

Research Article

Undefinable True Target Learning: Towards Learning with Democratic Supervision

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Assumptions regarding the true target (TT), which is a computationally equivalent transformation of the Ground Truth, are crucial for the formulation of diverse machine learning (ML) paradigms. In this article, we present a systematic review of TT assumptions across current ML paradigms and further explore a posited TT assumption and its correspondingly derived ML paradigm. Explicitly positing that the TT does not exist in the real world, we investigate the implications of this assumption and analyse how it may redefine our understanding of designing ML paradigms. These implications and analyses lead us to proposing undefinable true target learning (UTTL) towards learning with democratic supervision (LDS). We establish the definition of UTTL, illustrate its principles for revealing the undefinable TT, and discuss its practicability for LDS and its uniqueness compared with existing similar learning settings. Based on these, we summarize example UTTL principle-based solutions regarding existing works to show the practical value of UTTL in enabling LDS. This article offers a new ML paradigm towards LDS grounded in the non-existence assumption of TT.

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1. Introduction

The True Target (TT) is a computationally equivalent transformation of the Ground Truth. It serves as a fundamental concept in the formulation and deployment of machine learning (ML). Assumptions regarding the TT are crucial for the formulation of diverse ML paradigms. They intrinsically determine how learning models perceive, approximate, and generalize the underlying reality, thereby shaping both the theoretical foundations and practical limitations of ML systems. (In this article, the term TT is used instead of Ground Truth to facilitate practical discussion. The transformation from Ground Truth to a computational TT and its reverse are both essential in practice. Semantically, the two are equivalent.)

Unsupervised learning^{[1][2][3]}, supervised learning (SL)^{[4][5]}, weakly supervised learning (WSL)^[6], and reinforcement learning (RL)^{[7][8]} (Sutton et al. 2023) collectively constitute the four foundational paradigms of modern ML. An examination of the TT assumptions underlying these paradigms provides a valuable perspective for revisiting existing ML frameworks and evaluating their adequacy in transforming the underlying reality into predictive models.

This examination can probably enlighten explorations on new ML paradigm based on appropriately posited TT assumption. Motivated by this, in this article, we present a systematic review of TT assumptions across current ML paradigms and further explore new TT assumption and its correspondingly derived ML paradigm.

Based on our systematic review, the TT assumptions embedded in the current ML paradigms can be summarized as follows:

1. In unsupervised learning, the concept of TT is generally inapplicable;
2. In SL, it is typically assumed that the TT objectively exists in the real world;
3. In WSL, the prevailing mainstream assumption remains that the TT objectively exists, even though it may be partially missing, coarsely represented, or inaccurately observed in the available annotations;
4. In two prevalent sub-paradigms of inaccurate supervision (a typical type of WSL), including learning from noisy labels (LNL)^{[9][10]} and learning from multiple annotators (LMA)^{[11][12][13][14][15]}, there is an emerging skepticism toward the clarity, definability, and uniqueness of TT. This shift is particularly evident in LMA; and
5. For RL, the TT assumptions largely align with those of unsupervised learning, SL, and WSL.

Further elaboration and comparative analysis of these TT assumptions are provided in Section 2.

Particularly, recent studies in the two prevalent LNL and LMA sub-paradigms of inaccurate supervision have increasingly challenged the assumption of an objective TT^{[16][17][18][19][20][21][22][23][24][25][26][27][28][29][30][31]}. However, few works explicitly reject the premise that the TT objectively exists in the real world. Different from these works, in this article, we take a more radical stance by explicitly positing that the TT does not exist in the real world and exploring correspondingly derived ML paradigm, directly rejecting the objective existence of TT.

The explicitly posited non-existence assumption of TT originates from our previous works on inherently ambiguous TT segmentation in pathological images^{[20][28][30][32]}. While conducting these works, we gradually realized that assuming the objective existence of TT was fundamentally inadequate for tasks characterized by intrinsic indefinability of TT. For these situations, the more appropriate assumption should be that TT does not exist in the real world. Furthermore, insights gained from our systematic review of TT assumptions across current ML paradigms confirm the novelty and necessity of this non-existence assumption, which provide the conceptual foundation for the subsequent investigations.

From first principles, we conduct comparison between the non-existence and existence assumptions of TT to elucidate the implications of the assumption that TT does not exist in the real world towards learning with democratic supervision (LDS). Under the traditional existence assumption of TT, data prepared solely by domain expert dominates the learning process, leaving AI expert with limited influence (a situation we regard as undemocratic). In contrast, under the non-existence assumption of TT, both domain and AI experts collaboratively contribute to data preparation, forming a more balanced and participatory (democratic) framework. The existence assumption can thus be viewed as a special case of the non-existence assumption. The non-existence perspective

offers a broader conceptual space and more flexibility for developing new ML paradigms. These comparative insights suggest that new data preparation strategies grounded in expert democracy and appropriate learning paradigms with democratic supervision should be both explored. Together, these two explorations lead to an expanded scope towards LDS. More details are provided in Section 3.

Thus, grounded in the assumption that the TT does not exist in the real world, we propose undefinable true target learning (UTTL), which exemplifies the essence of LDS. We establish the definition of UTTL, illustrate its principles for revealing the undefinable TT, and discuss its practicability for LDS and its uniqueness compared with existing similar learning settings. Based on these, we also summarize example UTTL principle-based solutions regarding existing works to show the practical value of UTTL in realizing LDS. More details are provided in Sections 4 and 5.

To the best of our knowledge, this article is the first to explicitly posit the assumption that the TT does not exist in the real world and to investigate the corresponding UTTL framework as a pathway towards LDS. The contributions of this article include:

- Presenting a systematic review of TT assumptions cross current ML paradigms;
- Investigating the implications of the explicitly posited non-existence assumption of TT and analysing how it may redefine our understanding of designing ML paradigms;
- Proposing the UTTL framework to exemplify the essence of LDS;
- Summarizing example UTTL principle-based solutions regarding existing works to show the practical value of UTTL in realizing LDS;
- Offering a new ML paradigm (UTTL) towards LDS grounded in the non-existence assumption of TT.

The remainder of this article is organized as follows. Section 2 provides a systematic review of the fundamental TT assumptions underlying current ML paradigms. Section 3 compares the non-existence and existence assumptions of TT and elucidates the implications of assuming that the TT does not exist in the real world, particularly for LDS. Sections 4 and 5 introduce the UTTL framework and summarize example practical solutions derived from UTTL principles, respectively. Finally, Section 6 presents the discussion, conclusion, and directions for future work.

2. Systematic Review of Fundamental TT Assumptions Underlying Current ML Paradigms

In this section, we firstly categorize current ML paradigms and discuss their relations. Then, regarding the categorization of ML paradigms, we comprehensively review the fundamental TT assumptions under respective ML paradigms and related prevalent mainstream subtype paradigms. Finally, summarize the mainstream TT assumptions and trends underlying current ML paradigms.

2.1. Categorization of current ML paradigms and their relations

Depending on the availability of supervision information, the paradigms in current ML research can be broadly classified into unsupervised learning and learning with supervision. Unsupervised learning operates on data prepared without the use of TT (Ground Truth) labels^{[1][2][3]}. Learning with supervision can be further divided into supervised learning (SL) and weakly supervised learning (WSL), regarding the perfection or imperfection of the TT labels in the training data.

SL is founded on data with complete, exact, and accurate (i.e., perfect) TT labels, whereas WSL is based on data containing incomplete, inexact, or inaccurate (i.e., imperfect) TT labels^{[6][33]}. Both SL and WSL can be further subdivided into narrower categories. For instance, SL can be categorized into precisely supervised learning, moderately supervised learning, and hybrid forms that combine both^[23], reflecting different computational transformations of the TT derived from Ground Truth labels. WSL encompasses learning with incomplete supervision, inexact supervision, inaccurate supervision, and their cross-scenario^[6], corresponding to distinct forms of imperfection in the provided Ground Truth labels.

Another popular type of ML paradigm is reinforcement learning (RL)^{[7][8][34][35]}. RL is a learning framework in which an intelligent agent interacts with an environment and autonomously learns an optimal behaviour policy based on reward signals. Rather than relying on explicit supervision, RL learns through trial and error and feedback to maximize long-term returns.

Unsupervised learning, SL, WSL, and RL together constitute the four fundamental paradigms of modern ML. They form a series in terms of how learning signals are obtained and how strongly supervision is imposed. Unsupervised learning derives knowledge solely from the intrinsic structure and distribution of data, without relying on external TT labels. SL depends on complete, exact and accurate TT labels to directly learn the mapping between inputs and outputs. WSL bridges the gap between the two by leveraging incomplete, inexact, or noisy supervision to approximate the true supervision signal. RL, in contrast, learns through interaction with the environment, optimizing behavior based on reward feedback rather than explicit supervision. Details for the categorization of current ML paradigms are summarized in Table 1.

