Research Article

Undefinable True Target Learning

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The inability to define the true target (TT) precisely in a TT learning task is a common challenge across various artificial intelligence (AI) application scenarios. In this article, we define this challenge as undefinable TT learning (UTTL). We explicitly propose that the fundamental assumption underlying UTTL is that the TT does not exist in the real world. To justify the necessity of introducing UTTL, we conducted a series of studies aimed at rigorously addressing the intrinsic question: why is UTTL needed? These investigations affirm that, under the assumption that the TT is nonexistent in the real world, UTTL is both necessary and significant. From the perspectives of problem definition, alternative formulations, methodological development, and application scenarios, we present a formal theoretical foundation for UTTL to effectively address learning tasks where the TT cannot be precisely defined. In doing so, this article not only establishes the theoretical basis of UTTL grounded in the explicitly stated assumption but also reveals, from a theoretical standpoint, the potential benefits of noisy labels in enabling UTTL.

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1. Introduction

A common situation in various artificial intelligence (AI) application scenarios is that the true target (TT) for a TT learning task cannot be precisely defined. A TT learning task here involves implementing a predictive model based on machine learning (ML)-based AI technologies for automatically predicting the TT for future useful applications. For example, in the scenario of applying ML-based AI technologies to implement a tool for automatically segmenting tumor/lesion areas in whole slide histopathology images, the TT of the tumor/lesion areas for learning a predictive model to implement the tool is even impossible for pathological experts to label precisely [1][2][3]. In this article, we refer to this situation in AI application scenarios as the problem of undefinable TT learning (UTTL), which belongs to the realm of ML [4][5][6]. As the TT cannot be precisely defined in UTTL, only inaccurately labelled data can be provided to UTTL. This leads us to explicitly propose in this article that the fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

In the current literature on ML, UTTL is similar to learning with noisy labels (LWNLs) [7][8], which is a typical type of weakly supervised learning [9]. LWNLs consider the situation where the labels of the provided data contain certain noise, which leads to the inaccuracy of the labels in annotating the TT [7][8]. For the situation of LWNLs, inaccurately labelled data are provided mostly for the purpose of alleviating the labor-intensive labelling of the TT [10]. As the data prepared for the situations of UTTL and LWNLs can be identically inaccurate, UTTL shares a certain similarity with LWNLs. This seems to indicate that existing approaches for addressing LWNLs can be alternatively selected to address UTTL. A brief review of LWNLs is provided in Section 2.

However, for a TT learning task in the current literature of LWNLs or even in the current literature of the entire ML realm, the acquiescent assumption about the TT is that the TT exists in the real world. This means that although inappropriate, the assumption that the TT exists in the real world is still being used for situations where the TT for a TT learning task cannot be precisely defined. As a result, the assumption that the TT exists in the real world for the situation of LWNLs intrinsically indicates that existing approaches for addressing LWNLs are not suitable for handling UTTL, as the explicitly proposed fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

The existence of this issue can be proven with an underlying logic in ML, which is as follows: the assumption about the TT is the foundation for establishing the evaluation strategy, and the evaluation strategy established on the basis of the assumption about the TT will eventually affect the formation of the learning concept. In short, this underlying logic in ML is that the fundamental assumption about the TT will eventually determine the formation of the learning concept. In this work, we comprehensively illustrate how this underlying logic in ML is concluded and how the existing approaches for addressing LWNLs are not suitable for handling UTTL, which is proven with this underlying logic in ML. These serial works were conducted to provide a scrupulous answer to an intrinsic question of why we need to present UTTL. First, we discuss the definitions of labels and targets in ML. Second, we analysed the evaluation and learning procedures in ML. Third, we summarized existing assumptions for the TT in the evaluation procedure. Fourth, we organized the effects of different assumptions for TT on the evaluation procedure. Finally, we summarize an underlying logic in ML from the previous four serial works, which assumes that the TT will eventually determine the formation of the learning concept in ML, to prove the existence of the issue that existing approaches for addressing LWNLs are not suitable for handling UTTL. These serial works eventually led us to realize that it is indeed necessary and important to present UTTL on the basis of the explicitly proposed assumption that the TT does not exist in the real world. More information is provided in Section 3.

Because of the necessity and importance of presenting UTTL, in this article, we aim to formally present a theoretical foundation for UTTL on the basis of the explicitly proposed assumption that the TT does not exist in the real world. To achieve this goal, we systematically analysed UTTL from the perspectives of problem definition, alternative solutions, specific methods, and particular applications. Specifically, the definition for the UTTL problem is formally presented on the basis of the fundamental assumption that the TT for the UTTL problem does not exist in the real world. On the basis of the presented definition, the UTTL problem is transformed into a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. Referring to the presented alternative solution, specific methods such as one-step abductive multitarget learning (OSAMTL) and its extensions, which have been proposed in recent works [1][2][3][11], are presented for addressing the UTTL problem in different scenarios. Referring to the specific methods OSAMTL and its extensions, the implementation rules and techniques of these methods are summarized with respect to particular applications in real-world scenarios. With these works, we formally established a theoretical foundation for UTTL to handle situations where the TT for a TT learning task cannot be precisely defined. More information is provided in Section 4, Section 5, Section 6, and Section 7.

To the best of our knowledge, this article is the first to explicitly propose the fundamental assumption that the TT does not exist in the real world to formally present a theoretical foundation for UTTL to appropriately handle the situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios. In addition, as only inaccurately labelled data can be provided to UTTL, this article also naturally shows the benefits of noisy labels for realizing UTTL from a theoretical point of view while providing the theoretical foundation for UTTL on the basis of the explicitly proposed fundamental assumption that the TT does not exist in the real world. The rest of the contents of this article are structured as follows: In Section 2, we briefly introduce LWNLs and the similarities and differences between UTTL and LWNLs. In Section 3, we perform a series of works to answer the intrinsic question of why UTTL is needed. In Sections 4, 5, 6 and 7, we present the definition, alternative solution, specific method and particular application for the UTTL problem, respectively. Finally, in Section 8, we present a discussion, conclusion and future work for this article.

2. Related work

As the labels for the data prepared for the situations of UTTL and LWNLs can be identically inaccurate, UTTL shares a certain similarity with LWNLs. In this section, we briefly review approaches for the situation of LWNLs.

