Research Article

# Undefinable True Target Learning

**Yongquan Yang**[1]

1. Independent researcher

**The situation where the true target (TT) for a TT learning task cannot be precisely defined is quite common in various artificial intelligence (AI) application scenarios. In this article, we refer to this situation as undefinable TT learning (UTTL). We explicitly proposed that the fundamental assumption about the TT for UTTL is that the TT does not exist in the real world. We did a series of works to scrupulously answer the intrinsic question of why we need to present UTTL, which eventually shows that it is indeed necessary and important to present UTTL based on the explicitly proposed assumption that the TT does not exist in the real world. From the perspectives of problem definition, alternative solution, specific method, and particular application, we formally present a theoretical foundation for UTTL to appropriately handle the situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios. While providing the theoretical foundation for UTTL based on the explicitly proposed fundamental assumption that the TT does not exist in the real world, this article also naturally shows the benefits of noisy labels for realizing UTTL from a theoretical point of view.**

**Corresponding author:** Yongquan Yang, remy_yang@foxmail.com

## 1. Introduction

There is a quite common situation in various artificial intelligence (AI) application scenarios, which is that the true target (TT) for a TT learning task cannot be precisely defined. A TT learning task here is a task to implement a predictive model based on machine learning (ML)-based AI technologies for automatically predicting the TT for future useful application. For example, in the scenario of applying ML-based AI technologies to implement a tool for automatically segmenting tumour/lesion areas in whole slide histopathology images, the TT of tumour/lesion areas for learning a predictive model to implement the tool are even impossible for pathological experts to precisely label [1][2][3]. In this article, we refer to this situation in AI application scenarios as a problem of undefinable TT learning (UTTL), which belongs to the realm of ML [4][5][6]. As the TT cannot be precisely defined in UTTL, only inaccurately labelled data can be provided to UTTL. This leads us to explicitly propose in this article that the fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

Regarding the current literature of ML, UTTL is similar to learning with noisy labels (LWNLs) [7][8], which is a typical type of weakly supervised learning [9]. LWNLs consider the situation where the labels of the provided data contain certain noises which lead to the inaccuracy of the labels in annotating the TT [7][8]. For the situation of LWNLs, inaccurately labelled data are provided mostly due to the purpose of alleviating the labour-intensive labelling the TT [10]. As the data prepared for the situations of UTTL and LWNLs can be identically inaccurate, UTTL shares certain similarity with LWNLs. This seems to indicate that existing approaches for addressing LWNLs can be alternatively selected to address UTTL. A brief review of LWNLs is provided in Section 2.

However, for a TT learning task in the current literature of LWNLs or even in the current literature of the entire ML realm, the acquiescent assumption about the TT is that the TT exists in the real world. This means, though it is inappropriate, the assumption that the TT exists in the real world is still being used for the situation where the TT for a TT learning task cannot be precisely defined. As a result, the assumption that the TT exists in the real world for the situation of LWNLs intrinsically indicates that existing approaches for addressing LWNLs are not suitable for handling UTTL, as the explicitly proposed fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

The existence of this issue can be proved with an underlying logic in ML, which is the assumption about the TT is the foundation to establish the evaluation strategy, and the evaluation strategy established based on the assumption about the TT will eventually have the effect on the formation of the learning concept. In short, this underlying logic in ML is that the fundamental assumption about the TT will eventually determine the formation of the learning concept. In this article, we did a series of works to comprehensively illustrate how this underlying logic in ML is concluded and how the existence of the issue that existing approaches for addressing LWNLs are not suitable for handling UTTL is proved with this underlying logic in ML. These serial works were conducted in providing a scrupulous answer to an intrinsic question of why we need to present UTTL. Firstly, we discussed the definitions of label and target in ML. Secondly, we analysed the evaluation and learning procedures in ML. Thirdly, we summarized existing assumptions for the TT in the evaluation procedure. Fourthly, we organized the effects of different assumptions for TT on the evaluation procedure. Finally, we summarized an underlying logic in ML from the previous four serial works, which is the assumption that the TT will eventually determine the formation of the learning concept in ML, to prove the existence of the issue that existing approaches for addressing LWNLs are not suitable for handling UTTL. These serial works eventually led us to realize that it is indeed necessary and important to present UTTL based on the explicitly proposed assumption that the TT does not exist in the real world. More information is provided in Section 3.

Because of the necessity and importance of presenting UTTL, in this article, we aim to formally present a theoretical foundation for UTTL based on the explicitly proposed assumption that the TT does not exist in the real world. To achieve this, we systematically analysed UTTL from the perspectives of problem definition, alternative solution, specific method, and particular application. Specifically, the definition for the UTTL problem is formally presented based on the fundamental assumption that the TT for the UTTL problem does not exist in the real world. On the basis of the presented definition, the UTTL problem is transformed into mainly a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. Referring to the presented alternative solution, specific methods like one-step abductive multi-target learning (OSAMTL) and its extensions, which have been proposed in recent works [1][2][3][11], are summarized for addressing the UTTL problem in different scenarios. Referring to the summarized specific methods OSAMTL and its extensions, implementation rules and techniques of these methods are discussed regarding particular applications in real-world scenarios. With these works, we formally established a theoretical foundation for UTTL to handle the situation where the TT for a TT learning task cannot be precisely defined. More information is provided in Section 4, Section 5, Section 6, and Section 7.

As far as we know, this article is the first that explicitly proposed the fundamental assumption that the TT does not exist in the real world to formally present a theoretical foundation for UTTL to appropriately handle the situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios. In addition, as only inaccurately labelled data can be provided to UTTL, this article also naturally shows the benefits of noisy labels for realizing UTTL from a theoretical point of view while providing the theoretical foundation for UTTL based on the explicitly proposed fundamental assumption that the TT does not exist in the real world. The rest of the contents of this article are structured as follows: In Section 2, we briefly introduced LWNLs and the similarity and difference between UTTL and LWNLs; In Section 3, we did a series of works to scrupulously answer the intrinsic question of why we need to present UTTL. In Sections 4, 5, 6 and 7, we respectively presented the definition, alternative solution, specific method and particular application for the UTTL problem; Finally, in Section 8, we presented discussion, conclusion and future work for this article.

## 2. Related work

As the labels for the data prepared for the situations of UTTL and LWNLs can be identically inaccurate, UTTL shares certain similarity with LWNLs. In this section, we briefly review approaches for the situation of LWNLs.

In the literature on LWNLs, numerous approaches have been proposed to address this problem, including robust architectures, robust regularization, sample selection, and robust loss design [12]. Particularly, the objective of robust architectures [13][14][15][16][17][18][19][20] is to apply a noise adjustment layer over a deep neural network (DNN) to grasp how labels change or to construct a unique architectural design that accommodates a wider variety of label noise categories, which strive to hinder a DNN's tendency to overly adapt to incorrectly labelled examples through the implementation of training constraints. A key advantage of robust regularization [21][22][23][24][25][26] lies in its capacity to readily acclimate to novel scenarios with minimal adjustments. Sample selection strategies [27][28][29][30][31][32][33][34][35] endeavour to pinpoint and prioritize the samples deemed most plausible to be clean for the purpose of enhancing the optimization process. Robust loss [36][37][38][39][40][41][42][43][44][45][46][47][48][49][50][51] design seeks to calibrate the loss value in accordance with the certainty of a particular loss (or label) through various tactics, or devise a novel loss function tailored to cope with imprecise guidance. Typically, resilient loss functions incorporate a provision that imposes a penalty on predictions made with low confidence, which are more prone to result from noisy data points. For more information about the LWNLs problem and its alternative solutions, readers can refer to [7][8].

For a TT learning task in the current literature of LWNLs or even in the current literature of the entire ML realm, the acquiescent assumption about the TT is that the TT exists in the real world. Differently, in this article, the fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

## 3. Why do we need to present UTTL?

In this section, we systematically illustrate the necessity and importance of presenting UTTL based on the explicitly proposed fundamental assumption that the TT does not exist in the real world. This section

 doi.org/10.32388/KBK3P8.2 

is structured as follows: In Section 3.1, the definitions of label and target in ML are discussed; In Section 3.2, the evaluation and learning procedures in ML are analysed; In Section 3.3, existing assumptions for the TT in evaluation procedure are summarized; In Section 3.4, the effects of different assumptions for TT on evaluation procedure are organized; Finally, in Section 3.5, an underlying logic in ML was summarized from the previous four serial works, which eventually shows that it is indeed necessary and important to present UTTL based on the explicitly proposed assumption that the TT does not exist in the real world.

### *3.1. Label and target in ML*

In ML, a label or a target is usually associated with an instance. The instance and its corresponding label or target form a data point that can be collected to create a dataset for evaluation and learning of ML-based predictive models. The difference between a label and a target is that a label represents the mapping objective associated with an instance, while a target represents a transformation from the mapping objective for an instance that can be easily used for computation in specific procedures in ML.

### *3.2. Evaluation and learning procedures in ML*

In ML, two procedures play the decisive roles in evolving predictive models for specific applications: evaluation and learning procedures. The evaluation procedure aims to assess the performance of an ML-based predictive model, and the learning procedure aims to develop a predictive model based on specific ML algorithms. In ML, the evaluation procedure commonly has a close relation to the learning procedure. Their relation is that successful evaluation strategies for the evaluation procedure are critical to the learning procedure for developing ML-based predictive models, as successful evaluation strategies for the evaluation procedure in ML are generally used to build up the learning procedure in ML (i.e., if an evaluation strategy can be successfully used to evaluate an ML-based predictive model in the evaluation procedure, then it can also be generally used to learn an ML-based predictive model in the learning procedure).