ML Paradigm			Data Basis and Remarks
Unsupervised learning			Data without the use of TT (Ground Truth) labels ^{[1][2][3]}
Learning with supervision	Perfect supervision: supervised learning (SL)	Precisely supervised learning	Data with complete, exact, and accurate (i.e., perfect) TT labels ^[33] ; Narrower categorizations reflecting different computational transformations of the TT derived from Ground Truth labels
		Moderately supervised learning	
		Hybrid forms that combine both	
	Imperfect supervision: weakly supervised learning (WSL)	Incomplete supervision	Data containing incomplete, inexact, inaccurate, or their cross-scenario (i.e., imperfect) TT labels ^[6] ; Narrower categorizations corresponding to distinct forms of TT imperfection in the provided Ground Truth labels.
		Inexact supervision	
		Inaccurate supervision	
		Cross-scenario	
Reinforcement learning			Learning through trial and error and feedback to maximize long-term returns, without relying on explicit supervision ^{[7][8][34][35]}

Table 1. Categorization of ML paradigms

These four paradigms are complementary rather than isolated. Unsupervised learning supports SL and WSL paradigms by providing structural priors, robust representations, and auxiliary pseudo-supervision derived from unlabelled data^{[36][37][38][39][40]}. SL offers WSL with pretrained models, reliable loss functions, and well-established optimization strategies that serve as priors or initialization for learning from imperfect supervision^{[41][6][10]}. RL integrates unsupervised learning, SL, and WSL for decision-making and policy optimization in dynamic environments, forming a coherent and interdependent learning^{[8][42][34][43][44][45]}. The relations of the four paradigms are shown in Fig. 1.

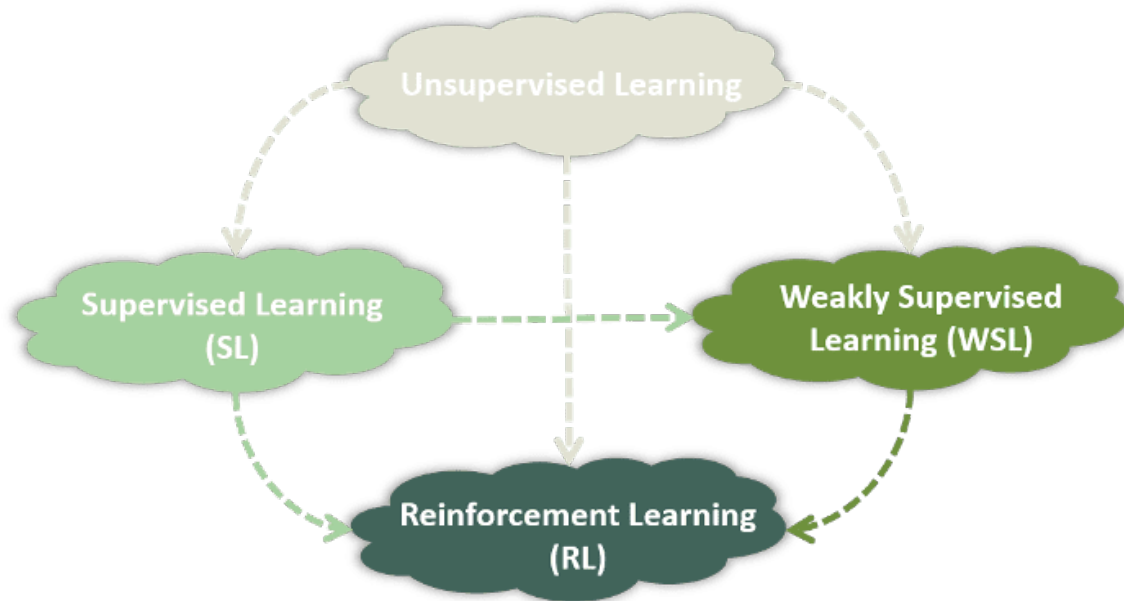


Figure 1. Relations of unsupervised learning, SL, WSL and RL.

2.2. Unsupervised learning

In unsupervised learning, the concept of TT generally does not apply. The learning process does not rely on labelled data or predefined target values. Instead, the goal is to uncover latent structures, patterns, or representations from the data itself. Ground truth may only be introduced post hoc for evaluation purposes (e.g., when comparing discovered clusters with known categories), but it is not inherent to the learning process^{[1][2][3]}.

2.3. Supervised learning

In supervised learning (SL), it is assumed that a true mapping exists between inputs x and their corresponding target outputs y . The mapping is often referred to as the TT function $f(x)$. Training data are viewed as observations of this underlying function. The goal of learning is to approximate f as closely as possible. This situation implies that every input has a well-defined and correct TT, even though in practice these labels may be imperfect due to annotation errors, ambiguity, or measurement noise^{[4][5]}.

SL has been further categorized into narrower subtypes based on the complexity involved in transforming the prepared Ground True labels into learnable true targets, including precisely supervised learning, moderately supervised learning, and hybrid forms that combine both^[33]. Although the formulations of the TT in these subtypes may differ, they share a common underlying assumption that the TT in the prepared data is complete, exact, and accurate. That is, the TT objectively exists in the real world.

2.4. Weakly supervised learning

The classic classification of weakly supervised learning (WSL) is the categories proposed by Zhi-Hua Zhou in "A brief introduction to weakly supervised learning"^[6]: incomplete supervision, inexact supervision, inaccurate supervision, and their cross-scenario. In this subsection, we summarize how the labels in these cases are "weak" and what mainstream assumptions are made about the TT, primarily based on recent highly cited survey works.

2.4.1. Incomplete supervision

Incomplete supervision refers to learning scenarios in which only a subset of training instances is labelled, while the remaining instances remain unlabelled^{[6][46]}. Typical forms of incomplete supervision include: semi-supervised learning^{[47][48]}, where only a subset of training samples are labelled and the remaining are unlabelled; positive-unlabelled learning^{[49][50]}, where only positive examples are labelled while all others remain unlabelled and potentially contain both positive and negative instances; and active learning^{[51][52]}, where the learner iteratively selects a small subset of unlabelled samples to be labelled by an oracle in order to minimize labelling cost while maximizing learning efficiency.

Under this paradigm, several assumptions about the TT are stated. For example, Zhou^[6] pointed out that the TT labels are assumed to exist for all samples, even if not all are annotated. Similarly, Ren et al^[46] treated labelled data as TT and considers unlabelled data as merely missing annotations. Zhang et al^[53] further discussed cases where annotated data may contain noise, yet still relies on the assumption that true labels exist for all samples, with only partial observation in the incomplete supervision setting.

2.4.2. Inexact supervision

Inexact supervision refers to learning settings where labels are available but are coarse-grained or imprecise^{[6][46]}. Typical examples include: image segmentation tasks, in which only image-level labels are provided rather than pixel- or object-level annotations^{[54][55]}; multi-instance learning (MIL) scenarios, where only bag-level labels are given without specifying which individual instances are responsible for the label^{[56][57][58]}; or situations in which class categories are defined at an insufficient level of granularity.

Under this paradigm, several assumptions about the TT are reflected. Zhou^[6] defined inexact supervision as cases in which "only coarse-grained label information is available," assuming the existence of unobserved fine-grained TT. Similarly, Yue et al.^[59] described remote sensing tasks where only image-level annotations are available, implicitly assuming the existence of pixel-level TT. In the literature on MIL, it is generally assumed that certain instances within each bag genuinely possess the class property, even though only the overall bag label is provided^[6]. Thus, again implies the existence of fine-grained TT that remains unobserved.

2.4.3. Inaccurate supervision

Inaccurate supervision refers to learning scenarios in which the provided labels are not always correct and may deviate from the TT due to labeling noise, human errors, or systematic biases^{[6][46]}. Typical forms of inaccurate supervision include: learning from noisy labels^{[9][10]}, where some of the provided labels are incorrect due to human or systematic annotation errors; and learning from multiple annotators (crowdsourcing)^{[11][12][13][14][15]}, where labels are collected from a group of annotators with varying expertise and reliability, leading to inconsistent or biased annotations.

Under this paradigm, several assumptions about the TT are presented. Zhou^[6] noted that, under inaccurate supervision, “the given labels are not always the ground truth” , which reflects the existence of TT. Zhang et al.^[53] posited that a small portion of labels may be noisy but assumed that the TT exists and can be recovered with the aid of unlabeled data and structured noise models. Similarly, Ren et al.^[46] treated inaccurate supervision as a setting where the provided labels may be erroneous, yet the TT labels are still regarded as existing and serve as the conceptual reference point for model learning.

2.4.4. Cross-scenario expansion

The three types of weak supervision can simultaneously appear in cross-scenario settings. For example, Zhang et al.^[53] proposed a unified framework for addressing learning challenges where data are only partially labelled or contain annotation errors. Yue et al.^[59] jointly discussed how the three forms of weak supervision (incomplete, inexact, and inaccurate) can facilitate optical remote sensing image understanding tasks such as classification, segmentation, change detection, and object detection. The former reflects the assumption that the TT exists but that some labels are missing or incorrect. The latter assumes that TT exists at all sample, pixel, or object levels, and that in practice, some annotations are missing, some are coarse-grained, and some are erroneous.