In the literature on LWNLs, numerous approaches have been proposed to address this problem, including robust architectures, robust regularization, sample selection, and robust loss design [12]. In particular, the objective of robust architectures [13][14][15][16][17][18][19][20] is to apply a noise adjustment layer over a deep neural network (DNN) to grasp how labels change or to construct a unique architectural design that accommodates a wider variety of label noise categories, which strives to hinder a DNN's tendency to overly adapt to incorrectly labelled examples through the implementation of training constraints. A key advantage of robust regularization [21][22][23][24][25][26] lies in its capacity to readily acclimate to novel scenarios with minimal adjustments. Sample selection strategies [27][28][29][30][31][32][33][34][35] endeavour to pinpoint and prioritize the samples deemed most plausible to be clean for the purpose of enhancing the optimization process. Robust loss [36][37][38][39][40][41][42][43][44][45][46][47][48][49][50][51] design seeks to calibrate the loss value in accordance with the certainty of a particular loss (or label) through various tactics or devises a novel loss function tailored to cope with imprecise guidance. Typically, resilient loss functions incorporate a provision that imposes a penalty on predictions made with low confidence, which are more prone to result from noisy data points. For more information about the LWNLs problem and its alternative solutions, readers can refer to [7][8].

For a TT learning task in the current literature of LWNLs or even in the current literature of the entire ML realm, the acquiescent assumption about the TT is that the TT exists in the real world. In contrast, in this work, the fundamental assumption about the TT for UTTL is that the TT does not exist in the real world.

3. Why is UTTL needed?

In this section, we systematically illustrate the necessity and importance of presenting UTTL on the basis of the explicitly proposed fundamental assumption that the TT does not exist in the real world. This section is structured as follows: In Section 3.1, the definitions of labels and targets in ML are discussed. In Section 3.2, the evaluation and learning procedures in ML are analysed. In Section 3.3, existing assumptions for the TT in the evaluation procedure are summarized. In Section 3.4, the effects of different assumptions for TT in the evaluation procedure are organized. Finally, in Section 3.5, an

doi.org/10.32388/KBK3P8.3

underlying logic in ML is summarized from the previous four serial works, which shows that it is indeed necessary and important to present UTTL on the basis of the explicitly proposed assumption that the TT does not exist in the real world.

3.1. Label and target in ML

In ML, a label or a target is usually associated with an instance. The instance and its corresponding label or target form a data point that can be collected to create a dataset for the evaluation and learning of ML-based predictive models. The difference between a label and a target is that a label represents the mapping objective associated with an instance, whereas a target represents a transformation from the mapping objective for an instance that can be easily used for computation in specific procedures in ML [52].

3.2. Evaluation and learning procedures in ML

In ML, two procedures play decisive roles in evolving predictive models for specific applications: evaluation and learning procedures. The evaluation procedure aims to assess the performance of an ML-based predictive model, and the learning procedure aims to develop a predictive model based on specific ML algorithms. In ML, the evaluation procedure is typically closely related to the learning procedure. Their relation is that successful evaluation strategies for the evaluation procedure are critical to the learning procedure for developing ML-based predictive models, as successful evaluation strategies for the evaluation procedure in ML are generally used to build up the learning procedure in ML (i.e., if an evaluation strategy can be successfully used to evaluate an ML-based predictive model in the evaluation procedure, then it can also generally be used to learn an ML-based predictive model in the learning procedure).

3.3. Existing assumptions for TT in the evaluation procedure

To reveal the existing assumptions for TT in the evaluation procedure in ML, we should first summarize various strategies proposed for the evaluation procedure for assessing the performance of an ML-based predictive model. For the evaluation procedure in ML, there are two usual types of evaluation strategies: usual evaluation with accurate ground-truth labels (AGTLs) and usual evaluation with inaccurate ground-truth labels (IAGTLs). Each of these two usual types also has specific subtypes regarding different preparations for evaluation. Usual evaluation with AGTLs can be classified into two subtypes:

doi.org/10.32388/KBK3P8.3

extracting TT from massive AGTLs [53][54][55] and extracting TT from limited AGTLs [56][57]. Usual evaluation with IAGTLs can be classified into two subtypes: selecting probable TTs from IAGTLs [58][59][60] and providing/estimating the TT error rate in IAGTLs [61]. More recently, a new evaluation strategy named the logical assessment formula (LAF) [62] was also proposed for evaluation with IAGTLs. LAF only requires the extraction of multiple TTs from IAGTLs for evaluation [63]. These classifications can be summarized in Table 1.

On the basis of the summarization of Table 1, we can analyse the underlying assumption for the TT in the three types of evaluation strategies for the evaluation procedure in ML. For the usual strategy of evaluation with AGTLs, the acquiescent assumption for the TT is that the TT exists in the provided labels, as the TT can be extracted from the provided massive or limited AGTLs. For the usual strategy of evaluation with IAGTLs, the acquiescent assumption for the TT is also that the TT exists in the provided labels, as the probable TT can be selected from the provided IAGTLs, or the TT error rate in the provided IAGTLs can be provided/estimated (i.e., if the TT does not exist in the provided labels, then no probable TT can be selected from the provided IAGTLs and no TT error rate in the provided IAGTLs can be provided/estimated). However, for the strategy of LAF for evaluation with IAGTLs, the fundamental assumption for the TT is not exclusive, as it only requires extracting multiple inaccurate TTs from the provided IAGTLs (i.e., the TT can exist or does not exist in the provided IAGTLs). As a result, the fundamental assumptions for the TT in the three types of evaluation strategies for the evaluation procedure in ML can be summarized in Table 2.

