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Research Article

Is the Observational Dark Energy Universe Completely a Coincidence?

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In this article, we propose a new cosmological model called 'fractal cosmology' based on two postulates that gives a 'not really' answer to the question in the title: At any epoch of the universe, for an arbitrary local observer living well below the scale of Hubble horizon, the observational universe centered on this observer appears to be accelerated expanding. The anthropic principle is unnecessary for our current observation of an accelerated expanding universe. We will argue how such a story is qualitatively compatible with the CMB and low-redshift observations on the expansion history. Moreover, fractal cosmology implies four characteristic signals that could substantially distinguish it from the standard ΛCDM cosmology and a family of models alike: 1) Unlike the prediction in ΛCDM, in fractal cosmology, the local Hubble rate will be positively correlated with regional matter overdensities. 2) In a conventional expansion history data analysis of modern cosmology, effectively, dynamical dark energy will show phantom behavior. 3) Over-aged high-redshift astronomical objects/events will generally exist, where 'over-aged' specifically means that the astronomically (local physics) derived event age is longer than the ΛCDM predicted universe age at the event redshift. 4) Astronomical events with a characteristic time, for example the supernovae light curves, are subject to a growing characteristic time scattering (variance) with their redshifts, even after being modulated by the $(1+z)$ factor expected in standard cosmology; On the contrary, in for example ΛCDM, no known effect would lead to such a redshift-dependent trend of the characteristic time variance of the same type of events. Each of those four signals has either inconclusively shown some hints in recent observation, or is feasible to be tested with current and near-future available data.

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I. Introduction

When asked why we happen to be living in an epoch of the universe where the negative pressure dark energy is taking the majority, $\sim 70\%$ of the cosmic fluid, the answer is usually the anthropic principle[1], i.e. a civilized observer needs to be born in an environment where the dense structure of the universe is diluted by dark energy. However, many find this explanation not satisfying enough, for its arbitrariness and lack of testable further implications. In this article, I will provide an alternative solution to this question, which appears to be less human-centric, and point to observational implications that can be tested in the near future.

The article is organized as follows. In section 2, we will reexplain the Einstein equation as a conservation law of the 4D spacetime substance volume, which naturally incorporates a positive metric term. In section 3, we try to connect the obtained positive metric term to the observational 'dark energy', or to be more specific, the accelerated expansion reality of our universe. In section 4, we will compare the 'fractal cosmology' picture proposed in this article with other theoretical candidates of beyond standard cosmology, and point out four intriguing implications of the fractal cosmology that can be tested against current and near-future astrophysical/cosmological observations.

We will use $(-+++)$ signature in this article. Differential manifold notations mainly follow[2], and Raychaudhuri's equation derivaiton and results are taken from[3].

II. Einstein equation with positive metric term

In the original Einstein's equation Λ was an extra term added with no good explanation within classical general relativity and thus has no prediction on its value. Here, we will try to give an alternative interpretation of the Einstein equation that naturally suggests the presence of such a negative pressure term and always positive energy.

An overview of the story is as follows: we will show how Einstein equation can be explained as a differential version of the 4D spacetime substance volume conservation law. Such a conservation law is applied to the flow along a vector field $F \in \mathfrak{X}(\mathcal{P})$, where $\mathcal{P} = (-\epsilon, \epsilon) \times \Sigma$ is a 4D submanifold in the spacetime, i.e. a spacetime/cosmological patch. The variations (of the volume and vector fields) is defined in terms of the pullbacks of the diffeomorphism given by the flow of F.

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On the other hand, to define the distance on a pseudo-Riemannian manifold with signature $(-+++)$, we need a metric $g : T_pM \times T_pM \to \mathbb{R}$, which in Cartan's coframe formalism can be expressed as $g = -\alpha_t \wedge \alpha_t + \sum \alpha_i \wedge \alpha_i$, where α are 1-forms. Denoting the base vector field dual to the coframes as $X_t, X_i \in \mathfrak{X}(\mathcal{P})$, we have them orthonormal with each other, and locally spanning $T_p\mathcal{P}$. However, they could all be non-commutative with F, thus the Lie derivative L_F is non-vanishing in every direction.