2.4.5. Mainstream TT assumption observed in WSL

The TT assumptions underlying the three primary forms of weak supervision, along with their cross-scenario extensions, are summarized in Table 2. Collectively, these summaries reveal a prevailing mainstream TT assumption in WSL: that the TT objectively exists in the real world, even though it may be partially missing, coarsely represented, or inaccurately observed in the available annotations.

WSL Category	Representative tasks / paradigms	TT Assumptions
Incomplete supervision	Semi-supervised learning ^{[47][48]}	Each sample possesses a unique TT label, although some samples remain unlabelled ^{[6][53][46]}
	Positive-unlabelled learning ^{[49][50]}	
	Active learning ^{[51][52]}	
Inexact supervision	Image-level supervised semantic segmentation ^{[54][55]}	Finer-grained (e.g., instance-level or pixel-level) TT labels exist, but the available annotations are aggregated or masked into coarse-grained labels ^{[6][59]}
	Multi-instance learning ^{[11][13][14][15]}	
Inaccurate supervision	Learning with noisy labels ^{[9][10]}	Each sample has a unique TT label, but the observed label may deviate from it due to noise or annotation errors ^{[6][53][46]}
	Learning from multiple annotators (crowdsourcing) ^{[11][13][14][15]}	
Cross-scenario	Learning with multiple types of weak labels ^{[53][59]}	TT label exists universally, but its observations in data are incomplete, coarse-grained, or corrupted ^{[53][59]}

Table 2. Taxonomy of WSL and its summarized TT assumptions

2.5. Two prevalent paradigms of inaccurate supervision

Inaccurate supervision is particularly challenging to the assumption that the TT objectively exists in the real world, since the available labels are either corrupted by noise or generated by multiple, potentially inconsistent annotators. Learning from noisy labels (LNL)^{[9][10]} and learning from multiple annotators (LMA)^{[11][12][13][14][15]} represent two prevalent paradigms of inaccurate supervision in WSL. Both cases can potentially raise questions about the appropriateness of the TT existence assumption. Therefore, in this subsection, we conduct a relatively comprehensive survey of the TT assumptions underlying these two forms of inaccurate supervision. This survey helps to reveal potential alternative assumptions that may deviate from the mainstream premise under WSL that the TT objectively exists in the real world.

2.5.1. Learning from noisy labels

In the early literature on LNL, researchers generally assumed that the TT objectively exists. For instance, Angluin and Laird^[60] directly modelled the observed labels as outcomes of independent noise flips applied to true labels. In this case, the annotator (or teacher) independently flips each label with a certain probability (the random classification noise model). Natarajan et al.^[9] introduced an unbiased loss estimator based on noise-rate correction. The validity of correction relies on a noise model assuming the existence of a single true label and class-conditional flip. That is,

observed labels are corrupted by class-conditional random noise (where the flip probability depends on the class), and the noise rate is either known or estimable. Patrini et al.^[61] further proposed forward and backward loss correction using an estimated noise transition matrix. Both their theoretical analysis and empirical studies are built upon the assumption that a true label distribution exists but is distorted by class-dependent noise, which can be represented by a transition matrix.

In subsequent works, researchers began to adopt more implicit assumptions about the existence of TT. Reed et al.^[62] posited the existence of a “true signal” learnable by the model. The corrected noisy labels through self-consistency between model predictions and observed labels (i.e., soft labels or bootstrapping). Han et al.^[41] assumed that a subset of “clean samples” exists and that deep networks first fit these clean samples due to the memorization effect. They proposed a co-teaching method to train two networks that mutually select small-loss samples to focus on clean data. Song et al.^[10] provided a systematic taxonomy of noise-robust learning approaches (e.g., loss correction, sample selection, robust loss). They analysed which methods explicitly rely on true labels or noise matrices and which can operate under weaker assumptions.

More recently, a number of studies have begun to explicitly question the clarity, definability and uniqueness of TT under noisy-label settings. Frénay and Verleysen^[17] summarized and categorized various noise models—such as random class-dependent noise, class-conditional noise, and instance and class-dependent noise models. They pointed out that, for subjective tasks, objective ground truths may be ambiguous. Plank^[21] and Yang et al.^[20]^[29] (2024b, a; Yang 2024a) further challenged the notion of a well-defined ground truth. These works suggested that label uncertainty may be an inherent property of certain tasks. Zhang et al.^[26] took a different approach by aligning instances with their noisy labels rather than correcting them. They treated the noisy labels as the targets to be aligned with. Conceptually, such approaches weaken the traditional assumption that a single, objective ground truth must exist.

2.5.2. Learning from multiple annotators

In the early literature on LMA, researchers typically assumed the existence of a consensual or latent TT. For instance, Dawid and Skene^[63] modelled each instance’s true label as a hidden latent variable. In this case, the observed noisy annotations were generated through individual annotators’ confusion matrices (i.e., error rates). Their Expectation–Maximization (EM) framework jointly estimated the annotators’ reliability parameters and the latent true labels. Raykar et al.^[12] explicitly treated the “true label” as a latent variable. They proposed a model that simultaneously learns both a classifier and annotator reliability (accuracy/bias). Their objective was to recover a single hidden truth in the absence of any gold standard. Whitehill et al.^[64] also assumed a single “correct answer” and proposed an ability-weighted label aggregation model. In this case, annotators’ votes were weighted according to their inferred competence to approximate the true label. Welinder et al.^[65] further extended this framework by incorporating image features and multi-dimensional annotator ability/bias factors, but the core assumption remained the estimation of a

single latent ground truth per instance. Sheng et al.^[66] investigated whether and how repeated labelling improves annotation quality. This case is still under the premise that aggregation strategies (e.g., majority voting) aim to approximate an underlying ground truth.

Subsequent studies introduced the notion of variable truth. Yan et al.^[67] argued that annotator expertise might depend on the instance itself (expertise conditional on instance). They emphasized that although a single ground truth is still commonly estimated, annotator reliability may vary across samples. This led to the view of a “single truth with instance–annotator interaction noise.” Nguyen et al.^[68] extended latent-truth modelling to sequential labelling tasks. They introduced more structured latent variables (e.g., token-level true labels) that capture dependencies among labels within sequences.

More recent works have begun to explicitly question the clarity, definability, and uniqueness of TT in multiple-annotator settings. For example, Mokhberian et al., Li et al., and Ibrahim et al.^{[22][24][31]} discussed the ambiguity of TT. They advocated modelling instance-dependent annotation noise or learning annotator-specific embeddings to preserve opinion diversity. Yang et al., Wang et al., and Zhang et al.^{[20][23][14][29]} (Yang et al. 2024b, a; Yang 2024a) questioned the definability of TT. They argued that when experts disagree, models should not blindly construct a gold standard but instead learn representations aligned with annotator consensus or preserve probabilistic and multi-perspective label structures. Snow et al., Subramanian et al., and Srinivasan and Chander^{[16][18][19]} challenged the uniqueness of TT. They noted that for inherently subjective or ambiguous tasks, a single gold label may not be appropriate. They emphasized that in subjective domains, such as affect recognition, aesthetics, readability, or perceived relevance, the “single truth” assumption may fail. They also underlined that modelling label distributions, multiple viewpoints, or population subgroups is more appropriate than forcing consensus as a proxy for truth.

2.5.3. Emerging trend of TT assumption observed in inaccurate supervision

Regarding representative timeline works, the TT assumptions underlying the two prevalent paradigms LNL and LMA within inaccurate supervision are summarized in Table 3. As observed from Table 3, both paradigms exhibit a clear evolutionary trajectory: early studies typically assumed the arbitrary existence of TT; intermediate works gradually relaxed this assumption; and recent research has increasingly questioned the clarity, definability, and uniqueness of TT. This progression indicates an emerging trend in which researchers are beginning to challenge the very existence of TT itself. Notably, this shift appears to be more radical in the LMA paradigm.