Evaluation strategy	Preparation for evaluation
Usual evaluation with AGTLs	Generating the TT from massive AGTLs
	Generating the TT from limited AGTLs
Usual evaluation with IAGTLs	Selecting the probable TT from IAGTLs
	Providing/estimating the TT error rate in IAGTLs
LAF for evaluation with IAGTLs	Extracting multiple inaccurate TTs from IAGTLs

Table 1. Summary of various strategies proposed for the evaluation procedure for assessing the performance of an ML-based predictive model

doi.org/10.32388/KBK3P8.3

Assumption for the TT	Evaluation strategy
The TT exists in the provided labels	Usual evaluation with AGTLs
	Usual evaluation with IAGTLs
The TT can exist or does not exist in the provided IAGTLs	LAF for evaluation with IAGTLs

Table 2. Summary of the fundamental assumptions for the TT in different types of evaluation strategies for the evaluation procedure in ML

3.4. Effects of assumptions for TT on the evaluation procedure

In fact, the fundamental assumptions for the TT are the causes that affect the emergence of various existing strategies for the evaluation procedure in ML. In other words, there are cause–and–effect relationships between the fundamental assumption for the TT and the various existing strategies for the evaluation procedure in ML. The detailed effects of the fundamental assumption for the TT on the evaluation procedure in ML can be summarized as follows: 1) The assumption that the TT exists in the provided labels is the foundation for establishing the two usual types of evaluation strategies, including usual evaluation with AGTLs and the usual evaluation with IAGTLs, for the evaluation procedure in ML; 2) the assumption that the TT can exist or does not exist in the provided IAGTLs is the foundation for establishing the LAF for evaluation with IAGTLS for the evaluation procedure in ML. The effects of different assumptions for the TT on the evaluation procedure in ML can be summarized in Table 3.

3.5. Necessity and importance of presenting UTTL

On the basis of the fact illustrated in Section 3.2 that successful evaluation strategies for the evaluation procedure in ML are generally used to build up the learning procedure in ML and the summarizations of Table 1, Table 2, and Table 3 presented in Section 3.3 and Section 3.4, in this subsection, we illustrate the necessity and importance of presenting UTTL.

As successful evaluation strategies for the evaluation procedure in ML are generally used to construct the learning procedure in ML, the assumption that the TT that has an effect on the evaluation procedure in ML will eventually also have an effect on the learning procedure in ML. As a result, the assumption that the TT exists in the provided labels has been affecting the learning procedure in the current literature of

doi.org/10.32388/KBK3P8.3

LWNLs or even in the literature of the entire ML realm, since the usual evaluation with AGTLs and the usual evaluation with IAGTLs are the two types of evaluation strategies most commonly used in ML. In other words, we can conclude that the acquiescent assumption about the TT for a TT learning task in the current literature of LWNLs or even in the literature of the entire ML realm is that the TT exists in the real world, even for situations where the TT cannot be precisely defined.

Cause	**Effects**		
Assumption for the TT	Evaluation strategy	Preparation for evaluation	Evaluation procedure
The TT exists in the provided labels	Usual evaluation with AGTLs	Generating the TT from massive or limited AGTLs	Evaluating on the generated TT
	Usual evaluation with IAGTLs	Selecting some probable TTs from IAGTLs, or providing/estimating rate of TT error in IAGTLs	Evaluating on the probable TT selected from IAGTLs, or evaluating on IAGTLs regarding to the provided/estimated rate of TT error
The TT can exist or does not exist in the provided IAGTLs	LAF for evaluation with IAGTLs	Extracting multiple inaccurate targets from IAGTLs	Evaluating on the multiple inaccurate targets extracted from IAGTLs

Table 3. Summarization of different assumptions for the TT on the evaluation procedure in ML

Recent works [62][63] have shown that the new evaluation strategy of LAF for evaluation with IAGTLs can be successfully established on the basis of the assumption that the TT can exist or does not exist in the provided IAGTLs. With the common logical sense that a successful evaluation strategy for the evaluation procedure in ML can be generally used to build up the learning procedure in ML, it is reasonable that we can explicitly propose the assumption that the TT does not exist in the real world and present UTTL on the basis of this assumption.

doi.org/10.32388/KBK3P8.3

In summary, a clear underlying logic in ML can be drawn from the serial works conducted in Sections 3.1 to 3.4, which assume that the TT is the foundation for establishing the evaluation strategy, and the evaluation strategy established on the basis of the assumption that the TT will eventually affect the formation of the learning concept. In short, the assumption about the TT will eventually determine the formation of the learning concept in ML. Regarding this underlying logic in ML, the two assumptions about the TT that the TT does not exist in the real world and that the TT exists in the real world will eventually lead to different learning concepts. Specifically, with the assumption that the TT does not exist in the real world, a new evaluation strategy of LAF for evaluation with IAGTLs is established, which can eventually lead to the formation of the new learning concept UTTL presented in this article. With the assumption that the TT exists in the real world, evaluation strategies of the usual evaluations with AGTLs or IAGTLs have been established, which affect the concept of TT learning in LWNLs or ML. The comparison of the two fundamental assumptions about the TT for establishing different evaluation strategies that eventually lead to the two learning concepts of UTTL and TT learning in LWNLs or ML is shown in Table 4.

Assumption about the TT	Evaluation strategy	Learning concept
The TT does not exist in the real world	LAF for evaluation with IAGTLs	UTTL
The TT exists in the real world	Usual evaluation with AGTLs	TT learning in LWNLs or ML
	Usual evaluation with IAGTLs	

Table 4. Comparison of the two fundamental assumptions about the TT for establishing different evaluation strategies, which eventually lead to different learning concepts

With the underlying logic in ML that the assumption about the TT will eventually determine the formation of the learning concept, Table 4 reasonably proves that existing approaches for addressing LWNLs are not suitable for handling UTTL. As a result, it is necessary and important to present UTTL on the basis of the explicitly proposed assumption that the TT does not exist in the real world to appropriately handle the situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios.

4. Definition of the UTTL

Let us consider the situation where the true target of a learning task cannot be precisely defined. In practice, this situation inevitably leads to a large problem in label preparation for the learning task, which is that the label prepared for an entity/event contains severe inaccuracy in representing the true target associated with the entity/event. Here, we refer to this situation as the problem of undefinable true target learning (UTTL). Since large inconsistencies usually appear among experts regarding an agreement on the true target for the UTTL problem, in this article, we explicitly propose the fundamental assumption about the true target for the UTTL problem, which is that the true target does not exist in the real world.

