum. The resulting parameter map can combine atomic-clock correlations, fifth-force bounds and scalar-polarization searches without assigning any one signal a prematurely unique interpretation.

The phenomenological hierarchy is naturally organized by the Tempon Compton wavelength. When m_Φ^{-1} exceeds a laboratory baseline, the field behaves coherently across a clock network and the leading observables are correlated frequency shifts and equivalence-principle tests. At intermediate ranges, spatial gradients and screening determine the fifth-force signal. For propagating relativistic excitations, the same conformal response enters scalar-polarization searches. Constraints from these experiments can be projected onto the common parameters $A'(\Phi)/A$, m_Φ and the background amplitude.

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The clock interpretation adds a further requirement beyond ordinary scalar phenomenology: the background must also lie inside the sign-preserving and timelike-gradient sector identified by the canonical analysis. Consequently, an experimentally allowed scalar parameter point is relevant to Tempon time only when it also satisfies the clock-viability inequalities. Combining those two filters—observational viability and relational-clock viability—is a distinctive output of the program and provides a direct way to connect foundational claims with measurable physics.

A particularly direct target is a coherent low-mass background. If $\Phi(t)=\bar{\Phi}+\Phi_1\cos(m\Phi t+\phi)$, the clock-network response is narrow-band at the Compton frequency, with detector-motion sidebands determined by the spatial coherence of the field. Their relative size follows from the background distribution and the conformal charges, making them a calculable target for a companion phenomenology study.

7.3. Quantitative clock quality and comparison criteria

Clock quality can be expressed through three dimensionless diagnostics. The reading uncertainty is

$$\sigma\tau \approx \Delta\Phi/|d\langle\Phi\rangle/d\tau|,$$

the branch quality is

$$Q_\Phi = |\langle\pi_\Phi\rangle|/\Delta\pi_\Phi,$$

and a convenient backreaction measure is

$$\varepsilon_{BR} = \rho_\Phi/\rho_{\text{tot}}$$

or, in the Born–Oppenheimer reduction,

$$\|\Delta_{BO}\|/\|H_{\text{eff}}\|.$$

A useful Tempon regime combines small σ_τ, large Q_Φ and $\varepsilon_{BR}\ll1$. These quantities turn the accuracy–disturbance balance into a calculable criterion.

Comparison with Brown–Kuchař dust or a geometric clock should use the same physical state and the same observables. One conditions on Φ and on the alternative clock in their overlapping domains, then compares dimensionless conditional probabilities, branch lifetime, resolution and backreaction. Equivalent predictions identify alternative perspectives; systematic differences isolate domain restrictions, inequivalent quantizations or operational advantages of matter-wide accessibility.

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The distinctive preferred-clock hypothesis is therefore precise: the universal conformal coupling may make Φ simultaneously readable by diverse subsystems while maintaining a longer sign-preserving branch and lower conditional uncertainty than competing clocks. Establishing that hierarchy in a concrete model would elevate the Tempon from a viable clock sector to a dynamically distinguished one.

8. Discussion

8.1. Established contribution

This work establishes a self-consistent four-dimensional starting point for a physical clock sector. The trace interaction is derived from a conformal matter metric; the matter momentum and Hamiltonian follow from the combined Legendre transform; and the resulting covariant, nondegenerate theory realizes the standard classical hypersurface-deformation algebra. The explicit scalar-sector bracket check identifies the cancellations behind that result. In homogeneous cosmology, the exact momentum balance supplies a calculable sign-preserving criterion and a turning-point boundary.

The quantum analysis establishes the corresponding reduction pathway. A full local Tempon field defines a multi-fingered relational structure, while homogeneous reduction or gauge fixing produces a single clock coordinate. Within a positive-frequency branch and a self-adjoint reduced domain, the Tempon supports a conserved conditional inner product and relational Schrödinger evolution. The Born–Oppenheimer expansion then identifies the nonadiabatic and backreaction parameters that control the approximation.