Prevalent Paradigm of Inaccurate Supervision	Representative Timeline Works	TT Assumptions
Learning from noisy labels (LNL)	Early-stage studies: foundational works ^{[60][9][61]}	Assuming that the TT objectively exists.
	Intermediate developments: expansion and methodological refinement ^{[62][41][10]}	Adopting more implicit assumptions about the existence of TT
	Recent advances: emerging trends and paradigm shifts ^{[17][20][21][26][29]} (Yang et al. 2024b, a; Yang 2024a)	Questioning the clarity, definability and uniqueness of TT under noisy-label settings
Learning from multiple annotators (LMA)	Early-stage studies: foundational works ^{[63][64][12][65]}	Assuming the existence of a consensual or latent TT
	Intermediate developments: expansion and methodological refinement ^{[67][68]}	Introducing the notion of variable truth
	Recent advances: emerging trends and paradigm shifts ^{[16][18][19][20][22][23][24][14][29][31]} (Yang et al. 2024b, a; Yang 2024a)	Questioning the clarity, definability, and uniqueness of TT in multiple-annotator settings

Table 3. Taxonomy of WSL and corresponding TT assumptions

2.6. Reinforcement learning

In reinforcement learning (RL), the reward signal serves as the core driving force. The efficient implementation of RL often relies on the representations, priors, and auxiliary signals provided by unsupervised learning, SL, and WSL. Specifically, unsupervised learning contributes to effective exploration and state representation through representation learning and intrinsic motivation mechanisms. It is helpful especially when external rewards are sparse, thereby improving generalization and sample efficiency^{[43][45]}. SL can provide policy priors or reward models through expert demonstrations or human feedback, substantially accelerating policy optimization^{[8][42]}. In addition, weakly supervised learning supports reward modelling and policy evaluation in cases where reward signals are sparse or noisy. It helps to construct approximate supervision signals derived from human preferences, partial annotations, or proxy feedback^{[34][44]}. Together, these learning paradigms reinforce the learnability and stability of the RL framework at different levels. Therefore, the assumptions regarding the TT are not specifically discussed for the RL paradigm in this section, as they can be referred to those already discussed for the other ML paradigms.

2.7. Summary

In summary, the assumptions of TT across current ML paradigms can be summarized as follows (as shown in Table 4). In unsupervised learning, the concept of TT is generally inapplicable. In SL, it is typically assumed that the TT objectively exists in the real world. In WSL, the prevailing mainstream assumption remains that the TT objectively exists, even though it may be partially missing, coarsely represented, or inaccurately observed in the available annotations. Notably, in two prevalent paradigms of inaccurate supervision (LNL and LMA), an emerging trend has been the growing scepticism toward the clarity, definability, and uniqueness of TT. This shift is particularly pronounced in LMA, where the assumption of an objective TT is increasingly being challenged. For RL, the TT assumptions are largely aligned with those in unsupervised learning, SL, and WSL.

ML Paradigm		TT assumptions
Unsupervised learning		None: the concept of TT generally does not apply (Section 2.2)
Supervised learning (SL)	Precisely supervised learning	The TT objectively exists in the real world (Section 2.3)
	Moderately supervised learning	
	Hybrid forms that combine both	
Weakly supervised learning (WSL)	Incomplete supervision	A prevailing mainstream TT assumption is observed in WSL-related surveys: the TT objectively exists in the real world, even though it may be partially missing, coarsely represented, or inaccurately observed in the available annotations (Section 2.4)
	Inexact supervision	
	Inaccurate supervision	
	Cross-scenario	
Two prevalent paradigms in inaccurate supervision	Learning from noisy labels (LNL)	Emerging trend of TT assumption is observed in more recent works in LNL and LMA: researchers are beginning to question the clarity, definability, and uniqueness of TT in both noisy-label and multiple-annotator setting; and the shift in LMA appears to be more radical (Section 2.5)
	Learning from multiple annotators (LMA)	
Reinforcement learning		Not specifically discussed, can be referred to the above assumptions (Section 2.6)

Table 4. Summarization of TT assumptions in current ML paradigms.

Different from the recent studies in the two prevalent LNL and LMA paradigms of inaccurate supervision, in this article, we take a more radical stance by explicitly positing the assumption that the TT does not exist in the real world and exploring correspondingly derived ML paradigm.

3. Implications of the Assumption that TT Does Not Exist in the Real World

In this section, from first principles, we conduct comparison between the non-existence and existence assumptions of TT to reveal the implications of the assumption that TT does not exist in the real world towards learning with democratic supervision (LDS).

3.1. Comparison between the Non-Existence and Existence Assumptions of TT

The comparison between the non-existence and existence assumptions of TT is summarized in Fig. 2.

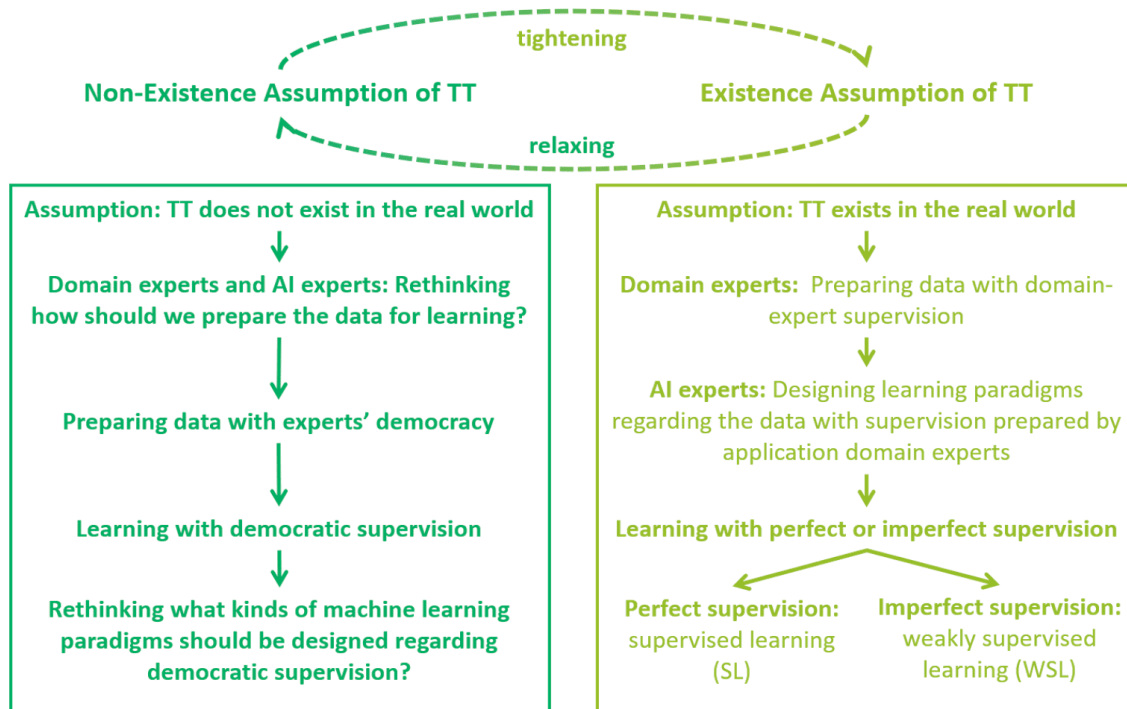


Figure 2. Summarization for comparison between the non-existence and existence assumptions of TT

Let us first consider the existence assumption of TT, where TT is assumed to objectively exist in the real world. Under this assumption, the typical process of designing ML paradigms proceeds as follows:

1. Data preparation: domain experts prepare datasets under expert supervision, as TT is presumed to exist and experts are regarded as professional and trustworthy identifiers of it;
2. Paradigm design: based on such supervised data, AI experts develop appropriate paradigms to transform the data into predictive models;
3. Paradigm establishment: depending on the degree of supervision perfection, paradigms such as SL with perfect supervision and WSL with imperfect supervision are typically adopted.

Now, let us consider the non-existence assumption of TT, where TT is assumed not to exist in the real world. In this case, the typical process of designing ML paradigms fundamentally changes:

1. Collaborative data conceptualization: domain experts and AI experts are placed on an equal footing to jointly rethink how data should be conceptualized and prepared for capturing the properties of the underlying reality, as TT does not objectively exist and domain experts are no longer the sole authority for identifying it;
2. Democratic data preparation: data are constructed through expert democracy, with both domain and AI experts equally participating in the supervision process;
3. Paradigm rethinking: given this data prepared under expert democratic supervision, it becomes necessary to reconsider what forms of ML paradigms can be designed to effectively learn from such data to reveal the underlying reality.

Under the existence assumption of TT, the data prepared under domain-expert supervision plays a decisive role, as the ML paradigms subsequently designed by AI experts to transform the underlying reality into predictive models are largely determined by the perfection of the expert-prepared data. This situation reflects a structural imbalance: AI experts seldom participate in data preparation. We regard this as undemocratic, since the experiential knowledge of AI experts, particularly their understanding of what kinds of data effectively capture the underlying reality for model learning, is often neglected.

Under the non-existence assumption of TT, the prepared data still retains its decisive role. However, the crucial difference is that it embodies expert democracy, where both domain experts and AI experts collaboratively (democratically) contribute their insights to more effectively capture and represent the underlying reality.

Furthermore, when the non-existence assumption of TT is gradually tightened, it naturally degenerates into the existence assumption; conversely, when the existence assumption is gradually relaxed, it evolves toward the non-existence assumption. In this sense, the existence assumption can be viewed as a special case of the non-existence assumption. This offers a broader conceptual space and more flexibility for inventing ML paradigms that aim to transform the underlying reality into predictive models by LDS.