On the basis of this fundamental assumption, the UTTL problem can be described as follows: on the basis of a collected number of data points, each of which consists of an entity/event and a prepared label that contains severe inaccuracy in representing the undefinable true target associated with the entity/event, a function that can map the entities/events into the undefinable true targets can be found. Notably, as the label prepared for the entity/event contains severe inaccuracy because the true target is undefinable, the properties of the label prepared for the entity/event inevitably cannot precisely represent the properties of the undefinable true target. Thus, the solution to the UTTL problem (i.e., the found function that can map the entities/events into the corresponding undefinable true targets) should be subject to the condition that the properties of the labels prepared for the entities/events are included in the properties of the undefinable true targets mapped from the entities/events.

Denote the collected number of data points as $H = \{d, l\}$, where d represents the entities/events, l represents the prepared labels associated with d that cannot precisely represent the undefinable true target, and the elements in d and l have a one-to-one correspondence. The function that can map the entities/events into the corresponding undefinable true targets is denoted as $f : d \longmapsto t$, where t represents the mapped examples of the undefinable true target and the elements in d and t also have a one-to-one correspondence. The mapping function f should be subject to the condition that the properties of l are included in the properties of t. The properties of l are denoted as $prop(l)$, the properties of t are denoted as $prop(t)$, and the relationships included are denoted as $\subseteq$. Now, the UTTL problem is formally defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(l) \subseteq prop(t). \tag{1}$$

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Here, Θ_f denotes the space of the mapping function f, and we do not constrain the specific formation for $prop(*)$, as it can be semantic, numerical or both to describe the properties of *.

5. Alternative solution to UTTL

On the basis of the definition of UTTL, we propose an alternative solution to the UTTL problem. Specifically, we first transform the UTTL problem into a combination of the machine learning (ML) problem and the logical reasoning (LR) problem and then propose an alternative solution to the transformed UTTL problem.

5.1. Common ML and LR methods

For the common ML problem, the prepared set of labels l is usually assumed to be able to precisely represent the true targets t corresponding to the set of entities/events d in the collected number of data points $H = \{d, l\}$. Thus, in this situation, the properties of l ($prop(l)$) are equal to the properties of t ($prop(t)$) compared with formula (1). Formally, the common ML problem can be defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(t) = prop(l). \tag{2}$$

Usually, the alternative solution to the common ML problem can be described as an optimized mapping function that can minimize the error between $t = f(d)$ and l, which can be formally expressed as

$$\tilde{f} = \arg\min_{f \in \Theta_f} \; o(t = f(d), l). \tag{3}$$

Here, o is a predefined loss function that can estimate the error between $t = f(d)$ and l.

For the common LR problem, in addition to the prepared set of entities/events d and the corresponding set of labels l, an accumulated knowledge base (KB) containing various prior knowledge facts about the true target is provided. The LR problem can be expressed as follows: a reasoning path (r) can be searched from the collected data points $H = \{d, l\}$ and KB to draw a set of conclusions (c) that are consistent with ($\cong$) some knowledge facts in KB. Formally, the common LR problem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\, r : \{d, l\}, KB \rightarrow c \qquad s.t. \quad c \cong KB. \tag{4}$$

Here, Θ_r denotes the space of the reasoning path r. Usually, the alternative solution to the common LR problem can be described as a validated logical path (a series of valid logical processes) that can maintain the consistency between $c = r < \{d, L\}, KB >$ and KB, which can be formally expressed as

$$\tilde{r} = \arg \underset{r \in \Theta_r}{maint}\ cons(c = r < \{d, l\}, KB >, KB). \tag{5}$$

Here, $cons$ is a predefined procedure that can reflect the consistency between $c = r < \{d, l\}, KB >$ and KB.

5.2. Transformed UTTL

A comparison of the UTTL problem definition (formula (1)) with the common ML problem definition (formula (2)) reveals that the true learning target for the common ML problem can be precisely known, whereas the true learning target for the UTTL problem cannot be precisely known. This fact reflects that if we directly take the alternative solution to the common ML problem (formula (3)) as a solution to the UTTL problem, the mapping function $\tilde{f}$ will suffer from severe inaccuracies in predicting the true target for the UTTL problem.

Referring to the common LR problem definition (formula (4)), we can observe that if we regard the conclusions c drawn from the provided data points $H = \{d, l\}$ and the accumulated knowledge base KB as some statements about the undefinable true target for the UTTL problem, then it is plausible that we can search a reasoning path that can draw some statements that are consistent with KB to be able to better describe the undefinable true target than the labels l in T for the UTTL problem. Thus, the alternative solution to the common LR problem (formula (5)) can probably be leveraged to propose a better alternative solution to the UTTL problem than naively employing formula (3).

We propose transforming the UTTL problem into a type of problem that is a combination of the ML problem and the LR problem. Specifically, the transformed problem for UTTL can be divided into the following three subproblems.

1. On the basis of a number of provided data points $H = \{d, l\}$ in which l cannot precisely describe the undefinable true target and an extra accumulated knowledge base KB, which contains various prior knowledge facts about the undefinable true target, the primary subproblem is to search for a reasoning path r that can draw some statements c about the undefinable true target. The drawn c should be consistent with KB to be able to better describe the undefinable true target for UTTL than the labels l provided in H. Formally, referring to formulas (1) and (4), this subproblem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\ r : \{d, l\}, KB \rightarrow c \qquad s.t. \quad prop(l) \subseteq c \cong KB. \tag{6}$$

doi.org/10.32388/KBK3P8.3

2. On the basis of $H=\{d,l\}$ and c from 6), the subsequent subproblem is to build a program (p) that can generate a new set of learning targets t^* corresponding to d. The properties of the generated t^* should be equal to c when describing the undefinable true target for UTTL. This subproblem can be formally defined as

$$\tilde{p} = \underset{p\in\Theta_p}{building}\, p : \{d,l\}\,, c \rightharpoonup t^* \qquad s.t. \quad prop\,(t^*\ ) = c. \tag{7}$$

Here, Θ_p denotes the space of the program p.

