

Review of: "Autonomous Second-Order ODEs: A Geometric Approach"

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From a geometrical perspective, differential equations can be viewed as manifolds that are best studied using the contact geometry of jet spaces.

In the case of autonomous second-order scalar ordinary differential equations (ODEs), the paper under review presents a way to associate a Riemannian metric with any such equation. Based on this, the paper suggests using Riemannian geometry as an additional tool to explore the properties of these ODEs.

However, a key issue with the approach taken in the paper is that the metric associated with an ODE of the considered type is not invariantly defined. Specifically, one can easily verify that the symmetries of an equation do not necessarily correspond to isometries of the associated Riemannian metric. For example, the Lie algebra of infinitesimal point symmetries of the equation $u_2 = u_1^2$ does not form a subalgebra of the Killing algebra of the corresponding metric.

I believe that for a geometric approach that could also allow a correct use of Riemannian geometry, it is necessary to remove the issue described above.