3.2. Suggesting new data preparation strategies regarding expert democracy

The non-existence assumption of TT suggests that new data preparation strategies should be explored regarding expert democracy, to which both domain and ML experts collaboratively contribute. Under the existence assumption of TT, the labelling rules are mostly designed by the domain expert annotators. In this situation, performing the data preparation will be intense, as the labelling rules should be strictly followed to effectively capture the underlying reality. However, under the non-existence assumption of TT, the labelling rules can be relaxed to that each one of the domain and AI expert annotators is focusing on capture a single or a few properties for describing the underlying reality. In this situation, performing the data will be relaxed, as the labelling rules should be democratically discussed,

in which we are able to even consider using existing large models to help capturing the underlying reality. Thus, new data preparation strategies are needed regarding expert democracy.

3.3. Suggesting new learning paradigm regarding data prepared under expert democracy

The non-existence assumption of TT also suggests that new learning paradigms should be explored regarding the data prepared with expert democracy. Under the existence assumption of TT, the learning paradigms are mostly designed to predict the TT from the data prepared by domain expert annotators. In this situation, usually a clearly single TT is expected to be predicted by learning from the domain expert prepared data, which possibly contain incomplete, inexact or inaccurate supervision information. However, under the non-existence assumption of TT, the learning paradigms can be relaxed to that the predictive model learnt from the data prepared with expert democracy does not necessarily output a clearly single TT but a prediction that possess the observed key properties of the underlying reality. Thus, new learning paradigms are needed regarding data prepared with expert democracy.

3.4. Expanding research scope towards LDS

Together, the suggested necessities for new data preparation strategies regarding expert democracy and new learning paradigms regarding data prepared with expert democracy eventually enlarge the research scope of the conventional learning with perfect or imperfect supervision, as the existence assumption of TT is a special case of the non-existence assumption of TT. Thus, the non-existence assumption of TT expands the research scope towards LDS.

3.5. Summary

In summary, the existence assumption can be viewed as a special case of the non-existence assumption, which implies that new data preparation strategies should be explored regarding expert democracy and new learning paradigms should be explored regarding the data prepared with expert democracy. Together, these two new requirements under the non-existence assumption of TT imply an expanded research scope of current learning with perfect or imperfect supervision towards LDS.

4. Undefinable True Target Learning

Grounded in the assumption that TT does not exist in the real world and its implications, this section introduces the conceptual framework of undefinable true target learning (UTTTL). UTTTL reinterprets data preparation and model learning processes under the premise that the TT does not objectively exist in the real world. It serves as a representative paradigm within the broader research scope of learning with democratic supervision (LDS). In the following subsections, we formally define UTTTL, reveal its principles for uncovering undefinable TT, and discuss its practicability for LDS and its uniqueness compared with the LMA setting.

4.1. Definition for methodology of UTTL

Let us consider a situation where the TT of a domain learning task is inherently undefinable. In practice, this poses a fundamental challenge: how should we prepare data and design learning paradigms for predictive modelling when the TT cannot be objectively defined? We present the UTTL paradigm for alleviating this challenge. Grounded in the implications of the non-existence assumption of TT, which calls for new data preparation strategies based on expert democracy and new learning paradigms built upon such data, the UTTL paradigm is decomposed into two key components: 1) data preparation with expert democracy, and 2) learning from data prepared under expert democracy.

4.1.1. Predefined input and output

The input of the UTTL paradigm is the entities/events collected from the domain, denoted as d . The output of the UTTL paradigms is the revealed TT, denoted as \tilde{t} , corresponding to d . The revealed \tilde{t} is expected to cover a variety of properties of the undefinable TT associated with d in the domain.

4.1.2. Data preparation with expert democracy

Domain experts (DE) can label the d with their expertise in identifying the undefinable TT to capture some of its properties (t_{DE}^*). This DE labelling process can be formally defined as

$$t_{DE}^* = DE_Label(d; \theta^{DE_Label}). \quad (1)$$

AI experts (AIE) can do some complementary works to refine the t_{DE}^* corresponding to the d with their expertise in predictive modelling to produce a refined representation (t_{AIE}^*) for identifying the undefinable TT. This AIE refinement can be formally defined as

$$t_{AIE}^* = AIE_Refine(d, t_{DE}^*; \theta^{AIE_Refine}). \quad (2)$$

As the TT is inherently undefinable, both t_{DE}^* and t_{AIE}^* can probably contain severe inaccuracy in capturing it. To address this, t_{DE}^* and t_{AIE}^* can be fused in to a unified TT presentation by reasonable operations to more effectively capture the properties of the undefinable TT. As the domain expert and AI expert equally contribute to the unified TT representation, it is prepared with expert democracy (PwED). We define that the resulted unified TT representation (t_{PwED}^*) contains multiple inaccurate targets (t^*), each of which partially represent certain properties of the undefinable TT. Thus, this fusing of expert democracy can be formally defined as

$$t_{PwED}^* = PwED_Fuse(t_{DE}^*, t_{AIE}^*; \theta^{PwED_Fuse}) = \{t_1^*, \dots, t_v^*\}. \quad (3)$$

Together, the prepared d and t_{PwED}^* constitute an expert-democratic observation of the underlying domain reality.

Here, θ^- denotes the hyperparameter for implementing Formulas (1)–(3) respectively.

4.1.3. Learning from data prepared under expert democracy

Regarding common ML, the learning from the data prepared with expert democracy (d and t_{PwED}^*) can be described as: 1) constructing a function (f), which can map the d into corresponding predicted TT ($t = f(d)$); and 2) defining a loss function (l), which estimates the error between $t = f(d)$ and the t_{PwED}^* , for optimizing the f to minimize the value of l . As t_{PwED}^* contains multiple inaccurate targets $\{t_1^*, \dots, t_v^*\}$, this learning process is a multiple inaccurate target learning procedure, which is formally defined as

$$\tilde{f} = \arg \min_{f \in F} l(t = f(d), t_{PwED}^* = \{t_1^*, \dots, t_v^*\}). \quad (4)$$

Here, F denotes the function space of f .

4.1.4. Methodological formation

Referring to the predefined input and output, Formulas (1)-(3) for data preparation (DP), and Formula (4) for learning procedure (LP), the methodological formation for UTTL is formally denoted as

$$UTTL \left\{ \begin{array}{l} \text{Input: } d \\ \text{Domain expert labelling:} \\ t_{DE}^* = DE_Label(d; \theta^{DE_Label}) \\ \text{AI expert refinement:} \\ t_{AIE}^* = AIE_Refine(d, t_{DE}^*; \theta^{AIE_Refine}) \\ \text{Fusion of expert democracy:} \\ t_{PwED}^* = PwED_Fuse(t_{DE}^*, t_{AIE}^*; \theta^{PwED_Fuse}) \\ \quad = \{t_1^*, \dots, t_v^*\} \\ \text{Multiple inaccurate target learning:} \\ LP: \left\{ \begin{array}{l} \tilde{f} = \arg \min_{f \in F} l(t = f(d), t_{PwED}^* = \{t_1^*, \dots, t_v^*\}) \\ \text{Output: } \tilde{t} = \tilde{f}(d) \end{array} \right. \end{array} \right. \quad (5)$$

4.2. Principles of UTTL for revealing undefinable TT

Denoting $prop(\cdot)$ as the set of properties of an entity for representing the undefinable TT, this subsection elucidates the underlying principles of UTTL for uncovering the undefinable TT. Here, we do not impose constraints on the specific form of $prop(\cdot)$, which may take semantic, numerical, or other forms or a combination thereof in characterizing the properties of an entity. Based on the definition of UTTL (Formula (5)), we derive and discuss the following theorems under feasible hypotheses.

Theorem 1. Let the mapping \mathcal{L} is consistent with Formula (1): $t = \mathcal{L}(\theta)$. If empirical knowledge set K provided by domain expert for identifying the undefinable TT is encoded as a hyperparameter θ^{DE_Label} and produces $t_{DE}^* = \mathcal{L}(\theta^{DE_Label})$ with regard to d , then the set of properties of t_{DE}^* is included in the set of properties of the undefinable target TT, denoted as $rop(t_{DE}^*) \subseteq K \subseteq prop(TT)$.

Theorem 2. Let the mapping \mathcal{R} is consistent with Formula (2): $t = \mathcal{R}(\theta)$. If AI expert uses their predictive modelling expertise under the domain expert's empirical knowledge set K to assign θ^{AIE_Refine} and obtains refined $t_{AIE}^* = \mathcal{R}(\theta^{AIE_Refine})$ with regard to d and t_{DE}^* , then $prop(t_{AIE}^*)$ is consistent with K , denoted as $prop(t_{AIE}^*) \simeq K$.