3. On the basis of d and t^* from 2), the final subproblem involves finding a mapping function that can map d onto the corresponding final predicted true targets t for UTTL. The properties of the final predicted t should be equal to the properties of t^*. Formally, referring to formula (2), this subproblem can be defined as

$$\tilde{f}\ = \underset{f\in\Theta_f}{finding}\, f : d \longmapsto t \qquad s.t. \quad prop(t) = prop\,(t^*)\,. \tag{8}$$

Referring to formulas (6), (7), and (8), the UTTL problem definition expressed in formula (1) can be transformed as follows

$$\begin{cases} 1)\ \tilde{r}\ = \underset{r\in\Theta_r}{searching}\, r : \{d,l\}\,, KB \rightarrow c \\ 2)\ \tilde{p} = \underset{p\in\Theta_p}{building}\, p : \{d,l\}\,, c \rightharpoonup t^* \\ 3)\ \tilde{f}\ = \underset{f\in\Theta_f}{finding}\, f : d \longmapsto t \end{cases} \qquad s.t. \quad prop(l) \subseteq prop(t) \cong KB. \tag{9}$$

Formula (9) shows that the subject condition for the transformed UTTL problem definition is now $prop(l) \subseteq prop(t) \cong KB$, which is different from the subject condition $prop(l) \subseteq prop(t)$ in the original UTTL problem definition expressed in formula (1). More details on how we obtain the subject condition in formula (9) from formulas (6), (7), and (8) are provided in Proof 1 of the Appendix.

5.3. Analyses of the transformed UTTL

From the subject condition of the transformed UTTL problem definition expressed in formula (9) ($prop(l) \subseteq prop(t) \cong KB$), we can observe that the properties of the labels L in the provided data points T ($prop(l)$) are included in ($\subseteq$) the properties of the final predicted true targets ($prop(t)$), and $prop(t)$ is also consistent with ($\cong$) the extra accumulated knowledge base KB, which contains various prior knowledge facts about the undefinable true target. This subject condition reflects not only that the final predicted true targets t are better able to represent the undefinable true target for UTTL than the labels in the provided data points but also that the properties of the final predicted true targets t are consistent with various prior knowledge facts about the undefinable true target for UTTL. This reflects that the transformed UTTL problem definition is better at finding the appropriate mapping function for predicting the undefinable true target than the original UTTL problem definition is.

Although the final predicted true targets t possess better properties, which are consistent with KB, compared with the labels l, we are still not sure about whether t can be precise enough to represent the undefinable true target for UTTL. With respect to the subject condition $prop(l) \subseteq prop(t) \cong KB$ in formula (9), we can deduce that how precisely t can represent the undefinable true target for UTTL will depend on how precisely the prior knowledge facts contained in KB can represent the undefinable true target. However, theoretically, with more knowledge facts iteratively accumulated in KB to represent the undefinable true target, the final predicted t can be iteratively more precise to represent the undefinable true target for UTTL. As a result, the transformed UTTL problem definition provides a promising foundation for approaching the undefinable true target for UTTL.

5.4. Alternative solution to the transformed UTTL

Referring to the transformed UTTL problem definition expressed in formulas (6), (7), and (8), the alternative solution to the transformed UTTL problem can also be divided into three subsolutions.

1. The first subsolution is the solution to formula (6), which can be expressed as formula (5).
2. The second subsolution is the solution to formula (7), which is to build a programme (p) to generate the learning targets t^* corresponding to d from $H = \{d, L\}$ and the c produced by the first subsolution. Formally, the second subsolution can be expressed as

$$\tilde{p} = \arg \underset{p \in \{\Theta_r \cup \Theta_f\}}{build} \; t^* = p\left(\{d, l\}, c\right). \tag{10}$$

Here, $p \in \{\Theta_r \cup \Theta_f\}$ indicates that the built program p can be in the space of the LR-based methods (Θ_r), in the space of the ML-based methods (Θ_f) or in the space of the combined LR and ML methods ($\Theta_r \cup \Theta_f$).

3. The third subsolution is the solution to formula (8), which can be expressed as formula (3) with the replacement of l with t^*.

In summary, the alternative solution to the transformed UTTL problem can be formally expressed as follows.

$$\begin{cases} 1)\; \tilde{r} = \arg \underset{r \in \Theta_r}{maint}\; cons(c = r < \{d, l\}, KB >, KB) \\ 2)\; \tilde{p} = \arg \underset{p \in \{\Theta_r \cup \Theta_f\}}{build} \; t^* = p\left(\{d, l\}, c\right) \\ 3)\; \tilde{f} = \arg\min_{f \in \Theta_f} \; o(t = f(d), t^*) \end{cases} \tag{11}$$

 doi.org/10.32388/KBK3P8.3

5.5. Additional notes

Notably, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in this section since the alternative solution here is proposed on the basis of the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is possible that a better problem transformation and corresponding solution for the UTTL problem defined in formula (1) can still be proposed on the basis of other original thoughts and perspectives.

6. Specific methods

Referring to the alternative solution presented for the transformed UTTL problem, which is summarized in formula (11), one-step abductive multitarget learning (OSAMTL) and its extensions have been proposed in recent works [1][2][3][11] to provide some specific methods for addressing the UTTL problem.

6.1. OSAMTL

As input materials, OSAMTL requires a number of collected data points containing labels that cannot precisely represent the undefinable true target and an extra accumulated knowledge base that contains various prior knowledge facts about the undefinable true target. In addition to the required input materials, the key components of OSAMTL correspond to the three subsolutions presented in formula (11), which include the component of one-step abductive logical reasoning, the component of generation of multiple types of learning targets and the component of multitarget learning.

6.1.1. Input materials

The input materials for the OSAMTL method include a number of collected data points $H = \{d, l\}$, where d represents the entities/events, l represents the prepared labels associated with d, which cannot precisely represent the undefinable true target, and an extra accumulated knowledge base (KB), which contains various prior knowledge facts about the undefinable true target.

More specifically, H can be expressed as

$$H = \{d, l\} = \{\{d_1, l_1\}, \ \ldots, \ \{d_n, l_n\}\}. \tag{12}$$

KB can be more specifically expressed as

$$KB = \{k_1, \ldots, \ k_m\}. \tag{13}$$

doi.org/10.32388/KBK3P8.3

In formula (12), n denotes the number of data points collected in H, and each element $\{d_n, l_n\}$ represents a collected data point that consists of an entity/event d_n and its corresponding label l_n. In formula (13), m denotes the number of prior knowledge facts, and each element k_m represents an accumulated knowledge fact about the undefinable true target.