8.2. Domain of validity and research frontier

The established domain is the covariant classical theory, its homogeneous clock sectors and the reduced semiclassical quantum branch. The next frontier consists of an inhomogeneous clock theorem, quantum anomaly control, a nonperturbative physical Hilbert space and a direct comparison with dust, York-time and clock-neutral descriptions. Phenomenological identification likewise requires a specified mass, background state, screening regime and ultraviolet completion.

These extensions form a focused research program rather than a diffuse list of caveats. A positive result would exhibit an invariant open region of phase space with a dynamically protected Tempon branch and favorable clock-quality measures. A complementary no-go result would locate the region where trace

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coupling forces turning points or excessive backreaction. Both outcomes are sharp tests of the same model.

8.3. Novelty and significance

The originality of the Tempon framework lies in the combination of ingredients analyzed within one covariant model: a dedicated four-dimensional scalar clock sector, matter-wide conformal trace coupling, an exact combined canonical derivation, explicit first-class closure, a dynamical clock-viability boundary and a controlled semiclassical reduction. Scalar clocks, trace-coupled fields and relational Schrödinger equations each have important precedents; their integration into this single viability framework provides the paper's distinctive contribution.

The framework therefore offers more than a new name for a scalar field. It supplies a common language in which canonical gravity, relational quantum mechanics and low-energy scalar phenomenology can be evaluated together. The canonical algebra tests whether the field is mathematically admissible, the momentum balance tests whether it can order change, the reduced inner product tests whether it supports quantum probabilities, and the phenomenological map tests whether the same coupling survives observation. A theory that passes all four stages would provide an unusually complete realization of a physical internal clock.

9. Conclusion

The Tempon framework represents the missing-clock question as a definite field theory. Matter couples to $A^2(\Phi)\, g_{\mu\nu}$, the ΦT interaction emerges at linear order, the exact scalar momenta and Hamiltonian follow from the combined Legendre transform, and the classical constraints realize the standard hypersurface-deformation algebra. These results establish the canonical admissibility of the trace-coupled clock sector.

The exact homogeneous momentum balance then determines when the admissible scalar becomes a working clock. Free and controlled monotonic sectors support open sign-preserving branches, whereas oscillatory potentials encounter a calculable turning-point boundary. In the reduced quantum theory, positive-frequency branch selection and self-adjointness yield a conserved conditional inner product and Schrödinger evolution relative to the Tempon. The same conformal coupling defines quantitative clock-quality measures and connects the theory to precision clocks, fifth-force tests and scalar gravitational-wave response.

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The central scientific claim is therefore both positive and testable: a self-consistent trace-coupled scalar can form a first-class canonical sector and can provide relational time on dynamically identifiable branches. The next calculations will determine the size and stability of those branches, their operational advantage over alternative clocks and the experimentally allowed parameter space. This creates a direct path from the foundational Problem of Time to canonical analysis, quantum clock comparison and precision phenomenology.

Appendices

Appendix A. Exact Trace Coupling and Legendre Transform

A.1. Variation of the conformal matter metric

Let $S_m[\tilde{g}, \Psi]$ be defined by the matter metric

$$\tilde{g}_{\mu\nu} = A^2(\Phi) g_{\mu\nu}.$$

The stress tensor with respect to $\tilde{g}$ is defined through

$$\delta S_m = \frac{1}{2} \int \sqrt{-\tilde{g}}\, \widetilde{T}^{\mu\nu}\, \delta \tilde{g}_{\mu\nu}.$$

For a variation of Φ at fixed $g_{\mu\nu}$,

$$\delta \tilde{g}_{\mu\nu} = 2AA'(\Phi) g_{\mu\nu} \delta\Phi = 2\frac{A'(\Phi)}{A} \tilde{g}_{\mu\nu} \delta\Phi.$$

Hence

$$\delta S_m = \int d^4x \sqrt{-\tilde{g}}\ \frac{A_{,\Phi}}{A} \widetilde{T} \delta\Phi.$$

After expressing the density and trace in the Einstein frame, this gives the source used in the Φequation. Expanding

$$A = \exp\left(\frac{\alpha\Phi}{M}\right) = 1 + \frac{\alpha\Phi}{M} + O(\alpha^2)$$

yields the linear ΦToperator. All equations in the main text use one stress-tensor convention consistently.