Theorem 3. Let the mapping \mathcal{F} is consistent with Formula (3): $t = \mathcal{F}(\theta)$. If reasonable operations are encoded as a hyperparameter θ^{PwED_Fuse} and produces $t_{PwED}^* = \mathcal{F}(\theta^{PwED_Fuse}) = \{t_1^*, \dots, t_v^*\}$ with regard to t_{DE}^* and t_{AIE}^* , then the resulted unified TT representation (t_{PwED}^*) is more consistent with K , denoted as $prop(t_{PwED}^*) \cong K$, than any individual representation.

Theorem 4. If the data is prepared under expert democracy, that is, given the input set d of data instances and the corresponding target set (the multiple inaccurate true targets of expert democracy fusion) $t_{PwED}^* = \{t_1^*, \dots, t_v^*\}$, then according to the conventional practice in machine learning, a mapping function f (hypothesis space F) can be constructed such that f maps the input d to the predicted true target: $t = f(d)$, and the optimal function is obtained by minimizing the inconsistency between the prediction and the target set (a certain loss function l), denoted as $\tilde{f} = \arg \min_{f \in F} l(f(d), t_{PwED}^*)$.

To prove Theorems 1-4, we introduce some natural hypotheses shown as Tables 5 and 6. These hypotheses should be verified in practical solution implementation of UTTL. More details about the introduced natural hypotheses and proofs based on them for theorems are provided in Supplementary 1.

Hypothesis No. and Abbrev.	Detailed description
Hypothesis (i) (Domain expert correctness)	The domain expert's empirical knowledge K is correct and $K \subseteq \text{prop}(TT)$
Hypothesis (ii) (Annotation fidelity)	The annotation/generation process faithfully converts domain expert facts into certain instance attributes, that is $\text{prop}(t_{DE}^*) \subseteq K$
Hypothesis (iii) (Conciseness and evidence compatibility)	New properties introduced or amplified by the AI expert during the refinement process (denoted as the set A) do not create semantic or logical conflicts with K , that is, $K \cup A$ is satisfiable
Hypothesis (iv) (Locality/partial representation)	Each partial inaccurate target t_v^* describes only a subset of the properties of TT, that is $\text{prop}(t_v^*) \subseteq \text{prop}(TT)$
Hypothesis (v) (Complementarity)	The multiple partial property sets $\{\text{prop}(t_1^*), \dots, \text{prop}(t_v^*)\}$ for fused representation are complementary in terms of their coverage of K , that is $ \text{prop}(t_{PwED}^*) \cap K \geq \max(\text{prop}(t_{DE}^*) \cap K , \text{prop}(t_{AIE}^*) \cap K)$
Hypothesis (vi) (Reasonable fusion does not introduce systematic inconsistencies)	The fusion process avoids retaining a large number of conflicting incorrect properties through "reasonable operations" such as weighting, denoising, conflict detection, or logical intersection and union
Hypothesis (vii) (Spaces and measurability)	D (input space) and T (target/prediction space) are topological spaces (usually subsets of R^m and R^k), every $f \in F$ is measurable, and the loss $l(d, t_{PwED}^*)$ is measurable in d for each fixed t_{PwED}^*
Hypothesis (viii) (Nonnegativity and continuity of loss)	$l(d, t_{PwED}^*) \geq 0$ for all arguments, and for each fixed t_{PwED}^* , the map $d \mapsto l(d, t_{PwED}^*)$ is continuous
Hypothesis (ix) (Compactness / closedness of mapping function space)	F is nonempty and is a compact subset of a topological vector space (or at least closed and bounded in a finite-dimensional parameterisation). Concretely, if $F = \{f_\theta : \theta \in \Theta\}$ with $\Theta \subset R^p$ then Θ assume is compact and $d \mapsto f_\theta(d)$ is continuous
Hypothesis (x)	When the targets t_{PwED}^* arise from repeated sampling $\left\{ \left(d_i, t_{i, PwED}^* \right) \right\}_{i=1}^n$ drawn i.i.d. from a distribution P , assume $l(f(x), t_{PwED}^*)$ is P -integrable for all $f \in F$

Hypothesis No. and Abbrev.	Detailed description
(i.i.d. sampling and integrability)	

Table 5. Introduced natural hypotheses for proving Theorems 1-4

Theorem	Associated hypotheses
Theorem 1	Hypotheses (i) and (ii)
Theorem 2	Hypotheses (i) and (iii)
Theorem 3	Hypotheses (i), (iv), (v) and (vi)
Theorem 4	Hypotheses (vii), (viii), (ix) and (x)

Table 6. Hypotheses respectively associated with Theorems 1-4

4.3. Representative paradigm towards LDS

The methodological definition of UTTL (Formula (5)) and its principles for revealing undefinable TT (Theorems 1-4) indicate that UTTL practically serves as a representative paradigm of LDS. The two core components of UTTL, data preparation with expert democracy and learning from data prepared under expert democracy, jointly embody the democratization of the supervision process. The data preparation component of UTTL integrates the perspectives of both domain and AI experts for identifying an underlying TT. The learning procedure of UTTL develops solutions that learn from collaboratively constructed supervision (multiple inaccurate true targets of expert democracy fusion) rather than a single authoritative truth. UTTL transforms supervision from an authority-driven process into a participatory and negotiated one, thereby exemplifying the essence of LDS.

4.4. Uniqueness of UTTL

The UTTL setting is unique compared with the LMA setting, although the learning procedures of two settings are identically based on multiple inaccurate TT representation set. The primary difference between the two is that the UTTL setting is established under the assumption that the TT does not exist in the real world while the LMA setting is largely grounded in the assumption that the TT objectively exists in the real world. From the implications of the non-existence assumption of TT, LMA under WSL is a special case regarding this assumption.

5. UTTL Principle-Based Practical Solutions for LDS

Regarding the definition of UTTL (Formula (5)) and its principles (Theorems 1-4) for revealing undefinable TT, in this section, we summarize UTTL principle-based practical solutions for LDS. Under the frame work of UTTL, the summarized solutions, which are based on AI application works in specific domain^{[20][28][30][32][69]} as well as representative works of learning from multiple annotators^{[14][15]}, detail the implementations for data preparation with expert democracy and learning from data prepared under expert democracy. As the hypotheses introduced for the proofs of Theorems 1-4 are natural, they can be typically satisfied in practice. Even if some of these hypotheses are not fully met, the conclusions of Theorems 1-4 would merely be weakened rather than invalidated. Therefore, in this section, we do not further specifically verify these natural hypotheses when we applying Theorems 1-4 for specific implementations.

5.1. Implementation for data preparation with expert democracy

Based on existing works^{[20][28][30][32][14][15][69]}, regarding the definition of UTTL and Theorems 1-3, we summarize the detailed implementation for data preparation with expert democracy in practice.

5.1.1. Domain expert labelling

The expertise of domain experts can be described as an accumulated knowledge (K) containing various prior proofed knowledge facts (k) about the undefinable TT. The K is regarded as the currently most appropriate information for approximating the undefinable TT, though it probably will be constantly changed with new accumulated domain knowledge. Formally, the K is expressed as

$$K = \{k_1, \dots, k_m\} \quad s.t. \quad K \subseteq prop(TT). \quad (6)$$

Thus, regarding Theorem 1, Formula (1) for labelling the entities/events (d) collected from the domain is implemented by using the K as hyperparameter

$$t_{DE}^* = DE_Label(d; \theta^{DE_Label} = K') = K(d) \quad s.t. \quad prop(t_{DE}^*) \subseteq K \subseteq prop(TT). \quad (7)$$

5.1.2. AI expert refinement

One expertise of AI experts can be described as searching a reasoning path (r) from the domain expert prepared data $\{d, t_{DE}^*\}$ under the condition of K to draw (\rightarrow) a set of logical conclusions (c) that are consistent with (\simeq) some knowledge facts in K . The drawn c can be helpful to refine t_{DE}^* for representing the underlying TT. Formally, this expertise of AI experts is defined as

$$\tilde{r} = \underset{r \in R}{searching} r : \langle \{d, t_{DE}^*\} | K \rangle \rightarrow c \quad s.t. \quad c \simeq K. \quad (8)$$

Here, R denotes the space of the reasoning path r .

Another expertise of AI experts can be described as building a program (p) to generate new TT representation (t_{AIE}^*) from $\{d, t_{DE}^*\}$ and logical conclusions c . The properties of the generated t_{AIE}^* should be equal ($=$) to c for describing the underlying TT. This expertise of AI experts is formally defined as

$$\tilde{p} = \underset{p \in P}{building} p : (\{d, t_{DE}^*\}, c) \Rightarrow t_{AIE}^* \quad s.t. \quad prop(t_{AIE}^*) = c. \quad (9)$$

Here, P denotes the space of the program p .