6.1.2. One-step abductive logical reasoning

On the basis of the input materials H and KB, the one-step abductive logical reasoning (OSALR) component of OSAMTL draws some statements/conclusions (c) that can more accurately describe the undefinable true target than the labels provided in H. Formally, referring to subsolution 1) of formula (11), this component can be expressed as

$$c = \tilde{r}(H, KB) = \{c_1, \cdots, c_w\}. \tag{14}$$

More specifically, the OSALR component consists of three substeps as follows.

From H, substep one extracts a list of groundings that can describe the logical facts contained in the given diverse noisy samples. This grounding extraction (GE) step can be expressed as

$$g = GE(H) = \{g_1, \cdots, g_s\}. \tag{15}$$

Via logical reasoning, substep two estimates the inconsistencies between the extracted groundings g and the prior knowledge accumulated in KB. Formally, this logical reasoning (R) step can be expressed as

$$ic = R(g, KB) = \{ic_1, \cdots, ic_u\}. \tag{16}$$

Substep three revises the groundings in g via logical abduction, which aims to reduce the estimated inconsistencies in ic. Formally, this logical abduction (LA) step can be expressed as

$$c = LA(ic) = \{c_1, \cdots, c_w\}. \tag{17}$$

With these three specific substeps (GE, R, LA) for implementing $\tilde{r}$ in formula (14), the final statements/conclusions drawn are revised groundings that are consistent with KB to better describe the undefinable true target than simply the groundings of the labels l provided in H.

6.1.3. Generation of multiple types of learning targets

The generation of multiple types of learning targets (GMTLT) components aims to leverage H and c drawn by the OSALR component to abduce multiple types of learning targets. Formally, referring to subsolution 2) of formula (11), this component can be expressed as

$$t^* = \tilde{p}(H, c) = \{t_1^*, \cdots, t_v^*\}. \tag{18}$$

doi.org/10.32388/KBK3P8.3

Formula (18) indicates that the built program $\tilde{p}$ can generate multiple types of learning targets ($\{t_1^*, \cdots, t_v^*\}$) from H and c that are associated with each data point of d in H. Usually, the program $\tilde{p}$ can be specifically implemented via logical reasoning and machine learning methods.

As the multiple types of learning targets ($\{t_1^*, \cdots, t_v^*\}$) can be generated from H with the help of the revised groundings (c) that are consistent with KB to better describe the undefinable true target, the generated multiple types of learning targets in formula (18) can also possess certain consistencies with our prior knowledge to better represent the undefinable true target.

6.1.4. Multitarget learning

The multitarget learning (MTL) component of OSAMTL is carried out on the basis of a specifically constructed machine learning [4][5][6] architecture (f) that can map entities/events (d) into corresponding predicted targets (t), which can be expressed as $t = f(d)$. Here, the MTL component of OSAMTL aims to optimize the parameters of f to minimize the error between the targets (t) predicted by f and the multiple types of targets (t^*) generated by the GMTLT component.

To estimate the error between t and t^*, a loss function (o) is typically needed. As t^* contains multiple types of targets, the error between t and the multiple types of targets in t^* can be estimated by the weighted sum of the errors between t and the respective t_v^* in t^*, which can be expressed as

$$o\left(t, t^*\right) = \sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right) \quad s.t. \quad \sum_{i=1}^{v} \alpha_i = 1. \tag{19}$$

Commonly, o in formula (19) can be implemented via cross-entropy for classification and least squares for regression. Furthermore, to produce the optimized machine learning model $\tilde{f}$, $o\left(t, t^*\right)$ should be minimized. Specifically, if f is constructed via state-of-the-art deep learning methods [64] based on neural networks, the minimization of $o\left(t, t^*\right)$ can be implemented via stochastic gradient descent variants.

As the multiple types of learning targets (t^*) generated by the GMTLT component possess certain consistencies with our prior knowledge to better represent the undefinable true target, the produced machine learning model $\tilde{f}$ can make reasonable predictions (t) about the undefinable true target by minimizing the error between t and t^*.

doi.org/10.32388/KBK3P8.3

6.2. Extensions of the OSAMTL

In Section 6.1, we present formulas (12)-(19) to denote the original OSAMTL method. However, the original OSAMTL method inevitably has limitations in handling some situations in real-world scenarios for UTTL, as the presented formulas only denote the basic components to concisely present the OSAMTL method. In this subsection, on the basis of the original OSAMTL method presented in Section 6.1, we discuss several extensions of OSAMTL to expand the usage range of OSAMTL in real-world scenarios for UTTL.

One extension of OSAMTL is that the data points provided for UTTL can be extended to diverse types instead of only a single type of data point. Unlike the original OSAMTL, we denote this type of extension as OSAMTL with diverse types of data points (DiTDP) (OSAMTL-DiTDP). Another extension of OSAMTL is that the label l_n corresponding to the entity/event d_n in formula (12) can be extended to diverse types instead of only a single type of label. In contrast with the original OSAMTL, we denote this type of extension as OSAMTL with diverse types of labels (DiTL) (OSAMTL-DiTL).

6.2.1. OSAMTL-DiTDP

For the situation of OSAMTL-DiTDP, referring to formula (12), the provided DiTDP can be expressed as

$$H = \{H_1, \ldots, H_k\} = \{\{d_1, l_1\}, \ldots, \{d_k, l_k\}\}$$
$$= \left\{\{\{d_{1,1}, l_{1,1}\},\ \ldots,\ \{d_{1,n_1}, l_{1,\,n_1}\}\}, \ldots, \{\{d_{k,1}, l_{k,1}\},\ \ldots,\ \{d_{k,n_k}, l_{k,\,n_k}\}\}\right\}. \tag{20}$$

Here, k denotes the number of DiTDPs, and n_k denotes the number of data points for each type.

In fact, DiTDP can increase the diversity of the provided data points, which eventually leads to labels in the provided data points representing diverse aspects of the undefinable true target. By comparing formula (20) with formula (12), we can deduce that if the sum of the numbers for the multiple types of data points in formula (20) is equal to the number of data points in formula (12) (i.e., $\sum_{i=1}^{k} n_i = n$), the complexity of preparing DiTDP can remain essentially unchanged when a single type of data point is prepared. As a result, this extension of DiTDP has the potential to significantly increase the diversity of the labels of the prepared data to represent the undefinable true target while maintaining the average complexity unchanged when a single type of data point is prepared for OSAMTL.