The above rather mathematical description can be understood in the following physics interpretation: a patch of the universe as a 4D spacetime (sub)manifold is like a fluid cylinder with the 3D space as the cross-sectional area, 1D time duration as the thickness of the fluid cylinder, and the vector field F the flow velocity (we will see later what makes X_t a little bit more special, by choosing synchronous gauge). Imagine that a bug, or a human, flowing in the fluid, or spacetime substance, is trying to construct a metric so that it can measure the things happening around it. It is natural for the bug to take the flow direction as special and build a metric anchored to the flow velocity, but in principle, it is a choice of the bug itself. Besides, when flowing in the fluid without any reference outside the flow, it is impossible for it to know the true flow velocity, thus to construct the exact 'right' metric that has one of the coframes commuting with the flow vector field. See figure 1 for a schematic illustration.

When enforcing the 4D volume conservation law for the variation along the vector field F flow, where the 4D volume form is defined conventionally as $\alpha_t \wedge \alpha_1 \wedge \alpha_2 \wedge \alpha_3$, such a conservation law puts constraints on the metric. We will see that locally they take the form of the Einstein's equation with a metric term, and the coefficient for the metric term is always positive, but could be a general scalar field instead of a constant. Just like a fluid cylinder could be stretched or compressed along the direction of flow, so could the scale of time of a patch of the universe be stretched or compressed. This metric term coefficient is an outcome, thus reflects how much a spacetime patch is stretched or compressed along its flow.

In practical discussion of cosmological observations and data analysis, one usually works in a hypothetically homogeneous FLRW background metric. Effectively, the operation here is to take a 3D volume average over an equal-time hypersurface foliation for the Einstein's equation obtained in the previous step.

To begin with, we start from a 4D pseodo-Riemannian manifold $\mathcal{M}$, and a vector field $F \in \mathfrak{X}(\mathcal{M})$. The physical meaning of them are the spacetime manifold we are living in and the flow of the substance of the spacetime [1]. Now we take a hypersurface $\Sigma \subset \mathcal{M}$, which is compact, integral, well-behaved and nowhere tangent to F, i.e. $F_p \notin T_p\Sigma$ for all $p \in \Sigma$.

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The restriction of F on Σ, $F|_\Sigma$ can specify a local coordinate chart that $\frac{\partial}{\partial\tau} \sim F$. Denote the 4D manifold $\mathcal{P} = (-\epsilon, \epsilon) \times \Sigma$ constructed by vector field straightening that is diffeomorphic (smooth and invertible mapped) to a submanifold $S \in \mathcal{M}$ as a patch.