Thus, regarding Theorem 2, Formula (2) is implemented by using the $\{\tilde{r}|K, \tilde{p}\}$ as hyperparameter

$$\begin{aligned} t_{AIE}^* &= AIE_Refine(d, t_{DE}^*; \theta^{AIE_Refine} = \{\tilde{r}|K', \tilde{p}'\}) \\ &= \tilde{p}(\{d, t_{DE}^*\}, c = \tilde{r} < \{d, t_{DE}^*\} | K >) \quad s.t. \quad prop(t_{AIE}^*) \simeq K \end{aligned} \quad (10)$$

5.1.3. Fusion of expert democracy

Logical operations (LO) can be conducted on t_{DE}^* and t_{AIE}^* to form a unified TT presentation (t_{PwED}^*) to more effectively capture the properties of the undefinable TT. The properties of the t_{PwED}^* are expected to be more consistent with (\simeq) the knowledge facts in K .

Regarding Theorem 3, Formula (3) is implemented by using the LO as hyperparameter

$$\begin{aligned} t_{PwED}^* &= PwED_Fuse(t_{DE}^*, t_{AIE}^*; \theta^{PwED_Fuse} = LO') \\ &= LO(t_{DE}^*, t_{AIE}^*) = \{t_1^*, \dots, t_v^*\} \quad s.t. \quad prop(t_{PwED}^*) \simeq K. \end{aligned} \quad (11)$$

5.1.4. Detailed solution in practice

The result of domain expert labelling t_{DE}^* from Formula (7) and its associated d form an initial data basis, denoted as $H = \{d, t_{DE}^*\}$. The data structures of H are diverse. As observed in existing works^{[32][15][69]} (Yang et al. 2020, 2024b, a; Zhang et al. 2023a), the structures of H are primarily denoted as

$$H = \{d, t_{DE}^*\} = \left\{ \left(d_1, t_{1,DE}^* \right), \dots, \left(d_n, t_{n,DE}^* \right) \right\}, \quad (12)$$

$$\begin{aligned} H &= \{H_1 = \{d_1, t_{1,DE}^*\}, \dots, H_k = \{d_k, t_{k,DE}^*\}\} \\ &= \left\{ H_1 = \left\{ \left(d_{1,1}, t_{1,1,DE}^* \right), \dots, \left(d_{1,n_1}, t_{1,n_1,DE}^* \right) \right\}, \dots, H_k = \left\{ \left(d_{k,1}, t_{k,1,DE}^* \right), \dots, \left(d_{k,n_k}, t_{k,n_k,DE}^* \right) \right\} \right\}, \end{aligned} \quad (13)$$

$$\begin{aligned} H &= \left\{ d, \left\{ t_{1,DE}^*, \dots, t_{m,DE}^* \right\} \right\} \\ &= \left\{ \left(d_1, \left\{ t_{1,1,DE}^*, \dots, t_{1,m_1,DE}^* \right\} \right), \dots, \left(d_n, \left\{ t_{n,1,DE}^*, \dots, t_{n,m_n,DE}^* \right\} \right) \right\}. \end{aligned} \quad (14)$$

Formula (12) indicates that each instance of d has a domain expert labelled t_{DE}^* (Yang et al. 2020, 2024b). Formula (13) signifies that diverse data samples $H = \{H_1, \dots, H_k\}$ are labelled by domain experts and each data sample may capture partial properties of the underlying TT in the domain (Yang et al. 2024a). Formula (14) notifies that each instance of d has multiple domain expert labelled targets $t_{DE}^* = \{t_{1,DE}^*, \dots, t_{m,DE}^*\}$ ^{[32][15][69]} (Zhang et al. 2023a). Together, Formulas (12)-(14) form the practical solutions for Formula (7).

The result of Formula (8) is a found reasoning path \tilde{r} that can draw from H under the condition of K a set of logical conclusions that are helpful to refine the domain expert labelled t_{DE}^* . The process of \tilde{r} is denoted as

$$c = \tilde{r}(H, K) = \{c_1, \dots, c_w\}. \quad (15)$$

Referring to existing works (Yang et al. 2020, 2024b, a), Formula (15) is solved based on abductive reasoning as follows:

1. Extract a list of groundings from H that can describe the logical facts contained in t_{DE}^* . This grounding extraction (E) step is expressed as

$$g = E(H) = \{g_1, \dots, g_s\}. \quad (16)$$

2. Estimate the inconsistencies between the extracted groundings g and the prior knowledge accumulated in K by logical reasoning. This logical reasoning (R) step is expressed as

$$ic = R(g, K) = \{ic_1, \dots, ic_u\}. \quad (17)$$

3. Revise the groundings in g via logical abduction to reduce the estimated inconsistencies in ic . The revised groundings (rg), which serve as the hypothesis for generating the reduced inconsistencies in the logical abduction, are the drawn statements/conclusions (c) that can more accurately describe the underlying TT than the t_{DE}^* provided in H . Referring to Formula (13), this logical abduction (A) step is expressed as

$$c = rg = A(ic, g) = \{rg_1, \dots, rg_w\} = \{c_1, \dots, c_w\} \quad (18)$$

Together, Formulas (15)-(18) form the practical solution for Formula (8).

The result of Formula (9) is a built program \tilde{p} that generates new TT representation t_{AIE}^* from $\{d, t_{DE}^*\}$ and logical conclusions c . The process of \tilde{p} is expressed as

$$t_{AIE}^* = \tilde{p}(H, c). \quad (19)$$

Referring to existing works^{[20][28][30][69]}, Formula (19) is differently solved with regard to the structures of H as follows:

1. When $H = \{d, t_{DE}^*\}$ is available, with regard to c , we can develop programs that extract a new TT from H that can be complementary to t_{DE}^* for representing the underlying TT^{[20][28]}.
2. When $H = \{H_1 = \{d_1, t_{1,DE}^*\}, \dots, H_k = \{d_k, t_{k,DE}^*\}\}$ is available, with regard to c , we can develop programs that explore mutual enhancements between different data samples $\{H_1, \dots, H_k\}$ and result new TT for better representations of the underlying TT^[30].
3. When $H = \{d, \{t_{1,DE}^*, \dots, t_{m,DE}^*\}\}$ is available, with regard to c , we can develop programs that explore mutual enhancements between multiple domain expert labelling $\{t_{1,DE}^*, \dots, t_{m,DE}^*\}$ and result new TT for better representations of the underlying TT^{[32][69]}.

Together, these solutions for Formula (19) form the practical solutions for Formula (9).

Referring to existing works^{[20][28][30]}, for the implementation of Formula (11), logical operations like union and intersection can be employed to fuse t_{DE}^* and t_{AIE}^* into a unified TT presentation (t_{PwED}^*). While union operations make the unified t_{PwED}^* representation more complete, intersection operations enable the unified t_{PwED}^* representation more accurate.

5.2. Implementation for learning from data prepared under expert democracy

Based on existing works^{[70][71][72][73][20][28][30][32][74][75]}, regarding the definition of UTTL and Theorem 4, we summarize the detailed implementation for learning from the data prepared with expert democracy in practice.

5.2.1. Multiple inaccurate target learning

Based on a constructed function f , the error between $t = f(d)$ and $t_{PwED}^* = \{t_1^*, \dots, t_v^*\}$ can be estimated and a predefined loss function l . As t_{PwED}^* contains multiple types of inaccurate true targets, the error between t and $\{t_1^*, \dots, t_v^*\}$ can be estimated by the weighted sum of the errors between t and the respective t_v^* . We can design a loss function within a multiple inaccurate target learning procedure, which is expressed as

$$l(t, t_{PwED}^*) = \sum_{i=1}^v \alpha_i l(f(d), t_i^*) \quad s.t. \quad \sum_{i=1}^v \alpha_i = 1. \quad (20)$$

Here, α is the weights for the multiple inaccurate TT set $\{t_1^*, \dots, t_v^*\}$.

By minimizing $l(t, t_{PwED}^*)$ with regard to f , a final optimized model \tilde{f} can be obtained for mapping d into the revealed TT ($\tilde{t} = \tilde{f}(d)$), which should submit to the condition $prop(\tilde{t} = \tilde{f}(d)) \approx prop(t_{PwED}^*) \cong K$ by common logical sense.

Thus, regarding Theorem 4, Formula (4) is implemented by a multiple inaccurate target learning procedure

$$\begin{aligned} \tilde{f} = \arg \min_{f \in F} & \left[l(t, t_{PwED}^*) = \sum_{i=1}^v \alpha_i l(f(d), t_i^*) \right] \\ s.t. & \quad \sum_{i=1}^v \alpha_i = 1 \text{ and } prop(\tilde{t} = \tilde{f}(d)) \approx prop(t_{PwED}^*) \cong K. \end{aligned} \quad (21)$$

5.2.2. Detailed implementation in practice

The mapping function f can be constructed via state-of-the-art deep learning methods^[71] based on neural networks. The loss function l can be defined using cross-entropy for classification and least squares for regression^{[72][74][75]}. With regard to the deep learning based f , the minimization of $l(t, t_{PwED}^*)$ is solved via stochastic gradient descent variants^{[70][73]}.