Moreover, this extension of OSAMTL is more complex to implement than the original OSAMTL is, as the extension of DiTDP increases the complexity in implementing the OSALR and GLTMT components of OSAMTL-DiTDP for particular applications. Specifically, for the OSALR component, formulas (15), (16), and (17) need to be carried out multiple times regarding the prepared DiTDP to produce the final revised grounds to better describe the undefinable true target. For the GLTMT component, formula (18) needs to be used to consider the possible associations among the prepared DiTDP and their corresponding revised groundings, which can make the implementation of the GLTMT component more complicated.

6.2.2. OSAMTL-DiTL

For the situation of OSAMTL-DiTL, DiTL can be expressed as $l_n = \{l_{n,1}, \ldots, l_{n,j}\}$, where j denotes the number of multiple types of labels included in l_n. Referring to formula (12), the provided data points with DiTDP can be expressed as

$$\begin{aligned} H = \{d, l\} = \{\{d_1, l_1\}, \ \ldots, \ \{d_n, l_n\}\} \\ = \{\{d_1, \{l_{1,1}, \ldots, \ l_{1,j}\}\}, \ \ldots, \ \{d_n, \{l_{n,1}, \ldots, \ l_{n,j}\}\}\}. \end{aligned} \tag{21}$$

In fact, DiTL can significantly reduce the complexity of the original OSAMTL method, as multiple types of targets can be reasonably extracted from DiTL provided in the data points to represent the undefinable true target. As a result, this extension of the OSAMTL can be less complex to implement than the original OSAMTL. Although OSAMTL-DiTL requires diverse labels for the data points, it is practical in real-world scenarios. This is because the diverse labels required can be inaccurate, which can make the label preparation procedure much easier.

6.3. Essence of OSAMTL

The fundamental assumption for the proposed OSAMTL is that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target. On the basis of this fundamental assumption, the three key components of the OSAMTL contribute to this assumption.

Primarily, from the input materials of data points H and the knowledge base KB, the OSALR component of OSAMTL draws some revised groundings (c) that are consistent with KB to be able to better describe the undefinable true target than simply the groundings of the labels l in H. Subsequently, leveraging the provided data points H and the revised groundings c drawn by the OSALR component, the GMTLT component of OSAMTL uses multiple types of learning targets containing information that is consistent with our prior knowledge KB about the undefinable true target. Finally, on the basis of a specifically constructed machine learning architecture (f), the MTL component of OSAMTL produces an optimized machine learning model $\tilde{f}$ that can make reasonable predictions about the undefinable true target by minimizing the error between the targets (t) predicted by f and the multiple types of targets (t^*) generated by the GMTLT component.

With these three key components of OSAMTL to realize the assumption that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target, the essence of OSAMTL is that it forces the machine learning architecture to learn from the weighted summarization of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable true target. More specifically, this essence of the OSAMTL reflects the following result.

Theorem 1. *For a classification or a regression task, the loss constructed by* $o\left(t, t^*\right)=\sum_{i=1}^{v} \alpha_i o\left(t, t_i^*\right)$ *can be theoretically expressed as* $o\left(t, t^*\right)=o\left(t, \sum_{i=1}^{v} \alpha_i t_i^*\right)+c$*, where* c *is a constant term.*

Detailed proofs for Theorem 1 are provided in Proofs 2 and 3 of Supplementary 1. Through Theorem 1, we can declare that OSAMTL is able to force the learning model reasonably to achieve logically rational predictions about undefinable targets by learning from the weighted summarization of multiple types of targets. Learning from the weighted summarization of multiple types of targets, which are consistent with our prior knowledge about undefinable true targets, can lead to a trade-off among the multiple types of targets and thus to a reasonable approximation of the undefinable true target.

7. Specific application

The proposed specific method OSAMTL and its extensions for UTTL have been successfully applied to address some tasks in medical histopathology whole slide image analysis (MHWSIA) [1][2][3][11]. In this section, we summarize the implementation rules and techniques of these specific methods for particular applications in practice. For simplicity, here, we provide the key points of implementing OSAMTL and its extensions for practice. More detailed illustrations of the application of these methods to specific tasks in MHWSIA are provided in Supplementary 2.

Referring to the contents of Section 6, OSAMTL and its two extensions, OSAMTL-DiTDP and OSAMTL-DiTL, can be visually summarized, as shown in Fig. 1. Fig. 1 shows that there are two key points for applying the three methods: 1) OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL differ primarily in the preparations of the data points with respect to the input materials; 2) OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL share the same abstract formulas from the perspectives of OSALR, GMTLT, and MTL,

doi.org/10.32388/KBK3P8.3

although these abstract formulas can differ in specific implementations regarding different methods and applications.