### *3.3. Existing assumptions for TT in evaluation procedure*

In order to reveal the existing assumptions for TT in the evaluation procedure in ML, we should firstly summarize various strategies proposed for the evaluation procedure for assessing the performances of an ML-based predictive model. For the evaluation procedure in ML, there are two usual types of evaluation strategies: usual evaluation with accurate ground-truth labels (AGTLs) and usual evaluation with inaccurate ground-truth labels (IAGTLs). Each of these two usual types also has specific subtypes regarding different preparations for evaluation. Usual evaluation with AGTLs can be classified into two subtypes: extracting TT from massive AGTLs [52][53][54] and extracting TT from limited AGTLs [55][56]. Usual evaluation with IAGTLs can be classified into two subtypes: selecting probable TT from IAGTLs [57][58][59], providing / estimating TT error rate in IAGTLs [60]. More recently, a new evaluation strategy named logical assessment formula (LAF) [61] was also proposed for evaluation with IAGTLs. LAF only requires extracting multiple TTs from IAGTLs for evaluation [62]. These classifications can be summarized as Table 1.

| **Evaluation strategy** | **Preparation for evaluation** |
|---|---|
| Usual evaluation with AGTLs | Generating the TT from massive AGTLs |
| | Generating the TT from limited AGTLs |
| Usual evaluation with IAGTLs | Selecting the probable TT from IAGTLs |
| | Providing/estimating the TT error rate in IAGTLs |
| LAF for evaluation with IAGTLs | Extracting multiple inaccurate TTs from IAGTLs |

**Table 1.** Summarization of various strategies proposed for the evaluation procedure for assessing the performances of a ML-based predictive model

Based on the summarization of Table 1, we now can analyse to reveal the underlying assumption for the TT in the three types of evaluation strategies for the evaluation procedure in ML. For the usual strategy of evaluation with AGTLs, the acquiescent assumption for the TT is obviously that the TT exists in the provided labels, as the TT can be extracted from the provided massive or limited AGTLs. For the usual strategy of evaluation with IAGTLs, the acquiescent assumption for the TT is also that the TT exists in the provided labels, as the probable TT can be selected from the provided IAGTLs, or the TT error rate in the provided IAGTLs can be provided/estimated (i.e., if the TT does not exist in the provided labels, then no probable TT can be selected from the provided IAGTLs and no TT error rate in the provided IAGTLs can be provided/estimated). However, for the strategy of LAF for evaluation with IAGTLs, the


doi.org/10.32388/KBK3P8.2

fundamental assumption for the TT is not exclusive, as it only requires extracting multiple inaccurate TTs from the provided IAGTLs (i.e., the TT can exist or does not exist in the provided IAGTLs). As a result, the fundamental assumption for the TT in the three types of evaluation strategies for the evaluation procedure in ML can be summarized as Table 2.

| **Assumption for the TT** | **Evaluation strategy** |
|---|---|
| The TT exists in the provided labels | Usual evaluation with AGTLs |
| | Usual evaluation with IAGTLs |
| The TT can exist or does not exist in the provided IAGTLs | LAF for evaluation with IAGTLs |

**Table 2.** Summarization of the fundamental assumptions for the TT in different types of evaluation strategies for the evaluation procedure in ML

### *3.4. Effects of assumptions for TT on evaluation procedure*

In fact, the fundamental assumptions for the TT are the causes that have effects on the emergence of various existing strategies for the evaluation procedure in ML. In other words, there are cause-and-effect relations between the fundamental assumptions for the TT and the various existing strategies for the evaluation procedure in ML. Detailed effects of the fundamental assumptions for the TT on the evaluation procedure in ML can be summarized as: 1) The assumption that the TT exists in the provided labels is the foundation to establish the two usual types of evaluation strategies, including usual evaluation with AGTLs and the usual evaluation with IAGTLs, for the evaluation procedure in ML; 2) The assumption that the TT can exist or does not exist in the provided IAGTLs is the foundation to establish the LAF for evaluation with IAGTLS for the evaluation procedure in ML. The effects of different assumptions for the TT on the evaluation procedure in ML can be summarized as Table 3.


doi.org/10.32388/KBK3P8.2

| Cause | Effects | | |
|---|---|---|---|
| Assumption for the TT | Evaluation strategy | Preparation for evaluation | Evaluation procedure |
| The TT exists in the provided labels | Usual evaluation with AGTLs | Generating the TT from massive or limited AGTLs | Evaluating on the generated TT |
| | Usual evaluation with IAGTLs | Selecting some probable TTs from IAGTLs, or providing/estimating rate of TT error in IAGTLs | Evaluating on the probable TT selected from IAGTLs, or evaluating on IAGTLs regarding to the provided / estimated rate of TT error |
| The TT can exist or does not exist in the provided IAGTLs | LAF for evaluation with IAGTLs | Extracting multiple inaccurate targets from IAGTLs | Evaluating on the multiple inaccurate targets extracted from IAGTLs |

**Table 3.** Summarization of different assumptions for the TT on the evaluation procedure in ML

### *3.5. Necessity and importance of presenting UTTL*

Based on the fact illustrated in Section 3.2 that successful evaluation strategies for the evaluation procedure in ML are generally used to build up the learning procedure in ML and the summarizations of Table 1, Table 2, and Table 3 presented in Section 3.3 and Section 3.4, in this subsection, we illustrate the necessity and importance of presenting UTTL.

As successful evaluation strategies for the evaluation procedure in ML are generally used to build up the learning procedure in ML, the assumption for the TT that has an effect on the evaluation procedure in ML will eventually also have an effect on the learning procedure in ML. As a result, the assumption that the TT exists in the provided labels has been affecting the learning procedure in the current literature of LWNLs or even in the literature of the entire ML realm, since the usual evaluation with AGTLs and the usual evaluation with IAGTLs are the two types of evaluation strategies most commonly used in ML. In other words, we can conclude that the acquiescent assumption about the TT for a TT learning task in the

 doi.org/10.32388/KBK3P8.2 

current literature of LWNLs or even in the literature of the entire ML realm is that the TT exists in the real world, even for the situation where the TT cannot be precisely defined.

Recent works [61][62] have shown that the new evaluation strategy of LAF for evaluation with IAGTLs can be successfully established based on the assumption that the TT can exist or does not exist in the provided IAGTLs. With the common logical sense that a successful evaluation strategy for the evaluation procedure in ML can be generally used to build up the learning procedure in ML, it is reasonable that we can explicitly propose the assumption that the TT does not exist in the real world and present UTTL based on this assumption.

In summary, a clear underlying logic in ML can be concluded from the serial works conducted in Sections 3.1 to 3.4, which is the assumption about the TT is the foundation to establish the evaluation strategy, and the evaluation strategy established based on the assumption about the TT will eventually have the effect on the formation of the learning concept. In short, the assumption about the TT will eventually determine the formation of the learning concept in ML. Regarding this underlying logic in, the two assumptions about the TT that the TT does not exist in the real world and the TT exists in the real world will eventually lead to different learning concepts. Specifically, with the assumption about the TT that the TT does not exist in the real world, the new evaluation strategy of LAF for evaluation with IAGTLs is established which can eventually lead to the formation of the new learning concept UTTL presented in this article; and with the assumption about the TT that the TT exists in the real world, the evaluation strategies of the usual evaluations with AGTLs or IAGTLs have been established which have had effects on the concept of TT learning in LWNLs or ML. The comparison of the two fundamental assumptions about the TT for establishing different evaluation strategies that eventually lead to the two learning concepts of UTTL and TT learning in LWNLs or ML is shown as Table 4.

| Assumption about the TT | Evaluation strategy | Learning concept |
|---|---|---|
| The TT does not exist in the real world | LAF for evaluation with IAGTLs | UTTL |
| The TT exists in the real world | Usual evaluation with AGTLs | TT learning in LWNLs or ML |
| | Usual evaluation with IAGTLs | |

**Table 4.** Comparison of the two fundamental assumptions about the TT for establishing different evaluation strategies, which eventually lead to different learning concepts

With the concluded underlying logic in ML that the assumption about the TT will eventually determine the formation of the learning concept, Table 4 reasonably proves the existence of the issue that existing approaches for addressing LWNLs are not suitable for handling UTTL. As a result, it is necessary and important to present UTTL based on the explicitly proposed assumption that the TT does not exist in the real world to appropriately handle the situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios.

## 4. Definition of UTTL

Let us consider the situation where the true target of a learning task cannot be precisely defined. In practice, this situation inevitably leads to a big problem in the label preparation for the learning task, which is that the label prepared for an entity/event contains severe inaccuracy in representing the true target associated with the entity/event. Here, we refer to this situation as a problem of undefinable true target learning (UTTL). Since large inconsistencies usually appear among experts regarding an agreement on the true target for the UTTL problem, in this article, we explicitly propose the fundamental assumption about the true target for the UTTL problem, which is that the true target does not exist in the real world.

Based on this fundamental assumption, the UTTL problem can be described as: based on a collected number of data points, each of which consists of an entity/event and a prepared label that contains severe inaccuracy in representing the undefinable true target associated with the entity/event, to find a function that can map the entities/events into the undefinable true targets. Notably, as the label prepared for the entity/event contains severe inaccuracy due to the fact that the true target is undefinable, the properties of the label prepared for the entity/event inevitably cannot precisely represent the properties of the undefinable true target. Thus, the solution to the UTTL problem (i.e., the found function that can map the entities/events into the corresponding undefinable true targets) should be subject to the condition that the properties of the labels prepared for the entities/events are included in the properties of the undefinable true targets mapped from the entities/events.

Denote the collected number of data points as $H = \{d, l\}$, where $d$ is the entities/events, $l$ is the prepared labels associated with $d$ that cannot precisely represent the undefinable true target, and the elements in $d$ and $l$ have a one-to-one correspondence. Denote the function that can map the entities/events into the corresponding undefinable true targets as $f : d \longmapsto t$, where $t$ is the mapped examples of the


doi.org/10.32388/KBK3P8.2

undefinable true target and the element in $d$ and $t$ as well have a one-to-one correspondence. The mapping function $f$ should be subject to the condition that the properties of $l$ are included in the properties of $t$. Denote the properties of $l$ as $prop(l)$, the properties of $t$ as $prop(t)$, and the relation of being included in as $\subseteq$. Now, the UTTL problem is formally defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(l) \subseteq prop(t) \tag{1}$$

Here, we do not constrain the specific formation for $prop(*)$, as it can be semantic, numerical or both to describe the properties of *.

## 5. Alternative solution to UTTL

We propose an alternative solution to the UTTL problem. Specifically, we firstly transform the UTTL problem into mainly a combination of the machine learning (ML) problem and the logical reasoning (LR) problem, and then we propose an alternative solution to the transformed UTTL problem.

### *5.1. Common ML and LR*

For the common ML problem, the prepared set of labels $l$ is usually assumed to be able to precisely represent the true targets $t$ corresponding to the set of entities/events $d$ in the collected number of data points $H = \{d, l\}$. Thus, in this situation, the properties of $l$ ($prop(l)$) are equal to the properties of $t$ ( $prop(t)$) compared with formula (1). Formally, the common ML problem can be defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(t) = prop(l). \tag{2}$$

Usually, the alternative solution to the common ML problem can be described as an optimized mapping function that can minimize the error between $t = f(d)$ and $l$, which can be formally expressed as

$$\tilde{f} = \arg\min_{f \in \Theta_f} \; o(t = f(d), l). \tag{3}$$

Here, $o$ is a predefined loss function that can estimate the error between $t = f(d)$ and $l$.

For the common LR problem, in addition to the prepared set of entities/events $d$ and the corresponding set of labels $l$, an accumulated knowledge base ($KB$) containing various prior knowledge facts about the true target is both provided. The LR problem can be expressed as: to search a reasoning path ($r$) that can from the collected data points $H = \{d, l\}$ and $KB$ to draw a set of conclusions ($c$) that are consistent with ($\cong$) some knowledge facts in $KB$. Formally, the common LR problem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\, r : \{d, l\}, KB \rightarrow c \qquad s.t. \quad c \cong KB. \tag{4}$$

Usually, the alternative solution to the common LR problem can be described as a validated logical path (a series of valid logical processes) that can maintain the consistency between $c = r < \{d, L\}, KB >$ and $KB$, which can be formally expressed as

$$\tilde{r} = \arg \underset{r \in \Theta_r}{maint}\, cons(c = r < \{d, l\}, KB >, KB). \tag{5}$$

Here, $cons$ is a predefined procedure that can reflect the consistency between $c = r < \{d, l\}, KB >$ and $KB$.