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A.2. Scalar matter ADM Lagrangian

For the explicit scalar matter model,

$$\sqrt{-g}=N\sqrt{h}$$

and

$$g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi=-v_\chi^2+h^{ij}D_i\chi D_j\chi.$$

Therefore,

$$\mathscr{L}_\chi=\frac{\sqrt{h}A^2}{2N}\left(\dot\chi-N^iD_i\chi\right)^2-\frac{N\sqrt{h}A^2}{2}D_i\chi D^i\chi-N\sqrt{h}A^4U.$$

Differentiation with respect to $\dot\chi$ gives

$$\pi_\chi=\sqrt{h}A^2v_\chi.$$

Solving for $\dot\chi$ and substituting into $\pi_\chi\dot\chi-\mathscr{L}_\chi$ gives

$$\pi_\chi\dot\chi-\mathscr{L}_\chi=N\left[\frac{\pi_\chi^2}{2\sqrt{h}A^2}+\frac{\sqrt{h}A^2}{2}(D\chi)^2+\sqrt{h}A^4U\right]+N^i\pi_\chi D_i\chi.$$

The Tempon sector is standard:

$$\pi_\Phi=\sqrt{h}v_\Phi,$$

and

$$\pi_\Phi\dot\Phi-\mathscr{L}_\Phi=N\left[\frac{\pi_\Phi^2}{2\sqrt{h}}+\frac{\sqrt{h}}{2}(D\Phi)^2+\sqrt{h}V\right]+N^i\pi_\Phi D_i\Phi.$$

Adding the two expressions produces the exact scalar Hamiltonian and momentum constraint given in Section 3. Expanding

$$A^2\simeq1+\frac{2\alpha\Phi}{M},A^{-2}\simeq1-\frac{2\alpha\Phi}{M},A^4\simeq1+\frac{4\alpha\Phi}{M},$$

shows that kinetic, gradient, and potential corrections combine into the complete trace interaction. Expanding only the kinetic term would omit the A^4 potential contribution.

A.3. Regularity and ghost conditions

The scalar kinetic matrix in normal velocities is

$$\mathrm{diag}(1, A^2).$$

Its determinant is

$$A^2,$$

so the Legendre map is invertible for $A \neq 0$. Positivity of the matter kinetic energy requires

$$A^2 > 0.$$

Since a real conformal factor already gives $A^2 \geq 0$, the nontrivial restriction is that Aremain finite and nonzero. Higher-derivative operators can introduce additional degrees of freedom or Ostrogradsky instabilities and are outside the model.

Appendix B. Explicit Scalar-Sector Bracket Check

B.1. Smeared generators

Let

$$H_s[N] = \int N \mathscr{H}_{\Phi\chi},$$

and

$$D_s[N^i] = \int N^i \left(\pi_\Phi D_i \Phi + \pi_\chi D_i \chi\right).$$

The fundamental brackets are

$$\{\Phi(x), \pi_\Phi(y)\} = \delta^3(x-y),$$

and

$$\{\chi(x), \pi_\chi(y)\} = \delta^3(x-y).$$

The momentum generator acts by Lie derivatives on both fields and their density-valued momenta. Consequently, $\{D_s[N], D_s[M]\}$and $\{D_s[N], H_s[M]\}$have the standard spatial-diffeomorphism form.

