

RESEARCH ARTICLE

Decay Characteristics of Neutron Excess Chromium Nuclei

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Abstract

A single-particle model is employed to calculate the properties of neutron excess chromium nuclei. The model predicts that $A = 57 - 78$ neutron excess chromium systems are bound and have half-lives in the range of 0.287 – 7.27 ms. The Japanese Nuclear Data Compilation predicts half-life values that are about a factor of four larger than the model proposed in this paper. Alpha, beta, positron, electron capture, and spontaneous fission decay modes are included in the model. However, neutron emission decay modes that have short half-lives are not evaluated. These short-lived neutron emission modes suggest that the model results could overestimate the half-lives of neutron excess chromium nuclei.

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1. Introduction

The construction of facilities and advances in experimental and theoretical physics has intensified interest in neutron excess nuclei^{[1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][25][26][27]}. A number of physical processes can produce neutron excess systems, but r-process nucleosynthesis often dominates. Investigation of neutron excess nuclei is important from a nuclear physics as well as astrophysics perspective. The production of these systems in neutron star and black hole mergers^{[1][2]} is a topic of active research interest.

Neutron excess chromium systems are investigated in this paper. This paper continues previous publications that addressed neutron excess systems having $Z = 9 - 23, 26$, and 30 ^{[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24]}. The systematics of these studies provides additional insight into the various nucleosynthesis mechanisms, and how these production modes vary with atomic and mass numbers.

2. Calculational Methodology

Refs^{[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24]} outline the method for investigating neutron excess nuclei. The

model utilizes the single particle approach, and incorporates the methodology of Lukasiak and Sobiczewski^[26] and Petrovich et. al.^[27]. The numerical methods of Refs^[28] and^[29] are utilized to determine the single particle energies of neutron excess nuclear systems.

The binding energy E_{NLSJ} of a nucleon in the field of a nuclear core is derived from the solution of the radial Schrödinger equation^{[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24]}

$$[(\hbar^2/2\mu) (d^2/dr^2 - L(L+1)/r^2) - E_{\text{NLSJ}} - V_{\text{LSJ}}(r)] U_{\text{NLSJ}}(r) = 0 \quad (1)$$

In Eq. 1, r is the radial coordinate, $V_{\text{LSJ}}(r)$ is the model interaction, and $U_{\text{NLSJ}}(r)$ is the radial wave function. The quantum numbers L , S , and J represent the orbital, spin, and total angular momentum, respectively. The radial quantum number (N) and the reduced mass (μ) complete the specification of the calculational model.

3. Nuclear Interaction

The Rost interaction^[30] forms the basis for the nuclear potential that has a central strength

$$V_0 = 51.6 [1 \pm 0.73 (N - Z)/A] \text{ MeV} \quad (2)$$

In Eq. 2, the positive (negative) sign applies to protons (neutrons). The spin-orbit interaction strength (V_{so}) is defined by the parameter γ ^[30]:

$$V_{\text{so}} = \gamma V_0 / 180 \quad (3)$$

The model interaction is completed with the inclusion of the pairing correction interaction of Blomqvist and Wahlborn^[31].

Refs.^[32] and^[33] note the difficulties in defining an appropriate interaction, and demonstrate that modifications are required to ensure that an accurate fit to the experimental energy levels and decay characteristics. Following the methodology from Refs.^[32] and^[33], the Rost central interaction strength (V_A) is modified in the following manner

$$V_A = V_0 \lambda [1 \pm a(A)] \text{ MeV} \quad (4)$$

In Eq. 4, a potential strength multiplier (λ) and a factor $[a(A)]$ adjust the potential strength as a function of A . For chromium, $\lambda = 1.5$ is selected for consistency with previous calculations^{[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24]}. This value was selected to ensure agreement with available data^{[34][35][36]}.

4. Model Limitations

Based upon previous calculations^{[8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][24][26][27][37]} alpha decay, beta decay, positron decay, electron capture, and spontaneous fission are represented reasonably well by calculations summarized in Sections 2 and 3. Although suitable for the aforementioned decay modes, single-particle models are not the best

approach to calculate neutron emission half-lives. These neutron emission decay modes are generally shorter than the previously noted decay modes. Accordingly, omission of neutron emission decay modes leads to results that tend to overestimate the calculated decay half-lives.

5. Results and Discussion

Table 1 summarizes the complete set of $78 \geq A \geq 57$ chromium isotopes considered in this paper. The $78 \geq A \geq 57$ chromium isotopes occupy the $1f_{5/2}$ ($^{57}\text{Cr} - ^{62}\text{Cr}$), $2p_{1/2}$ ($^{63}\text{Cr} - ^{64}\text{Cr}$), $1g_{9/2}$ ($^{65}\text{Cr} - ^{74}\text{Cr}$), and $2d_{5/2}$ ($^{75}\text{Cr} - ^{78}\text{Cr}$) neutron single-particle levels. Based on data summarized in Refs. [34][35][36], the heaviest observed system is ^{66}Cr . Extrapolations beyond $A > 66$ become more uncertain, because data is not available to guide the calculations.