### *5.2. Transformed UTTL*

Comparing the UTTL problem definition (formula (1)) with the common ML problem definition (formula (2)), we can note that the learning true target for the common ML problem can be precisely known while the learning true target for the UTTL problem cannot be precisely known. This fact reflects that if we directly take the alternative solution to the common ML problem (formula (3)) as a solution to the UTTL problem, the finally found mapping function $\tilde{f}$ will suffer from severe inaccuracy in predicting the true target for the UTTL problem.

Referring to the common LR problem definition (formula (4)), we can observe that if we regard the conclusions $c$ drawn from the provided data points $H = \{d, l\}$ and the accumulated knowledge base $KB$ as some statements about the undefinable true target for the UTTL problem, then it is plausible that we can probably search a reasoning path that can draw some statements which are consistent with $KB$ to be able to better describe the undefinable true target than the labels $l$ in $T$ for the UTTL problem. Thus, the alternative solution to the common LR problem (formula (5)) can probably be leveraged to propose a better alternative solution to the UTTL problem than naively employing formula (3).

We propose to transform the UTTL problem into a type of problem which is mainly a combination of the ML problem and the LR problem. Particularly, the transformed problem for UTTL can be divided into the following three sub-problems.

1. Based on a number of provided data points $H = \{d, l\}$ in which $l$ cannot precisely describe the undefinable true target and an extra accumulated knowledge base $KB$ which contains various prior knowledge facts about the undefinable true target, the primary sub-problem is to search a reasoning path $r$ that can draw some statements $c$ about the undefinable true target. The drawn $c$ should be consistent with $KB$ to be able to better describe the undefinable true target for UTTL

than the labels $l$ provided in the $H$. Formally, referring to formulas (1) and (4), this sub-problem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\, r : \{d, l\}, KB \rightarrow c \qquad s.t. \quad prop(l) \subseteq c \cong KB. \tag{6}$$

2. Based on $H = \{d, l\}$ and the $c$ from 6), the subsequent sub-problem is to build a programme ($p$) that can generate a new set of learning targets $t^*$ corresponding to $d$. The properties of the generated $t^*$ should be equal to $c$ in describing the undefinable true target for UTTL. Formally, this sub-problem can be defined as

$$\tilde{p} = \underset{p \in \Theta_p}{building}\, p : \{d, l\}, c \rightharpoonup t^* \qquad s.t. \quad prop\,(t^*\ ) = c. \tag{7}$$

3. Based on $d$ and the $t^*$ from 2), the final sub-problem is to find a mapping function that can map $d$ into the corresponding final predicted true targets $t$ for UTTL. The properties of the final predicted $t$ should be equal to the properties of $t^*$. Formally, referring to formula (2), this sub-problem can be defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(t) = prop\,(t^*)\,. \tag{8}$$

Referring to the formulas (6), (7), and (8), the UTTL problem definition expressed in the formula (1) can be transformed as follows

$$\begin{cases} 1)\ \tilde{r} = \underset{r \in \Theta_r}{searching}\, r : \{d, l\}, KB \rightarrow c \\ \qquad 2)\ \tilde{p} = \underset{p \in \Theta_p}{building}\, p : \{d, l\}, c \rightharpoonup t^* \\ \qquad\qquad 3)\ \tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \end{cases} \qquad s.t. \quad prop(l) \subseteq prop(t) \cong KB. \tag{9}$$

We can note from formula (9) that the subject condition for the transformed UTTL problem definition now is $prop(l) \subseteq prop(t) \cong KB$, which is different from the subject condition $prop(l) \subseteq prop(t)$ in the original UTTL problem definition expressed in the formula (1). More details on how we get the subject condition in formula (9) from the formulas (6), (7), and (8) are provided in Proof 1 of the Appendix.

### *5.3. Analyses of the transformed UTTL*

From the subject condition of the transformed UTTL problem definition expressed in the formula (9) ( $prop(l) \subseteq prop(t) \cong KB$), we can observe that the properties of the labels $L$ in the provided data points $T$ ($prop(l)$) are included in ($\subseteq$) the properties of the final predicted true targets ($prop(t)$), and $prop(t)$ is also consistent with ($\cong$) the extra accumulated knowledge base $KB$ which contains various prior knowledge facts about the undefinable true target. This subject condition reflects not only that the final predicted true targets $t$ are able to better represent the undefinable true target for UTTL than the labels in the provided data points, but also that the properties of the final predicted true targets $t$ are consistent with various prior knowledge facts about the undefinable true target for UTTL. This reflection indicates that the transformed UTTL problem definition is better at finding the appropriate mapping function for predicting the undefinable true target than the original UTTL problem definition

Though the final predicted true targets $t$ possess better properties, which are consistent with $KB$, compared with the labels $l$, we are still not sure about whether $t$ can be precise enough to represent the undefinable true target for UTTL. Regarding the subject condition $prop(l) \subseteq prop(t) \cong KB$ in formula (9), we can deduce that how precise $t$ can be to represent the undefinable true target for UTTL will depend on how precise the prior knowledge facts contained in $KB$ can be to represent the undefinable true target. However, theoretically, with more knowledge facts iteratively accumulated in $KB$ to represent the undefinable true target, the final predicted $t$ can be iteratively more precise to represent the undefinable true target for UTTL. As a result, the transformed UTTL problem definition provides a promising foundation to approach the undefinable true target for UTTL.

### *5.4. Alternative solution to the transformed UTTL*

Referring to the transformed UTTL problem definition expressed in the formulas (6), (7), (8), the alternative solution to the transformed UTTL problem can also be divided into three sub-solutions.

1. The first sub-solution is the solution to formula (6), which can be expressed as formula (5).
2. The second sub-solution is the solution to formula (7), which is to build a programme $(p)$ to generate the learning targets $t^*$ corresponding to $d$ from $H = \{d, L\}$ and the $c$ produced by the first sub-solution. Formally, the second sub-solution can be expressed as
$$\tilde{p} = \arg \underset{p \in \{\Theta_r \cup \Theta_f\}}{build} \; t^* = p\left(\{d, l\}, c\right) \tag{10}$$
Here, $p \in \{\Theta_r \cup \Theta_f\}$ indicates that the built programme $p$ can be in the space of the LR-based methods ($\Theta_r$), in the space of the ML-based methods ($\Theta_f$) or in the space of the combined LR and ML methods ($\Theta_r \cup \Theta_f$).
3. The third sub-solution is the solution to formula (8), which can be expressed as formula (3) with the replacement of $l$ with $t^*$.

In summary, the alternative solution to the transformed UTTL problem can be formally expressed as follows.


doi.org/10.32388/KBK3P8.2

$$\begin{cases} 1)\ \tilde{r} = \arg\underset{r\in\Theta_r}{maint}\ cons(c = r < \{d,l\}, KB >, KB) \\ 2)\ \tilde{p} = \arg\underset{p\in\{\Theta_r\cup\Theta_f\}}{build}\ t^* = p\left(\{d,l\}, c\right) \\ 3)\ \tilde{f} = \arg\min_{f\in\Theta_f}\ o(t = f(d), t^*) \end{cases} . \tag{11}$$

### 5.5. Additional notes

Notably, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in this section, since the alternative solution here is proposed based on the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is possible that a better problem transformation and corresponding solution for the UTTL problem defined in formula (1) can still be proposed based on other original thoughts and perspectives.

## 6. Specific method

Referring to the alternative solution presented for the transformed UTTL problem, which is summarized in the formula (11), one-step abductive multi-target learning (OSAMTL) and its extensions have been proposed in recent works [1][2][3][11] to provide some specific methods for addressing the UTTL problem.

### 6.1. OSAMTL

OSAMTL requires as input materials a number of collected data points containing labels that cannot precisely represent the undefinable true target and an extra accumulated knowledge base that contains various prior knowledge facts about the undefinable true target. In addition to the required input materials, the key components of OSAMTL are respectively corresponding to the three sub-solutions presented in the formula (11), which include the component of one-step abductive logical reasoning corresponding, the component of generation of multiple types of learning targets and the component of multi-target learning.

#### 6.1.1. Input materials

The input materials for the OSAMTL method include a number of collected data points $H = \{d, l\}$ where $d$ is the entities/events, $l$ is the prepared labels associated with $d$ that cannot precisely represent the undefinable true target, and an extra accumulated knowledge base ($KB$) which contains various prior knowledge facts about the undefinable true target.

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More specifically, $H$ can be expressed as

$$H=\{d,l\}=\{\{d_1,l_1\},\ \ldots,\ \{d_n,l_n\}\}. \tag{12}$$

And $KB$ can be more specifically expressed as

$$KB=\{k_1,\ldots,\ k_m\}. \tag{13}$$

In formula (12), $n$ denotes the number of data points collected in $H$, and each element $\{d_n,l_n\}$ represents a collected data point that consists of an entity/event $d_n$ and its corresponding label $l_n$. In formula (13), $m$ denotes the number of the prior knowledge facts, and each element $k_m$ represents an accumulated knowledge fact about the undefinable true target.

### 6.1.2. One-step abductive logical reasoning

Based on the input materials $H$ and $KB$, the one-step abductive logical reasoning (OSALR) component of OSAMTL draws some statements/conclusions $(c)$ that can more accurately describe the undefinable true target than the labels provided in $H$. Formally, referring to the sub-solution 1) of formula (11), this component can be expressed as

$$c=\tilde{r}(H,KB)=\{c_1,\cdots,c_w\}. \tag{14}$$

More specifically, the OSALR component consists of three sub-steps as follows.

From $H$, the sub-step one extracts a list of groundings that can describe the logical facts contained in the given diverse noisy samples. Formally, this grounding extract $(GE)$ step can be expressed as

$$g=GE(H)=\{g_1,\cdots,g_s\}. \tag{15}$$

Via logical reasoning, the sub-step two estimates the inconsistencies between the extracted groundings $g$ and the prior knowledge facts accumulated in $KB$. Formally, this logical reasoning $(R)$ step can be expressed as

$$ic=R(g,KB)=\{ic_1,\cdots,ic_u\}. \tag{16}$$

The sub-step three revises the groundings in $g$ by logical abduction, which aims to reduce the estimated inconsistencies in $ic$. Formally, this logical abduction $(LA)$ step can be expressed as

$$c=LA(ic)=\{c_1,\cdots,c_w\}. \tag{17}$$

With these three specific sub-steps $(GE,\ R,\ LA)$ for implementing $\tilde{r}$ in the formula (14), the finally drawn statements/conclusions are revised groundings that are consistent with $KB$ to be able to better describe the undefinable true target than simply the groundings of the labels $l$ provided in $H$.


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### 6.1.3. Generation of multiple types of learning targets

The generation of multiple types of learning targets (GMTLT) component aims to leverage $H$ and $c$ drawn by the OSALR component to abduce multiple types of learning targets. Formally, referring to the sub-solution 2) of the formula (11), this component can be expressed as

$$t^* = \tilde{p}(H, c) = \{t_1^*, \cdots, t_v^*\}. \tag{18}$$

The formula (18) indicates that the built program $\tilde{p}$ can generate multiple types of learning targets ( $\{t_1^*, \cdots, t_v^*\}$) from $H$ and $c$, that are associated with each data point of $d$ in $H$. Usually, the program $\tilde{p}$ can be specifically implemented by logical reasoning and machine learning methods.