B.2. Hamiltonian–Hamiltonian bracket

Decompose

$$\mathscr{H}_{\Phi\chi} = K_\Phi + G_\Phi + V_\Phi + K_\chi + G_\chi + V_\chi,$$

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with

$$K_\chi = \frac{\pi_\chi^2}{2\sqrt{h}A^2},$$

$$G_\chi = \frac{\sqrt{h}A^2}{2} D\chi \cdot D\chi,$$

and

$$V_\chi = \sqrt{h}A^4 U.$$

Brackets among ultralocal kinetic and potential terms cancel in the antisymmetrized smeared bracket. Terms containing spatial derivatives arise from the same-field kinetic–gradient brackets and from cross terms generated by the Φ-dependence of A.

The standard same-field terms give

$$\{H_\Phi[N], H_\Phi[M]\} = \int d^3x\, h^{ij}\,(N\partial_j M - M\partial_j N)\,\pi_\Phi D_i\Phi,$$

$$\Big\{H_\chi[N], H_\chi[M]\}_{A\ fixed} = \int d^3x\, h^{ij}\,(N\partial_j M - M\partial_j N)\,\pi_\chi D_i\chi.$$

When A depends on Φ, functional differentiation of K_χ, G_χ, and $V_\chi$produces terms proportional to $A'(\Phi)$. Terms without derivatives of the smearing functions cancel after antisymmetrization. The derivative terms arising from $D_i\chi$integrations by parts combine with the corresponding Φ-momentum terms, leaving the standard scalar momentum constraint.

$$\{H_s[N], H_s[M]\} = D_s\left[h^{ij}\,(N\partial_j M - M\partial_j N)\right].$$

Adding the gravitational bracket and the standard cross terms gives the total momentum constraint. The direct calculation confirms the covariance argument at the classical level; quantum realization of the same algebra is governed by regularization and factor ordering.

For clarity, the relevant functional derivatives are

$$\frac{\delta K_\chi}{\delta \pi_\chi} = \frac{N\pi_\chi}{\sqrt{h}A^2},$$

$$\frac{\delta K_\chi}{\delta \Phi} = -\frac{N\pi_\chi^2 A'(\Phi)}{\sqrt{h}A^3},$$

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and

$$\frac{\delta G_\chi}{\delta(D_i\chi)} = N\sqrt{\hbar}A^2 D^i\chi.$$

Integrating the χvariation of $G_\chi$by parts produces derivatives of the smearing $N$and of A^2. The latter contain

$2AA'(\Phi)D_i\Phi$ and are exactly the terms that would be missed if the A-dependence were ignored.

The cross bracket between $K_\Phi$and the Φ-dependence of $K_\chi + G_\chi + V_\chi$ is ultralocal in the smearing functions and cancels pairwise under antisymmetrization. The remaining non-ultralocal terms arise from spatial gradients. After integration by parts, derivatives of A^2multiplying $D_i\Phi$and $D_i\chi$combine into the scalar momentum density

$\pi_\Phi D_i\Phi + \pi_\chi D_i\chi$ with the expected structure function.

This calculation shows why the standard momentum constraint is necessary but insufficient by itself. Closure also depends on the exact relation between the kinetic, gradient, and potential coefficients $A^{-2},\ A^2,\ \ A^4$ inherited from the covariant action.

B.3. Boundary terms

Integrations by parts generate surface terms proportional to $\oint dS_i\left(ND^iM - MD^iN\right)F$. They vanish on compact slices. On noncompact slices, falloff conditions must make them finite and incorporate the associated boundary charge. The bracket statement assumes differentiable generators completed according to Regge–Teitelboim.

Appendix C. Homogeneous Dynamics and Clock-Sector Bounds

C.1. Energy density and pressure

For homogeneous fields, the scalar energy density and pressure are

$$\rho = \frac{\dot{\Phi}^2}{2} + V + \frac{A^2\dot{\chi}^2}{2} + A^4U,$$
$$p = \frac{\dot{\Phi}^2}{2} - V + \frac{A^2\dot{\chi}^2}{2} - A^4U.$$

The Friedmann equations are,

$$3M_p^2H^2 = \rho$$

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And,

$$-2M_p^2\dot{H} = \dot{\Phi}^2 + A^2\dot{\chi}^2.$$

These relations ensure regular evolution while the fields and Aremain finite; the sign of $\dot{P}_\Phi$follows from the separate momentum equation.