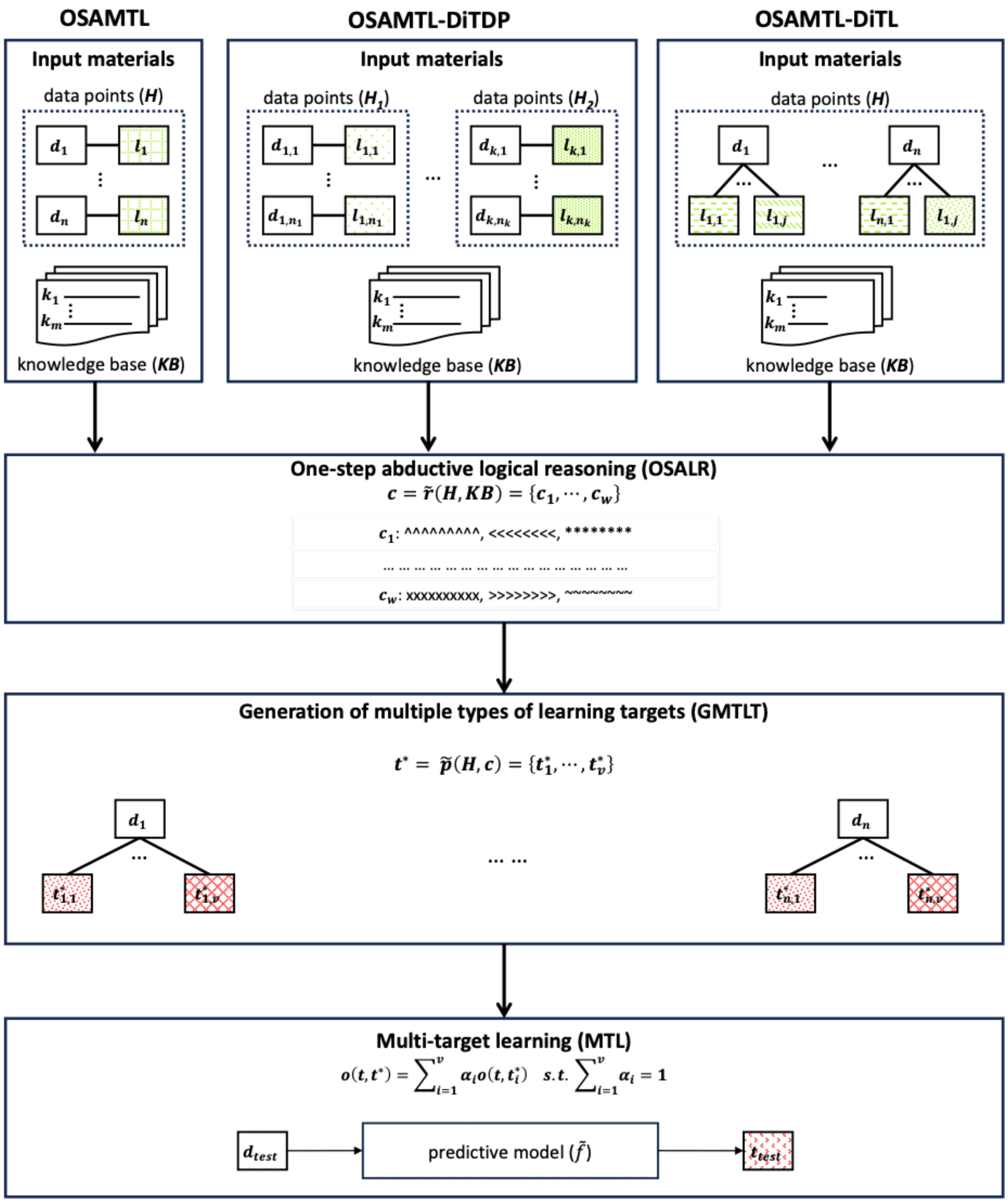

Figure 1. Summary of the OSAMTL and its two extensions, the OSAMTL-DiTDP and the OSAMTL-DiTL.

doi.org/10.32388/KBK3P8.3

8. Discussion, conclusion and future work

In this article, we explicitly propose the fundamental assumption that the TT does not exist in the real world to formally present the first theoretical foundation for UTTL to appropriately handle the common situation where the TT for a TT learning task cannot be precisely defined in various AI application scenarios.

To demonstrate the necessity and importance of presenting UTTL on the basis of the explicitly proposed fundamental assumption that the TT does not exist in the real world, we performed a series of works to address the intrinsic question of why we need to present UTTL. We discuss the definitions of labels and targets in ML, analyse the evaluation and learning procedures in ML, summarize existing assumptions for the TT in ML, organize the effects of different assumptions for TT on ML, and finally illustrate the necessity and importance of presenting UTTL.

To formally present a theoretical foundation for UTTL to handle the situation where the TT for a TT learning task cannot be precisely defined, we systematically analysed UTTL from the perspectives of problem definition, alternative solutions, specific methods, and particular applications. Owing primarily to the fundamental assumption that the true target for the UTTL problem does not exist in the real world, the definition for the UTTL problem is formally presented. On the basis of the presented definition, the UTTL problem is subsequently transformed into a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. In addition, with respect to the presented alternative solution, specific methods such as one-step abductive multitarget learning (OSAMTL) and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL) are summarized for addressing the UTTL problem in different scenarios. Finally, referring to the summarized OSAMTL and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL), the implementation rules and techniques of these methods are discussed with respect to particular real-world application scenarios. The discussions include applying the OSMTL to segment helicobacter pylori areas precisely in whole slide images [1][3], applying OSAMTL-DiTDP to tumor segmentation in HE-stained pretreatment biopsy images [2], and discussing the similarities and differences between the application of OSAMTL-DiTL and the applications of OSAMTL and OSAMTL-DiTDP to reveal the potential of the application of OSAMTL-DiTL [11].

In addition, as the TT cannot be precisely defined in UTTL, only inaccurately labelled data can be provided to UTTL. As a result, providing a theoretical foundation for UTTL based on the explicitly

doi.org/10.32388/KBK3P8.3

proposed fundamental assumption that the TT does not exist in the real world, this article also naturally shows the benefits of noisy labels in realizing UTTL from a theoretical point of view.

As we have analysed in Section 5.5, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in the article, since it is based on the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is probable that better problem transformations and corresponding solutions for the UTTL problem defined in formula (1) can still be proposed with respect to other original thoughts and perspectives. In addition, with the fundamental assumption that the TT does not exist in the real world, the concept of UTTL can also be applied in various other AI application scenarios to establish different perspectives for addressing related tasks.

Statements and Declarations

Funding

No funding was received for this research project.

Acknowledgements

The author, Yongquan Yang, especially thanks his Yang Family, Chengdu, China, for providing him with financial support and mental encouragement to do this research. The author established the conceptualization and did the initial work for this article while he was taking a vacation (from 2023.09--2024.02) in Chengdu, Sichuan, China. The rest of the work for the initial version of this article was done in his spare time when he was an invited contract employee (from 2024.03--2024.09) at Zhongjiu Flash Medical Technology Co., Ltd., Mianyang, Sichuan, China. The author truly appreciates the constructive comments received from the anonymous editor of Machine Learning and the reviewers Xingsi Xue and Ricardo Reier of Qeios, which provided the author with good opportunities to improve the quality of this updated version and more confidence and encouragement to continue moving forward with this research.

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Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.