As the multiple types of learning targets ($\{t_1^*, \cdots, t_v^*\}$) can be generated from $H$ with the help of the revised groundings ($c$) that are consistent with $KB$ to be able to better describe the undefinable true target, the generated multiple types of learning targets in the formula (18) can also possess certain consistencies with our prior knowledge to better represent the undefinable true target.

### 6.1.4. Multi-target learning

The multi-target learning (MTL) component of OSAMTL is carried out on the basis of a specifically constructed machine learning [4][5][6] architecture ($f$) that can map entities/events ($d$) into corresponding predicted targets ($t$), which can be expressed as $t = f(d)$. Here, the MTL component of OSAMTL aims to optimize the parameters of $f$, regarding minimizing the error between the targets ($t$) predicted by $f$ and the multiple types of targets ($t^*$) generated by the GMTLT component.

In order to estimate the error between $t$ and $t^*$, a loss function ($o$) is commonly required. As $t^*$ contains multiple types of targets, the error between $t$ and the multiple types of targets in $t^*$can be estimated by the weighted sum of the errors between $t$ and respective $t_v^*$ in $t^*$, which can be expressed as

$$o(t, t^*) = \sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right) \quad s.t. \quad \sum_{i=1}^{v} \alpha_i = 1. \tag{19}$$

Commonly, $o$ in the formula (19) can be implemented by cross-entropy for classification and least squares for regression. Further to produce the optimized machine learning model $\tilde{f}$, $o(t, t^*)$ should be minimized. Particularly, if $f$ is constructed by state-of-the-art deep learning methods [63] based on neural networks, the minimization of $o(t, t^*)$ can be implemented by stochastic gradient descent variants.

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As the multiple types of learning targets ($t^*$) generated by the GMTLT component possess certain consistencies with our prior knowledge to better represent the undefinable true target, the produced machine learning model $\tilde{f}$ can have reasonable predictions ($t$) about the undefinable true target by minimizing the error between $t$ and $t^*$.

### *6.2. Extensions of OSAMTL*

In Section 6.1, we presented the formulas (12)-(19) to denote the original OSAMTL method. However, the original OSAMTL method will inevitably have limitations in handling some situations in real-world scenarios for UTTL, as the presented formulas only denote the basic components to concisely present the OSAMTL method. In this subsection, based on the original OSAMTL method presented in Section 6.1, we discuss some extensions of OSAMTL to expand the usage range of OSAMTL in real-world scenarios for UTTL.

One extension of OSAMTL is that the data points provided for UTTL can be extended to diverse types instead of only a single type of data points. In contrast with the original OSAMTL, we denote this kind of extension as OSAMTL with diverse types of data points (DiTDP) (OSAMTL-DiTDP). Another extension of OSAMTL is that the label $l_n$ corresponding to the entity/event $d_n$ in the formula (12) can be extended to diverse types instead of only a single type of label. In contract with the original OSAMTL, we denote this kind of extension as OSAMTL with diverse types of labels (DiTL) (OSAMTL-DiTL).

#### *6.2.1. OSAMTL-DiTDP*

For the situation of OSAMTL-DiTDP, referring to the formula (12), the provided DiTDP can be expressed as

$$\begin{aligned} H &= \{H_1, \ldots, H_k\} = \{\{d_1, l_1\}, \ldots, \{d_k, l_k\}\} \\ &= \left\{\left\{\{d_{1,1}, l_{1,1}\},\ \ldots,\ \{d_{1,n_1}, l_{1,\,n_1}\}\right\}, \ldots, \left\{\{d_{k,1}, l_{k,1}\},\ \ldots,\ \{d_{k,n_k}, l_{k,\,n_k}\}\right\}\right\}. \end{aligned} \tag{20}$$

Here, $k$ denotes the number of DiTDP and $n_k$ denotes the number of data points for each type.

In fact, DiTDP can increase the diversity of the provided data points, which eventually leads to the labels in the provided data points representing diverse aspects of the undefinable true target. Comparing formula (20) with formula (12), we can deduce that if the sum of the numbers for the multiple types of data points in formula (20) is equal to the number of data points in formula (12) (i.e. $\sum_{i=1}^{k} n_i = n$), the complexity of preparing DiTDP can maintain averagely unchanged as preparing a single type of data points. As a result, this extension of preparing DiTDP has the potential to significantly increase the


doi.org/10.32388/KBK3P8.2

diversity of the labels of the prepared data to represent the undefinable true target while maintaining the complexity averagely unchanged as preparing a single type of data points for OSAMTL.

In the meantime, this extension of OSAMTL is more complex to implement than the original OSAMTL, as the extension of preparing DiTDP increases the complexity in implementing the OSALR and GLTMT components of OSAMTL-DiTDP for particular applications. Specifically, for the OSALR component, the formulas (15), (16), and (17) need to be carried out multiple times regarding the prepared DiTDP to produce the final revised grounds to better describe the undefinable true target. For the GLTMT component, the formula (18) needs to be carried out by considering the possible associations among the prepared DiTDP and their corresponding revised groundings, which can make the implementation of the GLTMT component more complicated.

### *6.2.2. OSAMTL-DiTL*

For the situation of OSAMTL-DiTL, DiTL can be expressed as $l_n = \{l_{n,1}, \ldots,\ l_{n,j}\}$, where $j$ denotes the number of the multiple types of labels included in $l_n$. Referring to the formula (12), the provided data points with DiTDP can be expressed as

$$\begin{aligned} H &= \{d, l\} = \{\{d_1, l_1\},\ \ldots,\ \{d_n, l_n\}\} \\ &= \{\{d_1, \{l_{1,1}, \ldots,\ l_{1,j}\}\},\ \ldots,\ \{d_n, \{l_{n,1}, \ldots,\ l_{n,j}\}\}\} \end{aligned} \tag{21}$$

In fact, DiTL can significantly reduce the complexity of the original OSAMTL method, as multiple types of targets can be reasonably extracted from DiTL provided in the data points to represent the undefinable true target. As a result, this extension of OSAMTL can be less complex to implement than the original OSAMTL. In the meantime, although OSAMTL-DiTL requires diverse labels for the data points, it is practical in real-world scenarios. This is because the required diverse labels can be inaccurate which can make the label preparation procedure much easier.


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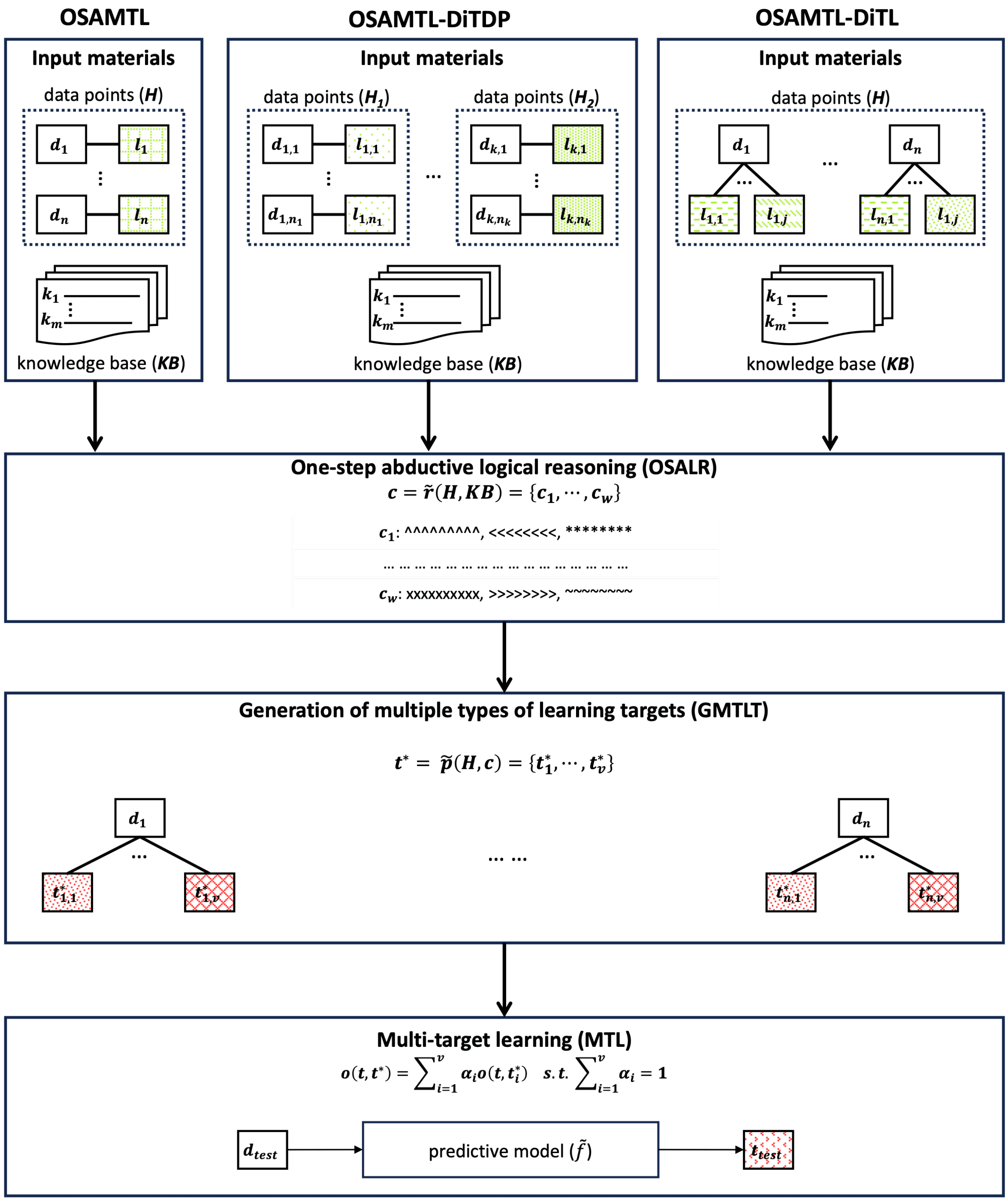


**Figure 1.** Summarization of OSAMTL and its two extensions OSAMTL-DiTDP and OSAMTL-DiTL.

### *6.3. Summarization of OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL*

The summarization of OSAMTL and its two extensions, OSAMTL-DiTDP and OSAMTL-DiTL, can be shown as Fig. 1. The three methods of OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL primarily differ in


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the preparations for the data points in the respective input materials. Because of the differences in the data points for the three methods, the complexities of implementing these three methods for UTTL tasks in real-world scenarios will also vary. Among the three methods, OSAMTL-DiTL theoretically is the easiest one to implement for real application, as the prepared data points already have similar structures to the results of the component GMTLT.

### *6.4. Essence of OSAMTL*

The fundamental assumption for the proposal of OSAMTL is that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target. Based on this fundamental assumption, the three key components of OSAMTL respectively make their contributions to realize this assumption.