5.1. $57 \geq A \geq 66$ Chromium Isotopes with Experimental Half-Life Data

Table 1 includes the half-life of the limiting decay mode (i.e., the transition that has the shortest decay half-life). For example, the ^{59}Cr model indicated five beta decay transitions (i.e., allowed $1f_{7/2}(n)$ to $1f_{7/2}(p)$ [8.05 s], allowed $2p_{3/2}(n)$ to $2p_{3/2}(p)$ [18.8 s], allowed $2p_{3/2}(n)$ to $2p_{1/2}(p)$ [8.27 min], allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ [460 ms], and allowed $1f_{5/2}(n)$ to $1f_{5/2}(p)$ [18.5 s]). For ^{59}Cr , the allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ [460 ms] transition is the limiting beta decay mode.

Table 1. Calculated Single-Particle and Experimental Decay Properties of Chromium Nuclei with $57 \leq A \leq 78$

Nuclide	a(A)	Half-Life (Decay Mode)	
		Experiment ^{a,b,c}	This Work
⁵⁷ Cr	-0.0587	21.1 s ^c	21.3 s (β) ^d
⁵⁸ Cr	-0.0582	7.0 s ^c	6.96 s (β) ^d
⁵⁹ Cr	-0.0240	460 ms ^c	460 ms (β) ^d
⁶⁰ Cr	-0.0356	490 ms ^c	490 ms (β) ^d
⁶¹ Cr	-0.0276	237 ms ^c	238 ms (β) ^d
⁶² Cr	-0.0334	206 ms ^c	206 ms (β) ^d
⁶³ Cr	-0.0246	113 ms ^c	113 ms (β) ^d
⁶⁴ Cr	+0.0018	43 ms ^c	43.0 ms (β) ^d
⁶⁵ Cr	+0.0131	27 ms ^c	27.0 ms (β) ^d
⁶⁶ Cr	+0.0570	10 ms ^c	10.0 ms (β) ^d
⁶⁷ Cr	+0.0683	e,f	7.27 ms (β) ^d
⁶⁸ Cr	+0.0902	f,g	4.68 ms (β) ^d
⁶⁹ Cr	+0.1120	e,h	3.15 ms (β) ^d
⁷⁰ Cr	+0.1339	e,i	2.21 ms (β) ^d
⁷¹ Cr	+0.1557	e,j	1.60 ms (β) ^d
⁷² Cr	+0.1776	e,k	1.18 ms (β) ^d
⁷³ Cr	+0.1994	e,l	0.895 ms (β) ^d
⁷⁴ Cr	+0.2213	e,m	0.692 ms (β) ^d
⁷⁵ Cr	+0.2431	e	0.545 ms (β) ^d
⁷⁶ Cr	+0.2650	e	0.434 ms (β) ^d
⁷⁷ Cr	+0.2868	e	0.352 ms (β) ^d
⁷⁸ Cr	+0.3087	e	0.287 ms (β) ^d

^a Ref. [\[34\]](#).

^b Ref. [\[35\]](#).

^c Ref. [\[36\]](#).

^d Allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transition.

^e No data provided in Refs. [\[34\]](#)[\[35\]](#)[\[36\]](#).

^f The Japanese data compilation [\[36\]](#) notes a calculated value of 31.1 ms for ⁶⁷Cr.

^g The Japanese data compilation [\[36\]](#) notes a calculated value of 17.2 ms for ⁶⁸Cr.

^h The Japanese data compilation [\[36\]](#) notes a calculated value of 11.6 ms for ⁶⁹Cr.

ⁱ The Japanese data compilation [\[36\]](#) notes a calculated value of 7.14 ms for ⁷⁰Cr.

^j The Japanese data compilation [\[36\]](#) notes a calculated value of 6.07 ms for ⁷¹Cr.

^k The Japanese data compilation^[36] notes a calculated value of 4.12 ms for ^{72}Cr .

^l The Japanese data compilation^[36] notes a calculated value of 3.80 ms for ^{73}Cr .

^m The Japanese data compilation^[36] notes a calculated value of 2.80 ms for ^{74}Cr .

As noted in Table 1, the model predicts the proper decay mode for the known $78 \geq A \geq 57$ chromium systems^{[34][35][36]}. The model half-lives are also consistent with data^{[34][35][36]}.