C.2. Integrated sign criterion

Define

$$F(t) = a^3\left[V'(\Phi) - \frac{A'(\Phi)}{A}T_\chi\right].$$

The exact solution is,

$$P_\Phi(t) = P_\Phi(t_0) - \int_{t_0}^{t} F(s)\, ds.$$

A sufficient positive-branch condition is,

$$\sup_{t\in I}\int_{t_0}^{t} F(s)\, ds < P_\Phi(t_0).$$

If $F \leq 0$, every initially positive $P_\Phi$is protected on the interval. If $F$is positive but integrable, sufficiently large initial momentum remains protected.

For,

$$V = \frac{m^2\Phi^2}{2}$$

and negligible trace source,

$$F = a^3m^2\Phi$$

changes sign during oscillation and $\dot{\Phi}$generically crosses zero repeatedly. For $V = 0$ with a radiation source of classical trace $T = 0$, $P_\Phi$is conserved. Dust or massive scalar matter produces a nonzero trace and shifts the balance according to the sign of $A'(\Phi)T$.

C.3. Perturbative stability question

Let

$$\Phi = \bar{\Phi}(t) + \delta\Phi(t, x).$$

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Even when $\bar{P}_\Phi$ is positive, a perturbation can make the local normal derivative vanish. Linear stability requires the perturbation energy and spatial gradients to remain small compared with $\dot{\bar{\Phi}}^2$.

A nonlinear result would require Sobolev estimates for the coupled Einstein–two-scalar system.

C.4. Representative dynamical sectors

Three limiting sectors illustrate the boundary. In a radiation-dominated background with $V = 0$and classically vanishing trace, $P_\Phi$is conserved and every homogeneous solution with $P_\Phi \neq 0$remains monotonic while the conformal factor and geometry stay regular. In a dust-dominated background, the trace source changes P_Φ secularly; depending on the sign of $A'(\Phi)$and the initial momentum, the source can protect the branch or drive it to a turning point. In a vacuum massive model, repeated turning points are generic.

A useful numerical study should scan dimensionless variables. With

$$A = \exp\left(\frac{\beta\Phi}{M_p}\right),$$

one may vary β, the initial kinetic-to-potential ratio, the matter fraction, and the equation of state. The output should record the first zero of P_Φ, loss of the timelike-gradient condition, violation of $A > 0$, and the accumulated number of e-folds. The resulting boundary separates successful and failed clock sectors.

The same scan can test robustness under perturbations by adding a small spatial Fourier mode. If the background branch remains positive but the perturbed normal derivative develops a local zero, homogeneous viability is not stable. Conversely, a bounded perturbation norm over a finite interval would provide evidence for an open inhomogeneous sector. Such evidence is weaker than a theorem but considerably stronger than an untested assertion of global monotonicity.

Appendix D. Quantum Reduction and Conditional Inner Product

D.1. Positive-frequency sector

Suppose the reduced constraint has the form

$$\left(-\frac{\partial^2}{\partial\Phi^2} + \Theta(\Phi)\right)\Psi = 0.$$

If $\Theta$is positive and self-adjoint on a fixed domain, instantaneous positive- and negative-frequency sectors can be defined. For Φ-independent Θ, the sectors are invariant and,

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$$\Psi_{+} = \exp\left(-i\sqrt{\Theta}\,\Phi\right)\Psi_{+}(0).$$

For Φ-dependent Θ, the projectors vary and branch mixing is controlled in an adiabatic regime.