Primarily, from the input materials of data points $H$ and the knowledge base $KB$, the OSALR component of OSAMTL draws some revised groundings ($c$) that are consistent with $KB$ to be able to better describe the undefinable true target than simply the groundings of the labels $l$ in $H$. Subsequently, leveraging the provided data points $H$ and the revised groundings $c$ drawn by the OSALR component, the GMTLT component of OSAMTL abduces multiple types of learning targets containing information consistent with our prior knowledge $KB$ about the undefinable true target. Finally, based on a specifically constructed machine learning architecture ($f$), the MTL component of OSAMTL produces the optimized machine learning model $\tilde{f}$ that can have reasonable predictions about the undefinable true target, via minimizing the error between the targets ($t$) predicted by $f$ and the multiple types of targets ($t^*$) generated by the GMTLT component.

With these three key components of OSAMTL to realize the assumption that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target, the essence of OSAMTL is that it forces the machine learning architecture to learn from the weighted summarization of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable true target. More specifically, this essence of OSAMTL reflects a result as follows.

**Theorem 1**. *For a classification or a regression task, the loss constructed by* $o(t, t^*) = \sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right)$*, can be theoretically expressed as* $o(t, t^*) = o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right) + c$*, where* $c$ *is a constant term.*

Detailed proofs for Theorem 1 are provided in Proof 2 and 3 of the Appendix. Through Theorem 1, we can declare that OSAMTL is able to reasonably force the learning model to achieve logically rational


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predictions about the undefinable target via learning from the weighted summarization of multiple types of targets. In fact, learning from the weighted summarization of multiple types of targets, which possess certain consistencies with our prior knowledge about the undefinable true target, can lead to a trade-off among the multiple types of targets and thus to a reasonable approximation of the undefinable true target.

# 7. Particular application

The proposed specific method OSAMTL and its extensions for UTTL have been successfully applied to address some tasks in medical histopathology whole slide image analysis (MHWSIA). In this section, we discuss the implementation rules and techniques of these specific methods in some tasks in MHWSIA.

## *7.1. Application of OSAMTL*

OSAMTL has been applied to the helicobacter pylori segmentation task. Precisely segmenting the helicobacter pylori areas in whole slide images digitalized from IHC slides is an unsolved task, as presenting high-quality labels to precisely annotate the helicobacter pylori areas in the whole slide images is very difficult even for pathology experts [1][3]. Taking the underlying true target of helicobacter pylori as the undefinable true target, the helicobacter pylori segmentation task can be transformed into a UTTL problem, and the OSAMTL method can just be applied to provide an alternative solution. In the following contents of this subsection, we briefly introduce the key information about the input materials required by OSAMTL and the results of the three components of OSAMTL, to illustrate the application of OSAMTL to the helicobacter pylori segmentation task.

### *7.1.1. Input materials*

Referring to the formulas (12) and (13), the input materials for the application of OSAMTL to the helicobacter pylori segmentation task include a number of collected data points that consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task. For the helicobacter pylori segmentation task, the entities of the collected data points are a number of image patches cropped from whole slide images digitalized from IHC slides and the corresponding labels are same-sized frames that contain polygons annotating the helicobacter pylori areas in the image patches. A mimic example of the collected data points and


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correspondingly related contents for illustration is provided in Fig. 2, and the accumulated knowledge base is shown as Table 5.

From Fig. 2 we can note that the label $l$ associated with the image patch $d$ is quite inaccurate to represent the underlying true target $t$, as the image '$l$ shown on $d$' shows that the provided label $l$ for the image patch $d$ includes many background areas as the target though it probably enclosed the entire underlying true target $t$. As the pieces of knowledge listed in Table 5 are provided by related experts for identifying the underlying true target of helicobacter pylori, the provided pieces of knowledge can to some extent describe the key features of the underlying true target, though they are semantic and unquantifiable.

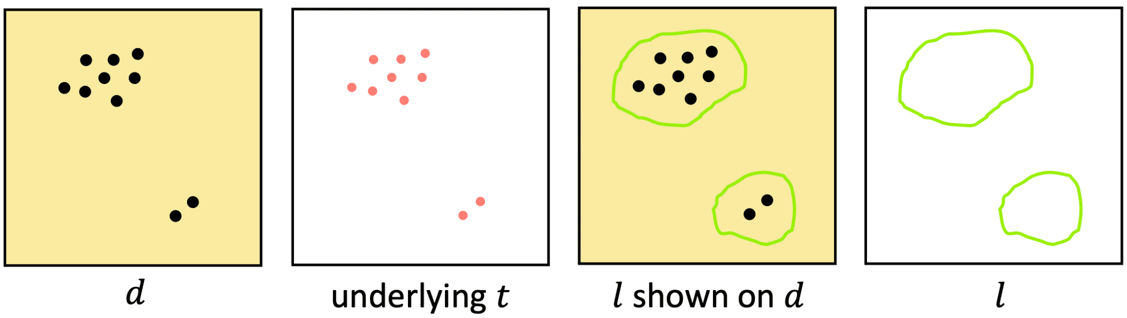


**Figure 2.** A mimic example of the collected data points and correspondingly related contents for illustration. The first and the last images ($d$ and $l$) constitute the example of the collected data points, which are an image patch cropped from a whole slide image digitalized from an IHC slide and its corresponding label that annotates the helicobacter pylori areas in the image patch. The second image (underlying $t$) is assumed to illustrate the underlying true target corresponding to the image patch $d$. The third image ($l$ shown on $d$) illustrates the helicobacter pylori areas annotated in the image patch.

| **Accumulated Knowledge Base** |
|---|
| $k_1$: Helicobacter pylori distributes in luminal areas |
| $k_2$: Helicobacter pylori are black dot-like regions |
| $k_3$: An obvious gradient exists between the location of helicobacter pylori and its neighbourhood |

**Tabel 5.** Details of the accumulated knowledge base [3]


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### 7.1.2. Results of OSALR

Based on the input materials, the OSALR component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via a series of logical reasoning processes [3]. The particularly implemented OSALR component of OSAMTL finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the collected data points of the input materials. Details of the revised groundings are shown in Table 6.

| **Revised Groundings** |
|---|
| $c_1$ : Pixels of images outside the polygons of labels are helicobacter pylori negatives |
| $c_2$ : Pixels of images inside the polygons of labels are helicobacter pylori positives |
| $c_3$ : Black dot-like pixels of images inside the polygons of labels which distribute in luminal areas and have an obvious gradient with their neighbourhood are true helicobacter pylori positives with high probability |

**Table 6.** Details of the revised groundings [3]

### 7.1.3. Results of GMTLT

Based on the revised groundings produced by the OSALR component of OSAMT, the GMTLT component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via a series of image processing algorithms and procedures [3]. The particularly implemented GMTLT component of OSAMTL finally resulted in two types of inaccurate targets to represent the underlying true target associated with the helicobacter pylori segmentation task. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patch are shown as Fig. 3.

From Fig. 3, we can observe that the target type $t_1^*$ can probably enclose the entire underlying true target while including many backgrounds as the target, just exactly like the labels provided in the input materials for the task. In addition, the target type $t_2^*$ can probably be accurate in representing the underlying true target while excluding some parts of the underlying true target as the background. In summary, the two types of targets are both inaccurate but complementary to each other. Thus, the union of the two types of inaccurate targets is reasonable to represent the underlying true target.


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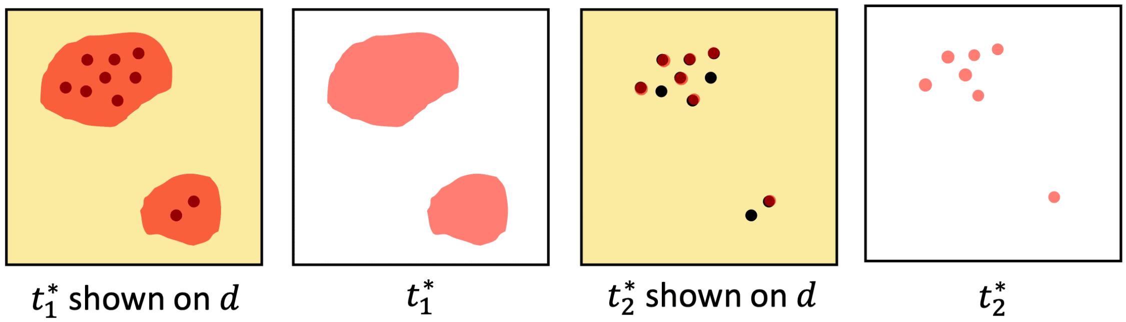


**Figure 3.** Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patch. The second and fourth images are examples of the produced two types of inaccurate targets $t_1^*$ and $t_2^*$. The first and the third images are the masks of $t_1^*$ and $t_2^*$ shown on the corresponding image patch.

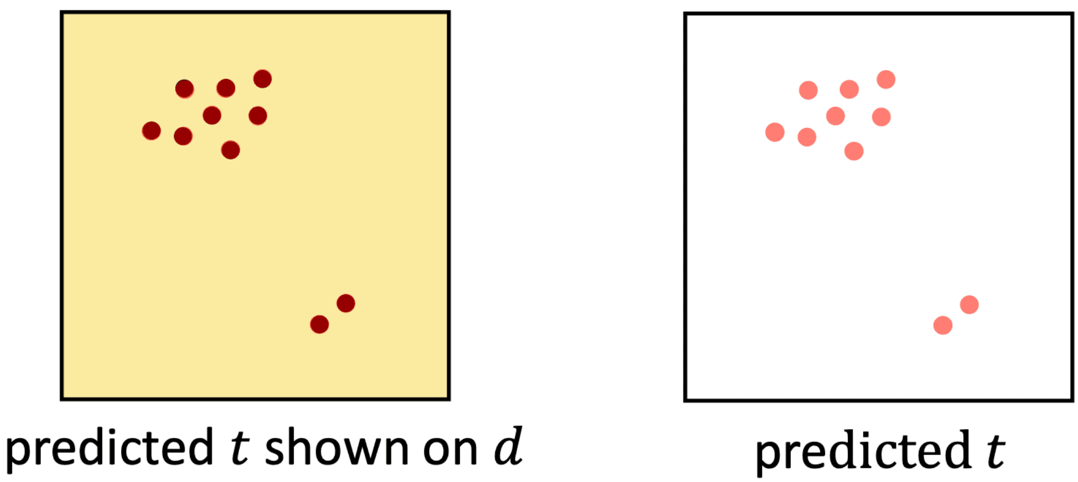


**Figure 4.** Mimic example of the predicted target and its mask shown on the corresponding image patch.

### *7.1.4. Results of MTL*

Based on the two types of inaccurate targets generated by the component GLTMT of OSAMTL and their corresponding image patches, the MTL component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network [3]. The particularly implemented MTL

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component of OSAMTL finally produced a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the two types of inaccurate targets. A mimic example of the predicted target and its mask shown on the corresponding image patch is shown as Fig. 4.

### *7.1.5. Summarization*

Regarding the helicobacter pylori segmentation task as a UTTL problem, the application of OSAMTL to this task can be summarized as follows.