The $^{57}\text{Cr} - ^{62}\text{Cr}$ systems occupy the $1f_{5/2}$ neutron shell, and decay via beta emission through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transitions. Model predictions for the half-lives of $^{57}\text{Cr} - ^{62}\text{Cr}$ are within about 1% of the experimental half-lives^[36]. The calculated beta decay modes for $^{57}\text{Cr} - ^{62}\text{Cr}$ are in agreement with Ref^[36].

The $^{63}\text{Cr} - ^{64}\text{Cr}$ nuclei fill the $2p_{1/2}$ neutron shell. These systems also decay via beta emission through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transitions. The $^{63}\text{Cr} - ^{64}\text{Cr}$ half-lives predicted by the model are in agreement with the experimental half-lives^[36]. The model's decay modes for $^{63}\text{Cr} - ^{64}\text{Cr}$ are consistent with Ref^[36].

^{65}Cr and ^{66}Cr partially fill the $1g_{9/2}$ neutron shell. The decay mode and half-life for these chromium systems are consistent with the data^[36]. These $1g_{9/2}$ systems decay via beta emission through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transitions.

5.2. $78 \geq A \geq 67$ Chromium Isotopes without Experimental Half-Life Data

The $a(A)$ values for $67 \geq A \geq 78$ chromium isotopes were derived from a linear fit based on the half-lives of $^{62}\text{Cr} - ^{66}\text{Cr}$. These extrapolated $a(A)$ values are provided in Table 1.

The $^{67}\text{Cr} - ^{74}\text{Cr}$ systems complete filling the $1g_{9/2}$ neutron shell, and have beta decay half-lives between 0.692 – 7.27 ms. These chromium systems decay through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transitions. The calculations summarized in the Japanese Data Compilation^[36] for $^{67}\text{Cr} - ^{74}\text{Cr}$ are about a factor of 4 larger than the model results.

The $^{75}\text{Cr} - ^{78}\text{Cr}$ nuclei partially fill the $2d_{5/2}$ neutron shell and decay through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transitions. The $^{75}\text{Cr} - ^{78}\text{Cr}$ half-lives decrease from 0.545 to 0.287 ms, respectively. None of these $2d_{5/2}$ systems are predicted to exist in Japanese Data Compilation calculations^[36].

No chromium systems with $A > 78$ are predicted by either the model or the Japanese Data Compilation calculations^[36]. This model limitation occurs because only 54 neutrons are bound in chromium system.

6. Conclusions

Neutron excess chromium isotopes terminate with ^{78}Cr . The $67 \leq A \leq 78$ chromium systems have predicted beta decay half-lives in the range of 0.287 – 7.27 ms. These neutron excess chromium systems decay through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transitions. Since the model does not include neutron emission decay modes, it likely overestimates

the actual half-life values.

References

1. ^{a, b}Siegel DM, Metzger BD. "Phys. Rev. Lett." 119, 231102 (2017).
2. ^{a, b}National Academy of Sciences Report No. 11796, *Scientific Opportunities with a Rare-Isotope Facility in the United States*, Washington DC, National Research Council (2007).
3. ^aFukuda N, et al. "Identification of New Neutron-Rich Isotopes in the Rare-Earth Region Produced by 345 MeV/nucleon 238U". *J. Phys. Soc. Jpn.* 87, 014202 (2018).
4. ^aShimizu Y, et al. "Observation of New Neutron-rich Isotopes among Fission Fragments from In-flight Fission of 345 MeV/nucleon 238U: Search for New Isotopes Conducted Concurrently with Decay Measurement Campaigns". *J. Phys. Soc. Jpn.* 87, 014203 (2018).
5. ^aKurcewicz J, et al. "Discovery and cross-section measurement of neutron-rich isotopes in the element range from neodymium to platinum with the FRS". *Phys. Lett. B* 717, 371 (2012).
6. ^aBaumann T, et al. "Discovery of 40Mg and 42Al suggests neutron drip-line slant towards heavier isotopes". *Nature* 449, 1022 (2007).
7. ^aTarasov OB, et al. "Production cross sections from 82Se fragmentation as indications of shell effects in neutron-rich isotopes close to the drip-line". *Phys. Rev. C* 87, 054612 (2013).
8. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Calcium Nuclei". *Physics Essays* 31 (4), 462 (2018).
9. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Iron Nuclei". *Physics Essays* 32 (2), 175 (2020).
10. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Fluorine Nuclei". *Qeios* 24XLL9, 1 (2020). doi:10.32388/24XLL9.
11. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Zinc Nuclei". *Qeios* JZ11LG, 1 (2020). doi:10.32388/JZ11LG.
12. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Neon Nuclei". *Qeios* 1WR291, 1 (2021). doi:10.32388/1WR291.
13. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Sodium Nuclei". *Qeios* 1Y819A, 1 (2021). doi:10.32388/1Y819A.
14. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Magnesium Nuclei". *Qeios* KIB58L, 1 (2021). doi:10.32388/KIB58L.
15. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Aluminum Nuclei". *Qeios* LCAO3W, 1 (2022). doi:10.32388/LCAO3W.
16. ^{a, b, c, d, e, f}Bevelacqua JJ. "Decay Characteristics of Neutron Excess Silicon Nuclei". *Qeios* Y6HDZF, 1 (2022). doi:10.32388/Y6HDZF.

17. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Phosphorous Nuclei". *Qeios Z16MGO*, 1 (2023).
[doi:10.32388/Z16MGO](#).
18. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Sulfur Nuclei". *Qeios QO9K3E*, 1 (2023).
[doi:10.32388/QO9K3E](#).
19. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Chlorine Nuclei". *Qeios HXV1XN*, 1 (2023).
[doi:10.32388/HXV1XN](#).
20. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Argon Nuclei". *Qeios JDLHDL*, 1 (2023).
[doi:10.32388/JDLHDL](#).
21. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Potassium Nuclei". *Qeios RBF GK2*, 1 (2024).
[doi:10.32388/RBF GK2](#).
22. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Scandium Nuclei". *Qeios 25NGQR*, 1 (2024).
[doi:10.32388/25NGQR](#).
23. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Titanium Nuclei". *Qeios NFSVCP*, 1 (2024).
[doi:10.32388/NFSVCP](#).
24. [a, b, c, d, e, f](#) Bevelacqua JJ. "Decay Characteristics of Neutron Excess Vanadium Nuclei". *Qeios 9VY02M*, 1 (2024).
[doi:10.32388/9VY02M](#).
25. [^] Terasawa M, Sumiyoshi K, Kajino T, Mathews GJ, Tanihata I. "New Nuclear Reaction Flow during r-Process Nucleosynthesis in Supernovae: Critical Role of Light Neutron-Rich Nuclei" (2001).
<https://cds.cern.ch/record/509832/files/0107368.pdf>.
26. [a, b, c](#) Lukasiak A, Sobiczewski A. "Estimations of half-lives of far-superheavy nuclei with $Z \approx 154 - 164$ ". *Acta Phys. Pol. B6*, 147 (1975).
27. [a, b, c](#) Petrovich F, Philpott RJ, Robson D, Bevelacqua JJ, Golin M, Stanley D. "Comments on Primordial Superheavy Elements". *Phys. Rev. Lett.* 37, 558 (1976).
28. [^] Brown GE, Gunn JH, Gould P. "Effective mass in nuclei." *Nucl. Phys.* 46, 598 (1963).
29. [^] Fox L, Godwin ET. "Some new methods for the numerical integration of ordinary differential equations". *Proc. Cambridge Philos. Soc.* 45, 373 (1949).
30. [a, b](#) Rost E. "Proton Shell-Model Potentials for Lead and the Stability of Superheavy Nuclei". *Phys. Lett.* 26B, 184 (1968).
31. [^] Blomqvist J, Wahlborn S. Shell model calculations in the lead region with a diffuse nuclear potential. *Ark Fys.* 16: 545 (1959).
32. [a, b](#) Ray L, Hodgson PE. Neutron densities and the single particle structure of several even-even nuclei from Ca40 to Pb208. *Phys Rev C.* 20: 2403 (1979).
33. [a, b](#) N. Schwierz, I. Wiedenhöver, and A. Volya, Parameterization of the Woods-Saxon Potential for Shell-Model Calculations, *arXiv:0709.3525v1 [nucl-th]* 21 Sep 2007.
34. [a, b, c, d, e, f](#) Baum EM, Ernesti MC, Knox HD, Miller TR, Watson AM. *Nuclides and Isotopes – Chart of the Nuclides*,

17th ed. Knolls Atomic Power Laboratory (2010).

35. [a](#), [b](#), [c](#), [d](#), [e](#), [f](#) National Nuclear Data Center, Brookhaven National Laboratory. NuDat3 (Nuclear Structure and Decay Data) (2024). <http://www.nndc.bnl.gov/nudat3/>.
36. [a](#), [b](#), [c](#), [d](#), [e](#), [f](#), [g](#), [h](#), [i](#), [j](#), [k](#), [l](#), [m](#), [n](#), [o](#), [p](#), [q](#), [r](#), [s](#), [t](#), [u](#), [v](#) Koura H, et al. Chart of the Nuclides 2018. Japanese Nuclear Data Committee and Nuclear Data Center, Japanese Atomic Energy Agency (2018).
37. [^]Wong CY. Additional evidence of stability of the superheavy element 310126 according to the shell model. *Phys Lett.* 21: 688 (1966).