D.2. Self-adjointness and measure

The reduced measure $d\mu_{red}$ arises after gauge fixing or construction of Dirac observables. Boundary conditions at singular configurations determine self-adjoint extensions. The conditional Hamiltonian is unitary on a domain preserved by the evolution.

D.3. BO current

For

$$\Psi = A\ \exp(iMS_0)\psi,$$

the Wheeler–DeWitt current expanded along the classical Tempon trajectory gives a leading conditional norm and corrections from $\partial_\Phi A$, $\partial_\Phi \psi$, and backreaction. Choosing the WKB transport equation removes the leading nonconserved prefactor. Residual nonadiabatic terms are higher order in the expansion parameter.

A nonperturbative completion will combine a physical Hilbert space for the full constraints, quantum-anomaly control, and transformations between clock perspectives. The reduced analysis identifies the precise structures that such a completion must preserve.

D.4. Order-by-order Born–Oppenheimer structure

Let the reduced Wheeler–DeWitt operator be

$$\widehat{H} = -\frac{1}{2M}\partial_\Phi^2 + MW(\Phi) + \widehat{H}_f(\Phi),$$

where M is the large semiclassical scale. Substituting

$$\Psi = \exp(iMS_0)A\psi$$

gives the Hamilton–Jacobi equation

$$\frac{\left(S_0'\right)^2}{2} + W = 0$$

at leading order. The next order yields the WKB transport equation and

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$$iS_0'\partial_\Phi \psi = \widehat{H}_f \psi$$

after a phase convention is chosen. Higher orders contain $\partial_\Phi^2 \psi$and Berry-connection terms.

The effective time derivative is directional in configuration space:

$$\partial_\tau = S_0' \partial_\Phi .$$

It becomes singular at a turning point $S_0' = 0$, matching the classical failure of P_Φ. Near such a point, the WKB branches mix and a single Schrödinger description is replaced by a connection formula, a two-branch treatment, or a periodic-clock construction.

Norm conservation at leading order follows from the transport equation and self-adjointness of $\widehat{H}_f$ on the reduced domain. At the next order, the corrected inner product may require a Φ-dependent metric operator so that the effective Hamiltonian is Hermitian with respect to it. This is analogous to adiabatic corrections in molecular Born–Oppenheimer theory. The correction is calculable only after a concrete minisuperspace model, ordering, and boundary condition are fixed.

Appendix E. Reproducible Viability Protocol

A reproducible Tempon study should report five layers separately. First, the action and frame convention must be fixed, including $A(\Phi)$, $V(\Phi)$, all matter potentials, and the definition of the stress tensor. Second, the full velocity Hessian and canonical momenta must be displayed. Third, the constraint algebra must be established either by an explicit calculation or by a stated covariance theorem whose hypotheses are verified. Fourth, the clock domain must be defined through the causal character of $\nabla\Phi$, the sign of π_Φ, and the absence of singular conformal factors. Fifth, the quantum reduction must state the gauge, frequency branch, ordering, measure, and operator domain.

Numerical work should report raw norm drift together with convergence under time-step and grid refinement. A genuine decoherence test uses the Φ-basis reduced density matrix or influence functional, and clock accuracy is extracted from $\Delta\Phi$, $\Delta\pi_\Phi$, branch lifetime, and backreaction rather than from generic oscillator evolution.

Analytic theorems should distinguish dynamical consequences from assumptions defining the chosen branch. Once a comparison inequality is derived from the field equations, it yields a genuine sign-preservation result. Turning-point and no-go results are equally informative because they map the viability boundary of the clock sector.

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Data Availability

No datasets were generated or analyzed during the current study. This work is theoretical and all relevant derivations are included in the manuscript.

Use of Generative AI

During the preparation of this work, the author used ChatGPT (GPT-5.5, OpenAI) for language editing, paraphrasing, proofreading, and checking the clarity and consistency of mathematical expressions. All intellectual content, scientific arguments, analyses, and conclusions were developed, reviewed, edited, and verified by the author.

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