1. One type of data point is prepared, in which one type of labels for annotating the underlying true target of helicobacter pylori areas are associated with corresponding image patches. Pieces of knowledge from related experts for identifying the underlying true target of helicobacter pylori are collected. The one type of labels in the prepared data points is quite inaccurate to represent the true helicobacter pylori areas in the corresponding image patches. The collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target of helicobacter pylori, though they are semantic and unquantifiable. Particularly, the labels in the prepared one type of data points include many background areas as the helicobacter pylori areas in the corresponding image patches.
2. Based on the input materials, the OSALR component of OSAMTL particularly implemented via a series of logical reasoning processes, finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the collected data points of the input materials.
3. Based on the revised groundings, the GMTLT component of OSAMTL particularly implemented via a series of image processing algorithms and procedures, finally resulted in two types of inaccurate targets to represent the underlying true target associated with the image patches in the collected data points for the helicobacter pylori segmentation task. The two types of targets are both inaccurate but complementary to each other.
4. Based on the two types of inaccurate targets and their corresponding image patches, the MTL component of OSAMTL, particularly implemented via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network, finally produced a predictive model that can map an image patch into the predicted target. The predicted target can


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more reasonably represent the underlying true target than the two types of inaccurate targets for the helicobacter pylori segmentation task.

More details of the application to the helicobacter pylori segmentation task in MHWSIA can be found in [1][3].

### *7.2. Application of OSAMTL-DiTDP*

OSAMTL-DiTDP has been applied to the tumour segmentation task for breast cancer. Precisely segmenting the tumour areas for breast cancer in whole slide images digitalized from IHC slides is also an unsolved task, since presenting high-quality labels to precisely annotate the tumour areas for breast cancer in the whole slide images is very difficult even for pathology experts [2]. Identically, taking the underlying true target of tumour for breast cancer as the undefinable true target, the tumour segmentation task for breast cancer can also be transformed into a UTTL problem, and the OSAMTL-DiTDP method can just be applied to provide an alternative solution. In the following contents of this subsection, we briefly introduce the key information about the input materials required by OSAMTL-DiTDP and the results of the three components of OSAMTL-DiTDP, to illustrate the application of OSAMTL-DiTDP to the tumour segmentation task for breast cancer. Particularly, for simplicity, the illustration is based on the task of tumour segmentation in HE-stained pre-treatment biopsy images [2].

#### *7.2.1. Input materials*

Referring to the formulas (20) and (13), the input materials for the application of OSAMTL-DiTDP to the tumour segmentation task for breast cancer include a number of collected two types of data points that respectively consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task. For the tumour segmentation task for breast cancer, the entities for each type of the collected data points are a number of image patches cropped from whole slide images digitalized from IHC slides and the corresponding labels are same-sized frames that contain polygons annotating the tumour areas for breast cancer in the image patches. Two mimic examples respectively for the collected two types of data points and correspondingly related contents for illustration are provided in Fig. 5, and the accumulated knowledge base is shown as Table 7.

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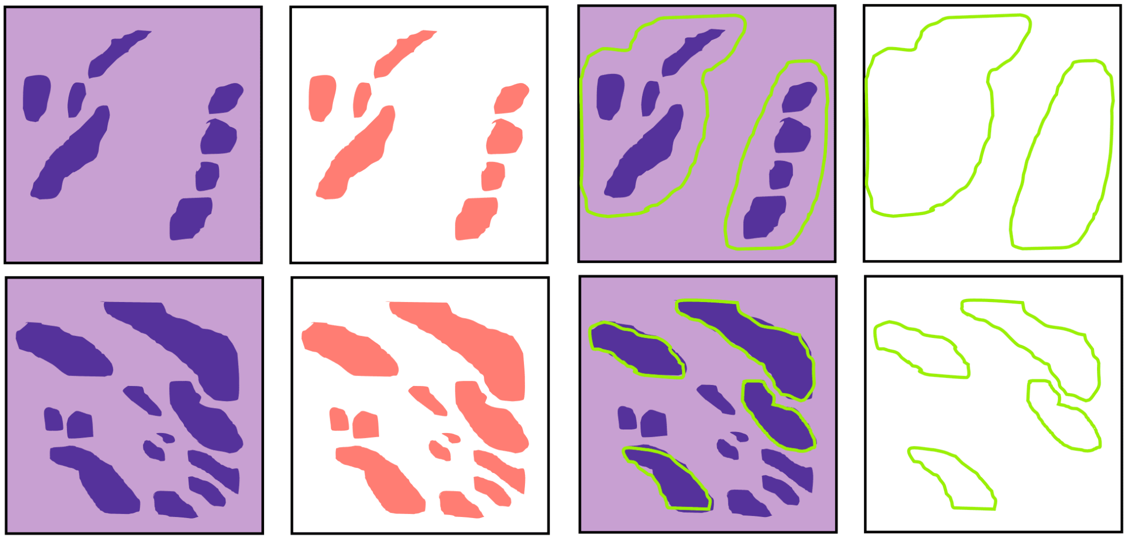


**Figure 5.** Two mimic examples respectively for the two types of collected data points and their correspondingly related contents for illustration. The top row is for type one of the collected data points, and the bottom row is for type two of the collected data points.

From Fig.4 we can note that, for each type of the collected data points, the label $l$ associated with the image patch $d$ is quite inaccurate to represent the underlying true target $t$. The image '$l$ shown on $d$' for type one of the collected data points shows that the provided label $l$ for the image patch $d$ included many background areas as the target though it probably enclosed the entire underlying true target $t$. On the contrary, the image '$l$ shown on $d$' for type two of the collected data points shows that the provided label $l$ for the image patch $d$ excluded some target areas as the background though it probably eliminated the entire background. The labels respectively prepared for the two types of collected data points are complementary to each other in representing the underlying true target. Identically, as the pieces of knowledge listed in Table 7 are also provided by related experts for identifying the underlying true target of tumour for breast cancer, the provided pieces of knowledge can to some extent describe the key features of the underlying true target though they are semantic and unquantifiable.

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| Accumulated Knowledge Base |
| --- |
| $k_1$: Tumour is composed of tumour cells. |
| $k_2$: Tumour cells may be arranged in cords, clusters, and trabeculae. |
| $k_3$: Some tumours are characterized by a predominantly solid or syncytial infiltrative pattern with little associated stroma. |
| $k_4$: The cytoplasm of a tumour cell is eosinophilic and vacuolated. |
| $k_5$: The nuclei of tumour cells are enlarged and chromatin of tumour cells is vacuolated. |
| $k_6$: The nuclei of tumour cells are degenerated. |

**Tabel 7.** Details of the accumulated knowledge base [2]

### *7.2.2. Results of OSALR*

Based on the input materials, the OSALR component of OSAMTL-DiTDP was particularly implemented for the tumour segmentation task for breast cancer via a series of logical reasoning processes [2]. The particularly implemented OSALR component of OSAMTL-DiTDP finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the two types of collected data points of the input materials. Details of the revised groundings are shown in Table 8.


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| Revised Groundings |
| --- |
| $c_1$ : Pixels of type-one images outside the polygons of type-one labels are tumour negatives |
| $c_2$ : Pixels of type-one images inside the polygons of type-one labels are tumour positives |
| $c_3$ : Pixels of type-one images outside the polygons of type-one labels are not exactly true tumour negatives |
| $c_4$ : Pixels of type-one images inside the polygons of type-one labels are not exactly true tumour positives |
| $c_5$ : Pixels of type-two images inside the polygons of type-two labels are tumour positives |
| $c_6$ : Pixels of type-two images outside the polygons of type-two labels are tumour negatives |
| $RG_7$ : Pixels of type-two images inside the polygons of type-two labels are not exactly true tumour positives |
| $RG_8$ : Pixels of type-two images outside the polygons of type-two labels are not exactly true tumour negatives |

**Table 8.** Details of the revised groundings [2]

### *7.2.3. Results of GMTLT*

Based on the revised groundings produced by the OSALR component of OSAMT-DiTDP, the GMTLT component of OSAMTL-DiTDP was particularly implemented for the tumour segmentation task for breast cancer via a series of logical reasoning and machine learning procedures [2]. The particularly implemented GMTLT component of OSAMTL-DiTDP finally resulted in two types of inaccurate targets to represent the underlying true target associated with the tumour segmentation task for breast cancer. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patches are shown in Fig. 6.

From Fig. 6, we can observe that the target type $t_1^*$ can probably enclose the entire underlying true target while including many background pixels as the target, just like the type-one labels provided in the input materials for the task. In addition, the target type $t_2^*$ can probably be accurate to represent the underlying true target while excluding some parts of the underlying true target as the background, just like the type-two labels provided in the input materials for the task. In summary, the two types of targets are both inaccurate but complementary to each other. Thus, the union of the two types of inaccurate targets is reasonable to represent the underlying true target.

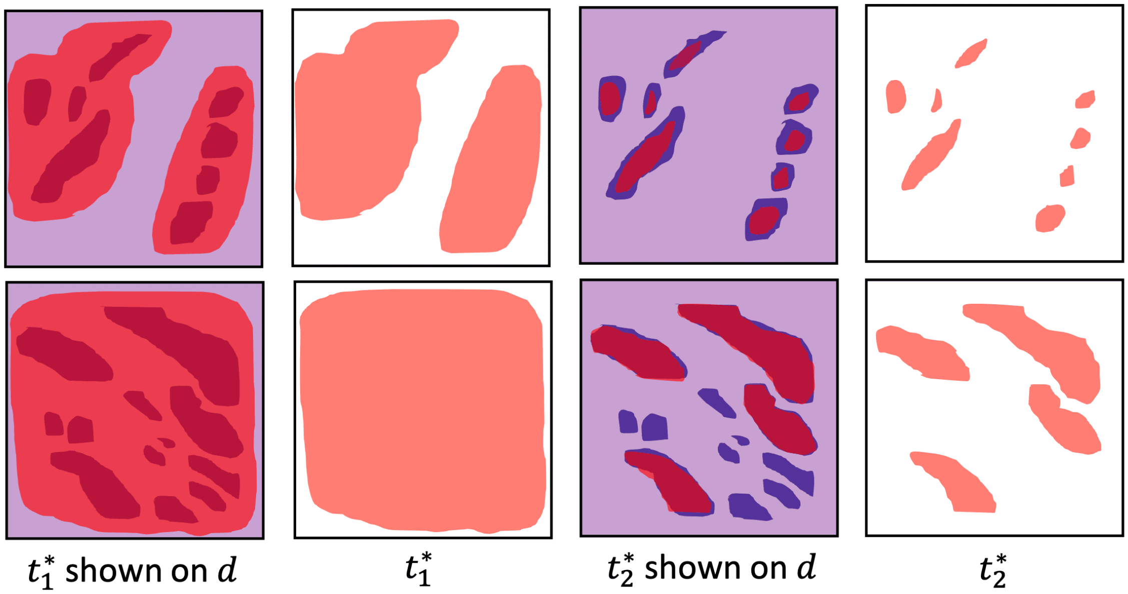


**Figure 6.** Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patches. The second and fourth column images are examples of the produced two types of inaccurate targets $t_1^*$ and $t_2^*$. The first and the third column images are the masks of $t_1^*$ and $t_2^*$ shown on the corresponding image patches.

