Review of: "Probability spaces identifying ordinal and cardinal utilities in problems of an economic nature: new issues and perspectives"

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The objective of the study under review is unclear to me. The study states some results that can be deemed to be already existing in, perhaps some alternate specific mathematical parlance. It is not clear to me, however, whether the study seeks to review the literature or seeks to formulate itself as a pedagogical tool. In stated respect, I do not infer any particularly new result - there might be, but the exposition is not clear enough for me to assert otherwise. In light of the foregoing, the exposition of the study that is under review still leaves much room for improvement. Relatedly, it seems to me that it is important that the study under review define it's audience, because in the manuscript's current form, only enthusiasts of either mathematics or the mathematical fundamentals of asset pricing are able to understand the study's articulations, an outcome which if the study were to fashion a well defined objective along the lines that I recommend would be such a waste. If English is not the author's first language, a professional edit of the revised manuscript might add significant value.

What seems to me most interesting is the highlight of a result that already is established in Obrimah (2023), namely that risk seeking preferences (rs) are rational `if and only if', relative to the returns $r()$ on a well diversified portfolio (dp), expected returns, μ satisfy,

$$\mu(rs) \geq \mu(dp) + 2 \times \sigma[r(dp)],$$

hence the insight that risk neutrality is not premised on identical returns to each of a globally risk averse agent (a fully diversified portfolio) and a risk seeking agent. I find the stated insight to be interesting, because the relation between risk neutrality, risk seeking preferences, and risk aversion can tend to be sorely misconstrued. In stated respect, the communicated insight rules out the notion that risk neutrality revolves around some assumed time invariance of the conditional risk free rate, as opposed to an intertemporal indifference between taking up, as appropriate, either global risk aversion or risk seeking preferences. Rational agents are indifferent to switches back and forth between the two preferences, because the application of heterogeneous realizations of discount rates (risk premiums), δ towards the valuation of cash flows that all have specification as certainty equivalents induces indifference between the two sets of risk preferences. Given the study under review already incorporates certainty equivalents into it's framework, it is amenable to such an exposition. If the study under review were to cast itself as either of a literature review of, or a pedagogical study of the stated phenomenon and how it contradicts the conventional and ubiquitous linkage of risk neutrality to the evolution of the risk free rate, it becomes more focused and streamlined, and it seems to me readers will better appreciate it's

objective.

   In addition to the foregoing, it seems to me that the relations that subsist between ordinal and cardinal utility could use some clarification along the following lines. Canonically, whereas ordinal utility is a qualitative measure of utility, regardless it is required to be measurable in the sense that, in the presence of two alternatives, α and β, it can be asserted that, $U(α) > U(β)$. In presence of measures that, independent of U, are appropriate to a comparison of α and β, for instance, discount rates, δ, the measure, $U(α) > U(β)$ can be distanced. Given asset prices, P and δ are duals, all else equal (all firms issuing only *1* unit of equity), asset prices, P become a measure of the utility distance, $U(α) – U(β)$, yet U itself remains qualitative and non-cardinal. In stated respect, as the study under review states, the assertion that P is concave in expected returns is a qualitative, as opposed to a quantitative assertion, because until the parameters of the concave function are specifically established, P is qualitative, that is, ordinally ordered, not cardinal, as such, non-parametric. Ditto the statement that P is a strictly convex function of δ. I do not find that the study under review makes clear that the notion of `distance' results from the measurability of α and β that is induced by, say δ, and that is independent of the fact that U itself remains ordinal. Naturally, δ becomes parametric in the context of the introduction of specific realizations for δ into a pricing function that is derived from the modeling of the martingale that, qualitatively, populates and propagates δ. An important fallout of the foregoing is, with either P or δ as the parametric parameterization of $U(α) > U(β)$, there is arrival at the redundancy of any and all efforts for the transformation of an ordinal U into some cardinal equivalent. Consider then that the modeling of cardinal utility for the determination of asset prices, an ubiquity which generates prices that are directly parametrically derived, as such is non-robust to the emergence of a perturbation to the parameters of an economy (the shape of the concave or strictly convex functions have to alter, and the new parameters only can be assumed, as such feasibly embed error that has materiality), is not as robust as the approach to P through a pricing function that is the parametric formulation of a pricing martingale, an approach that in my reading is promoted by the study under review. For concreteness, since a pricing function that is derived from a martingale is robust to any and all states of the world, contrary to the necessity of a reformulation of the pricing kernel in the context of cardinal utility, consequent on the emergence of a perturbation to the parameters of an economy, a pricing function remains robust to the pricing of all assets. In aggregate, the absence of any demand for changes to the parameters of the pricing function renders the derivation of a pricing function more robust than the adoption of cardinal specifications of utility functions. Whereas both still are preprints, I have a study, Obrimah (2023b) and a simulation supplement, Obrimah (2023c) of that study that the author might find relevant and helpful. For evidence that there still is some confusion as to the relations that subsist between risk neutrality, risk preferences, discount rates, and information, see Cochrane (2011).

Cochrane, J.H. (2011). Presidential address: Discount rates. Journal of Finance 66, 1047-1108.

Obrimah, O.A. (2023). Outside of a sole globally risk averse agent, all other agents in markets are risk seeking agents. Finance Research Letters 54, 103715.

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