Specifically, when cross-entropy loss is used for classification or least square loss is used for regression, the loss constructed by $l(t, t_{PwED}^*) = \sum_{i=1}^v \alpha_i l(t, t_i^*)$ can be theoretically expressed as $l(t, t_{PwED}^*) = l(t, \sum_{i=1}^v \alpha_i t_i^*) + c$, where c is a constant term^{[20][28][30]}. This indicates that the multiple inaccurate target learning procedure is able to

force the mapping model f reasonably to achieve logically rational predictions for undefinable targets by learning from the weighted summarization of multiple types of inaccurate targets.

5.3. Summary for implementing UTTL solutions

Regarding the definition of UTTL and its principles for revealing undefinable TT, we leverage detailed implementations of some existing works^{[20][28][30][32][14][15][69]} to detail the example implementations for data preparation with expert democracy and learning from data prepared under expert democracy under the framework of UTTL. Based on these implementations (Formula (6)-(21)), the example implementation of UTTL principle-based practical solutions (UTTL(S)) for LDS is abstractly summarized as

$$UTTL(S) \left\{ \begin{array}{l} \text{Input: } d \\ DP: \left\{ \begin{array}{l} t_{DE}^* = K(d), \quad prop(t_{DE}^*) \subseteq K \subseteq prop(TT) \\ t_{AIE}^* = \tilde{p}(\{d, t_{DE}^*\}, c = \tilde{r} < \{d, t_{DE}^*\} | K >), \quad prop(t_{AIE}^*) \simeq K \\ t_{PWED}^* = LO(t_{DE}^*, t_{AIE}^*) = \{t_1^*, \dots, t_v^*\}, \quad prop(t_{PWED}^*) \cong K \end{array} \right. \\ LP: \left\{ \begin{array}{l} \tilde{f} = \arg \min_{f \in F} [l(t, t_{PWED}^*) = \sum_{i=1}^v \alpha_i l(f(d), t_i^*)], \\ \sum_{i=1}^v \alpha_i = 1 \text{ and } prop(\tilde{t} = \tilde{f}(d)) \approx prop(t_{PWED}^*) \cong K \end{array} \right. \\ \text{Output: } \tilde{t} = \tilde{f}(d), \quad prop(\tilde{t}) \approx prop(t_{PWED}^*) \cong K \subseteq prop(TT) \end{array} \right. \quad (22)$$

Regarding Formula (6)-(21), Formula (22) for summarizing the example implementation of UTTL(S) in practice can be visually interpreted as Fig. 3.

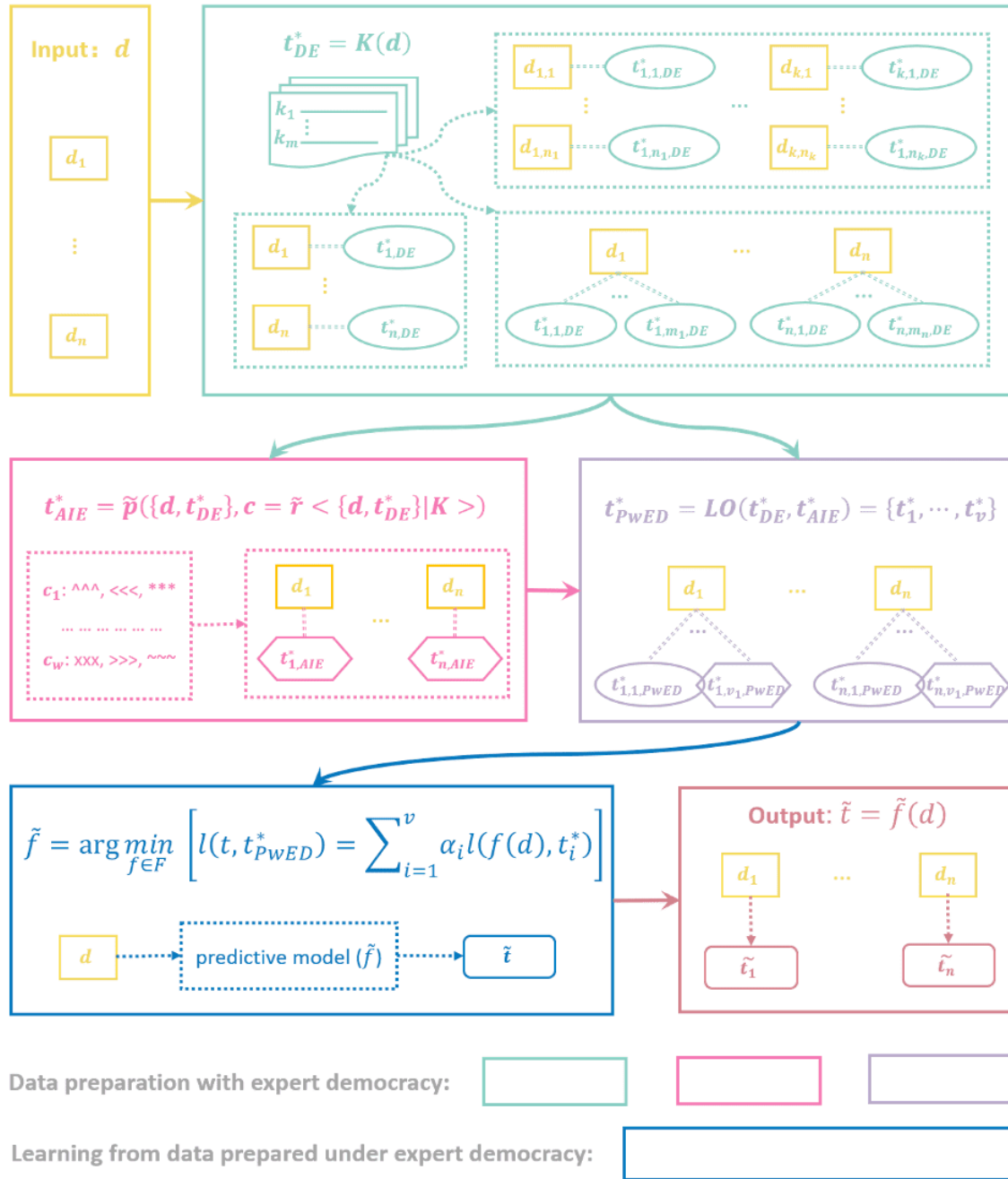


Figure 3. Visual interpretation of Formula (22) for summarizing the example implementation of UTTL(S) in practice.

5.4. Example UTTL(S) implementations in real world applications

Regarding the summary for implementing UTTL(S), more detailed discussions on example UTTL(S) implementations in real world applications, including helicobacter pylori segmentation^{[20][28]}, tumour segmentation for breast cancer^[30] and learning from multiple annotators in practice^{[32][69]}, are provided in Supplementary 2.

6. Discussion, conclusion and future work

Presenting a systematic review of TT assumptions across current ML paradigms, in this article, we explicitly posited the assumption that the TT does not exist in the real world and investigated the corresponding UTTL framework as a pathway towards LDS.

Our previous works on inherently ambiguous TT segmentation in pathological images^{[20][28][30][32]} inspired us to posit the non-existence assumption of TT, as they progressively revealed that, for tasks in which the TT is intrinsically indefinable, assuming an objectively existing TT is fundamentally inadequate. The novelty and necessity of this non-existence assumption of TT were confirmed by the insights gained from our systematic review of TT assumptions across current ML paradigms.

The implications of the non-existence assumption of TT and analyses on how this assumption may redefine our understanding of designing ML paradigms that transform underlying reality into predictive models suggested that: 1) new data preparation strategies grounded in expert democracy and appropriate learning paradigms with democratic supervision should be both explored; and 2) these two explorations could lead to an expanded scope towards LDS.

Thus, grounded in the assumption that the TT does not exist in the real world, we proposed UTTL to exemplify the essence of LDS. We established the definition of UTTL, illustrated its principles for revealing the undefinable TT, and discussed its practicability for LDS and its uniqueness compared with existing similar learning settings. Based on these, example UTTL principle-based solutions were also summarized regarding existing works^{[20][28][30][32][69]} to show the practical value of UTTL in enabling LDS.

In conclusion, this article offers a new ML paradigm (UTTL) towards LDS grounded in the non-existence assumption of TT.

The current formulation of UTTL for enabling LDS still exhibits several limitations. First, the LDS setting within UTTL is presently confined to the democracy of human experts. A natural question arises: Can the scope of LDS be extended beyond experts, for example, to non-experts or AI agents^[76]; and, if so, how should such an extension be rigorously defined and realized? Second, although the practical value of UTTL is partially demonstrated through example UTTL principle-based solutions that have been implemented in real world applications, the applicability boundary of UTTL in broader practical scenarios remains insufficiently understood. Addressing these limitations will require systematic future investigations, including theoretical generalization, methodological expansion, and more empirical studies.

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Conflicts of Interest

The authors declare that they have no conflict of interest.

Data Availability

We do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

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