### *7.2.4. Results of MTL*

Based on the two types of inaccurate targets generated by the component GLTMT of OSAMTL-DiTDP and their corresponding image patches, the MTL component of OSAMTL- DiTDP was particularly implemented for the tumour segmentation task for breast cancer via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network [2]. The particularly implemented MTL component of OSAMTL-DiTDP finally produced a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the two types of inaccurate targets. Mimic examples of the predicted targets and their masks shown on the corresponding image patches are shown as Fig. 7.

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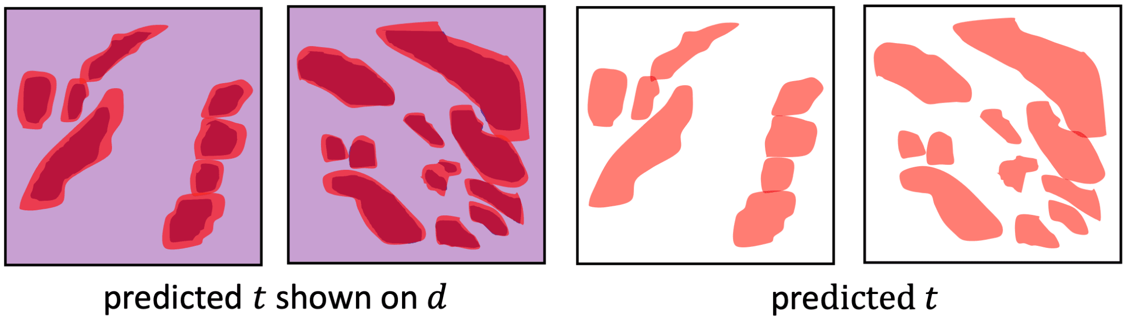


**Figure 7.** Mimic examples of the predicted targets and their masks shown on the corresponding image patches.

### *7.2.5. Summarization*

Regarding the tumour segmentation task for breast cancer as a UTTL problem, the application of OSAMTL-DiTDP to this task can be summarized as follows.

1. Two types of data points are prepared, respectively in which one type of labels for annotating the underlying true target of tumour areas for breast cancer is associated with corresponding image patches. And pieces of knowledge from related experts for identifying the underlying true target of tumours for breast cancer are collected. Each one type of labels in the prepared two types of data points is quite inaccurate to represent the true tumour areas for breast cancer in the image patches. And the collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target of tumours for breast cancer, though they are semantic and unquantifiable. Particularly, the labels in the prepared type-one data points include many background areas as the tumour areas for breast cancer, and the labels in the prepared type-two data points exclude some tumour areas as background areas in corresponding image patches.
2. Based on the input materials, the OSALR component of OSAMTL-DiTDP particularly implemented via a series of logical reasoning processes finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the two types of collected data points of the input materials.
3. Based on the revised groundings, the GMTLT component of OSAMTL-DiTDP particularly implemented via series of logical reasoning and machine learning procedures finally resulted in two types of inaccurate targets to represent the underlying true target associated with the image patches


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in the collected data points for the tumour segmentation task for breast cancer. The two types of targets are both inaccurate but complementary to each other.

4. Based on the two types of inaccurate targets and their corresponding image patches, the MTL component of OSAMTL-DiTDP particularly implemented via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network finally produced a predictive model that can map an image patch into the predicted target. The predicted target can more reasonably represent the underlying true target than the two types of inaccurate targets for the tumour segmentation task for breast cancer.

More details of the application to the tumour segmentation task for breast cancer in MHWSIA can be found in [2].

### 7.3. Application of OSAMTL-DiTL

To the best of our knowledge, there is no specific work that explored OSAMTL-DiTL in a real-world application for the UTLL problem. In this subsection, we focus more on discussing the similarities and differences between the application of OSAMTL-DiTL and the applications of OSAMTL and OSAMTL-DiTDP to reveal the potential of the application of OSAMTL-DiTL [11].

#### 7.3.1. Input materials

Identical to the former two applications of OSAMTL and OSAMTL-DiTDP to the two image segmentation tasks in MHWSIA, the input materials (referring to the formulas (20) and (13)) of the application of OSAMTL-DiTL to a real-world task also include a number of collected data points that respectively consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task.

The accumulated knowledge base is similar to the knowledge bases for the former two applications of OSAMTL and OSAMTL-DiTDP which are shown as Table 5 and Table 7. But, different from the former two applications of OSAMTL and OSAMTL-DiTDP, in which each entity in the collected data points only has one inaccurate label, each entity in the collected data points for OSAMTL-DiTL has multiple (more than one) inaccurate labels that can describe partial properties of the underlying true target. A mimic example of three inaccurate labels assigned to the same entity for the collected data points is shown as Fig. 8.

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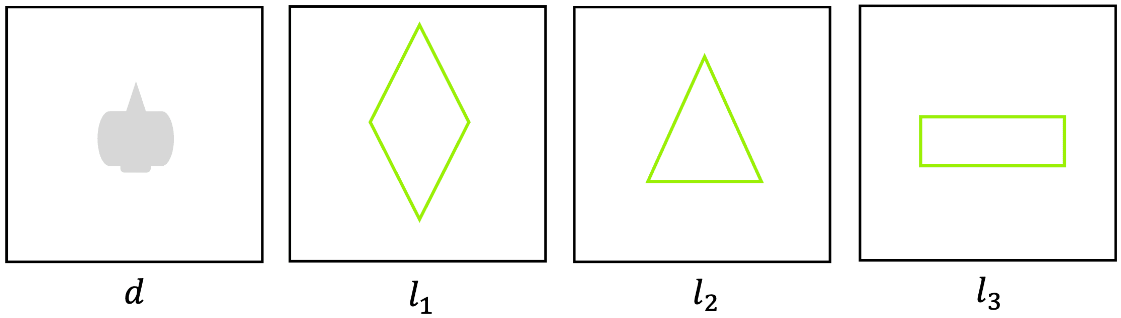

**Figure 8.** A mimic example of three inaccurate labels assigned to the same entity for the collected data points. The first column $d$ is the entity, and the rest three columns $l_1$, $l_2$ and $l_3$ are the inaccurate labels assigned to $d$.

### *7.3.2. Results of OSALR*

Identical to the former two applications of OSAMTL and OSAMTL-DiTDP to the two image segmentation tasks in MHWSIA, the OSALR component of OSAMTL-DiTL can be particularly implemented for a specific task via series of logical reasoning processes and some other possible procedures on the basis of the input materials. The particularly implemented OSALR component of OSAMTL-DiTL finally resulted in a number of revised groundings that can more accurately describe the undefinable true target than the multiple types of labels provided in the collected data points of the input materials.

Via some basic logical reasoning processes based on the current mimic input materials, the revised groundings can possibly contain contents like Table 9, in addition to the groundings contained in the three inaccurate labels.


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| Revised Groundings |
| --- |
| $c_1, \ldots, c_3$ |
| $c_4$: The union of the three inaccurate labels ($l_1$, $l_2$ and $l_3$) can probably contain the entire underlying true target while including some background areas as the target |
| $c_5$: The intersection of the three inaccurate labels ($l_1$, $l_2$ and $l_3$) can probably be accurate to represent the underlying true target while excluding some parts of the target as the background |

**Table 9.** Possible revised groundings

The contents in Table 9 can reflect that the implementation of the OSALR component of OSAMTL-DiTL can be much easier than the implementations of the OSALR components of OSAMTL and OSAMTL-DiTDP.

### *7.3.3. Results of GMTLT*

Based on the revised groundings produced by the OSALR component of OSAMTL-DiTL, the GMTLT component of OSAMTL-DiTL can be particularly implemented for a specific task via some specifically designed procedures. The particularly implemented GMTLT component of OSAMTL-DiTL will finally result in multiple types of inaccurate targets to represent the underlying true target associated with a specific task. Two series of possible mimic examples for the produced multiple inaccurate targets can be shown as Fig. 9.

Three types of inaccurate targets are presented in the top row series, and two types of inaccurate targets are presented in the bottom row series. In summary, the two possible series of inaccurate target types are inaccurate but complementary to each other. Thus, the union of the multiple types of inaccurate targets in the respective series can also be reasonable to represent the underlying true target.

From the top row series of multiple inaccurate targets, we can note that they are just exactly like the inaccurate labels provided in the collected data points for the input materials. And, from the bottom row series of multiple inaccurate targets, we can note that they are some results of logical processes based on the top row series of multiple inaccurate targets. These facts can reflect that the implementation of the


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GMTLT component of OSAMTL-DiTL can be much easier than the implementations of the GMTLT components of OSAMTL and OSAMTL-DiTDP.

### *7.3.4. Results of MTL*

Based on one series of the multiple types of inaccurate targets generated by the component GLTMT of OSAMTL-DiTDP and their corresponding entities, similar to the former two applications, the MTL component of OSAMTL-DiTL can be particularly implemented for a specific task via minimizing the summary error between the multiple types of inaccurate targets and the predictions of the entities corresponding to the multiple types of inaccurate targets from a machine learning model. The particularly implemented MTL component of OSAMTL-DiTL can finally produce a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the multiple types of inaccurate targets. A mimic example of the predicted target can be shown as Fig. 10.

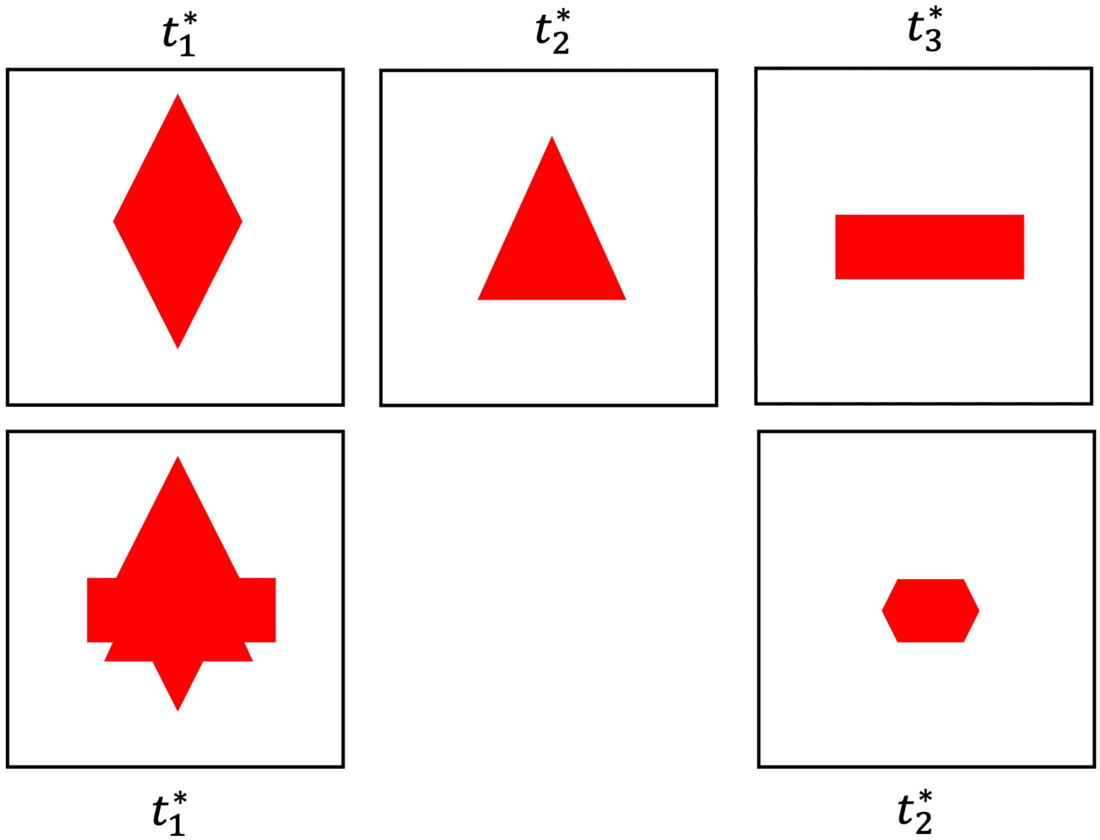


**Figure 9.** Two series of possible mimic examples for the produced multiple types of inaccurate targets. The top row series contain three types of inaccurate targets and the bottom row series contain two types of inaccurate targets.

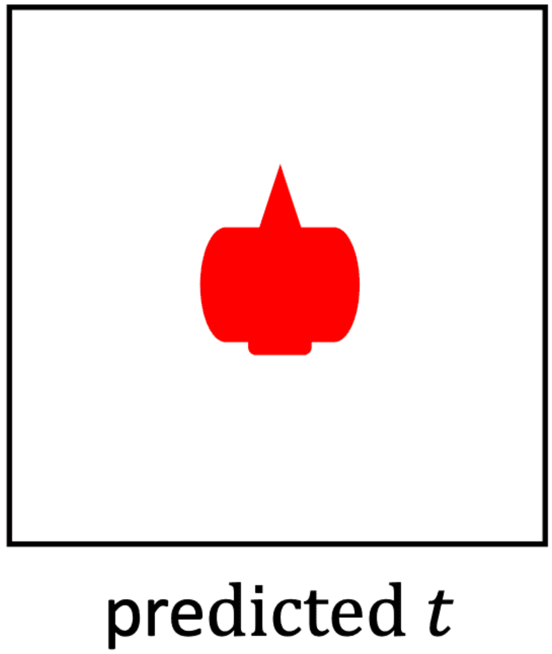


**Figure 10.** A mimic example of the predicted target.

### *7.3.5. Summarization*

The possible application of OSAMTL-DiTL for a UTTL problem can be summarized as follows.

1. One type of data point is prepared, in which multiple types of labels for annotating the underlying true target are associated with the corresponding entities. Pieces of knowledge from related experts for identifying the underlying true target are collected. Each one type of the multiple types of labels in the prepared data points can be inaccurate in representing the true target associated with the corresponding entities. The collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target though they can be semantic and unquantifiable.
2. Based on the input materials, the OSALR component of OSAMTL-DiTL, particularly implemented via a series of logical reasoning processes can finally result in a number of revised groundings that more accurately describe the undefinable true target.
3. Based on the revised groundings, the GMTLT component of OSAMTL-DiTL, particularly implemented via some specifically designed procedures, can finally result in two or more types of inaccurate targets to represent the underlying true target associated with the entities in the


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collected data points for a UTTL problem. The two or more types of targets are inaccurate but can be complementary to each other.

4. Based on the two or more types of inaccurate targets and their corresponding entities, the MTL component of OSAMTL-DiTL can be particularly implemented via minimizing the summary error between the two or more types of inaccurate targets and the predictions of the entities corresponding to the two or more types of inaccurate targets from a learning algorithm, which finally produces a predictive model that can map an entity into the predicted target. The predicted target can more reasonably represent the underlying true target than the two or more types of inaccurate targets for a UTTL problem.

## 8. Discussion, conclusion and future work

In this article, we explicitly proposed the fundamental assumption that the TT does not exist in the real world to formally present the first theoretical foundation for UTTL to appropriately handle the common situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios.

To show the necessity and importance of presenting UTTL based on the explicitly proposed fundamental assumption that the TT does not exist in the real world, we did a series of works for scrupulously answering the intrinsic question of why we need to present UTTL. We discussed the definitions of label and target in ML, analysed the evaluation and learning procedures in ML, summarized existing assumptions for the TT in ML, organized the effects of different assumptions for TT on ML, and finally illustrated the necessity and importance of presenting UTTL.

To formally present a theoretical foundation for UTTL to handle the situation where the TT for a TT learning task cannot be precisely defined, we systematically analysed UTTL from the perspectives of problem definition, alternative solution, specific method, and particular application. Primarily, based on the fundamental assumption that the true target for the UTTL problem does not exist in the real world, the definition for the UTTL problem is formally presented. Subsequently, on the basis of the presented definition, the UTTL problem is transformed into mainly a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. In addition, referring to the presented alternative solution, specific methods like one-step abductive multi-target learning (OSAMTL) and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL) are summarized for addressing the UTTL problem in different scenarios. Finally, referring to the summarized OSAMTL and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL), implementation rules and techniques of these methods are discussed regarding particular real-world application scenarios. The discussions include applying OSMTL to precisely segmenting the helicobacter pylori areas in whole slide images [1][3] and applying OSAMTL-DiTDP to tumour segmentation in HE-stained pre-treatment biopsy images [2], and discussing the similarities and differences between the application of OSAMTL-DiTL and the applications of OSAMTL and OSAMTL-DiTDP to reveal the potentials of the application of OSAMTL-DiTL [11].

In addition, as the TT cannot be precisely defined in UTTL, only inaccurately labelled data can be provided to UTTL. As a result, providing a theoretical foundation for UTTL based on the explicitly proposed fundamental assumption that the TT does not exist in the real world, this article also naturally shows the benefits of noisy labels in realizing UTTL from a theoretical point of view.

As we have analysed in Section 5.5, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in the article, since it is based on the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is probable that better problem transformations and corresponding solutions for the UTTL problem defined in formula (1) can still be proposed, regarding other original thoughts and perspectives. In addition, with the fundamental assumption that the TT does not exist in the real world, the concept of UTTL can also be applied in various other AI application scenarios to establish different perspectives for addressing related tasks.

## Appendix

**Proof 1**. From the formulas (6), (7), and (8), we have the following subject conditions:

$$prop(l) \subseteq c \cong KB, \tag{1}$$

$$prop\left(t^{*}\ \right) = c, \tag{2}$$

$$prop(t) = prop\left(t^{*}\right). \tag{3}$$

Referring to the subject conditions (2) and (3), we have

$$c = prop(t). \tag{4}$$

Substituting the subject condition (4) into (1), we have the final subject condition

$$prop(l) \subseteq prop(t) \cong KB. \tag{5}$$

**Proof 2**. When we use average cross entropy (ACE) to estimate the error between two elements for a two-class classification task, the basic loss function $o(\bullet\ , \bullet)$ can be denoted by


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$$
\begin{aligned}
o\left(t, t_0^*\right) = -\left[t_0^{*,f} \log(t) + \left(1 - t_0^{*,f}\right) \log(1-t)\right] \\
s.t. \;\; t_0^{*,f} \cup \;\left(1 - t_0^{*,f}\right) = t_0^*.
\end{aligned} \tag{1}
$$

Here, $t_0^{*,f}$ is the foreground class of the target $t^*{}_0$, and $1 - t_0^{*,f}$ is the background class of the target $t_0^*$. Referring to formula (1), we rewrite $o\left(t, t^*\right) = \sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right)$ by

$$
\begin{aligned}
o\left(t, t^*\right) &= \sum_{i=1}^{v} \alpha_i \left\{-\left[t_i^{*,f} \log(t) + \left(1 - t_i^{*,f}\right) \log(1-t)\right]\right\} \\
&= -\left[\sum_{i=1}^{v} \alpha_i t_i^{*,f} \log(t) + \sum_{i=1}^{v} \alpha_i \left(1 - t_i^{*,f}\right) \log(1-t)\right].
\end{aligned} \tag{2}
$$

Plugging $t^*{}_0 = \sum_{i=1}^{v} \alpha_i t_i^*$ and substituting into formula (1), we have

$$
o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right) = -\left[\sum_{i=1}^{v} \alpha_i t_i^{*,f} \log(t) + \sum_{i=1}^{v} \alpha_i \left(1 - t_i^{*,f}\right) \log(1-t)\right]. \tag{3}
$$

Comparing formula (3) with formula (2), theoretically we can have

$$
o\left(t, t^*\right) = o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right). \tag{4}
$$

**Proof 3**. When we use the mean squared error (MSE) to estimate the error between two elements for a regression task, the basic loss function $o(\bullet, \bullet)$ can be denoted by

$$
o\left(t, t_0^*\right) = \left(t - t_0^*\right)^2. \tag{1}
$$

Referring to formula (1), we rewrite $o\left(t, t^*\right) = \sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right)$ by

$$
\begin{aligned}
o\left(t, t^*\right) &= \sum_{i=1}^{v} \alpha_i \left(t - t_i^*\right)^2 \\
&= \left(t - \sum_{i=1}^{v} \alpha_i t_i^*\right)^2 + \sum_{i=1}^{v} \alpha_i {t_i^*}^2 - \left(\sum_{i=1}^{v} \alpha_i t_i^*\right)^2 \\
&= \left(t - \sum_{i=1}^{v} \alpha_i t_i^*\right)^2 + D\left(t^*\right).
\end{aligned} \tag{2}
$$

Here, $D\left(t^*\right) = \sum_{i=1}^{v} \alpha_i {t_i^*}^2 - \left(\sum_{i=1}^{v} \alpha_i t_i^*\right)^2$ is the variance for the multiple targets of $t^*$ and is a constant. Plugging $t^*{}_0 = \sum_{i=1}^{v} \alpha_i t_i^*$ and substituting into the formula (1), we have

$$
o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right) = \left(t - \sum_{i=1}^{v} \alpha_i t_i^*\right)^2. \tag{3}
$$

Comparing formula (3) with formula (2), theoretically we can have

$$o(t,t^*) = o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right) + D(t^*). \tag{4}$$

# Statements and Declarations


### *Acknowledgements*

The author, Yongquan Yang, established the conceptualization and did the initial work for this article while he was taking a vacation (from 2023.09 to 2024.02) in Chengdu, Sichuan, China. The rest of the work for the initial version of this article was done in his spare time when he was an invited contract employee (starting from 2024.03 to 2024.09) at Zhongjiu Flash Medical Technology Co., Ltd., Mianyang, Sichuan, China. The author really appreciates the constructive comments received from the anonymous editor of Machine Learning and the reviewers Xingsi Xue and Ricardo Reier of Qeios, which provided the author with good opportunities to improve the quality of this article and more confidence and encouragement to continue to move forward with this research.

### *Funding*

No fund received for this research project.


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## Declarations

**Funding:** No specific funding was received for this work.

**Potential competing interests:** No potential competing interests to declare.