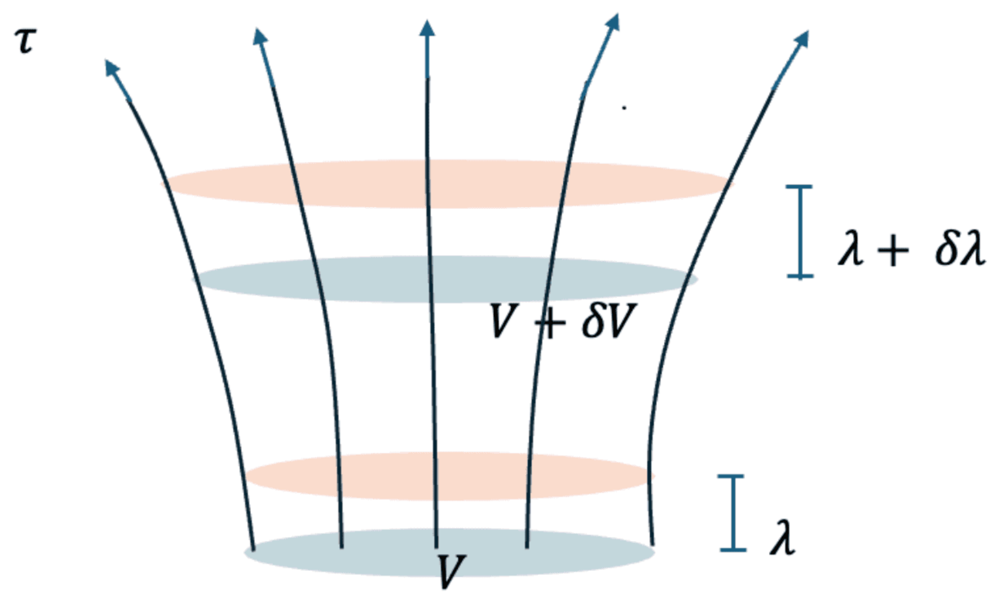

Figure 1. An illustration of time-like congruence. A 4D cylinder confined by red and blue hyperfurfaces λ away from each other has volume $U = V\lambda$, and the discussion in section 2 is focused on how the evolution along a congruence vary this 4D volume $\delta U = V\delta\lambda + \lambda\delta V$.

Now as a standard next step to do any physics involving distances and volumes on this spacetime patch, we need a metric $g : T_p\mathcal{P} \times T_p\mathcal{P} \to \mathbb{R}$. In general, it can be decomposed into a Cartan's coframe expression: $g = -\alpha_t \wedge \alpha_t + \sum \alpha_i \wedge \alpha_i$, with dual frame vector fields X_t, X_i defined by $\langle \alpha_a, X_b \rangle = \delta_{ab}$. In the most general case, $[\frac{\partial}{\partial\tau}, X_t]$ and $[\frac{\partial}{\partial\tau}, X_i]$ could be all non-vanishing. However, we have the freedom to choose synchronous gauge by requiring $[\frac{\partial}{\partial\tau}, X_t] \propto \frac{\partial}{\partial\tau}$, or even requiring $X_t \propto \frac{\partial}{\partial\tau}$. Note that there is nothing physical happening in this step, simply a gauge choice – by rotating the dual frame we are guaranteed to find such a X_a among the four, and we just need to call the coframe dual to it as α_t. Namely, the physical spacetime substance flow direction determines the time coordinate of our metric, but only the direction of it. There is no theory that can guarantee the point-to-point identification between the spacetime substance flow vector field and the time-like frame of the metric.

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A volume form corresponding to the metric is $\Gamma = \alpha_t \wedge \alpha_1 \wedge \alpha_2 \wedge \alpha_3$. Let us look into the variation of the element volume of submanifold $\mathcal{U} \in \mathcal{P}$ along the flow of $\frac{\partial}{\partial \tau}$.

Lie derivative on volume form Γ has the property:

$$L_{\frac{\partial}{\partial \tau}} \Gamma = (L_{\frac{\partial}{\partial \tau}} \alpha_t) \wedge \alpha_V + \alpha_t \wedge (L_{\frac{\partial}{\partial \tau}} \alpha_V) \tag{1}$$

where $\alpha_V = \alpha_1 \wedge \alpha_2 \wedge \alpha_3$ is a 3-form, the volume form of the spacial hypersurface.

Now we gradually transfer from the differential manifold language to the languages more familiar to the general relativistic physicists, so that we can use some of the well-established geometrical results and facilitate physical interpretation towards the end. We will denote the Lie derivative $L_{\frac{\partial}{\partial \tau}}$ as variation $\frac{\delta}{\delta \tau}$. We take the 4D volume of an element of the spacetime substance as $U = \lambda V$, where λ, V are element volumes on $\frac{\partial}{\partial \tau}$ direction 1D submanifold and on 3D submanifold Σ.

Based on the Lie derivative property acting on volume forms in equation (1), and the notation introduced above, we can write the variation of the 4D volume along the flow of $\frac{\partial}{\partial \tau}$ as:

$$\delta U = \lambda \delta V + V \delta \lambda. \tag{2}$$

Suppose The 4D volume of the spacetime substance is conserved along its flow:

$$\delta U + \delta Q = 0, \tag{3}$$

where δQ is the in/output of spacetime substance along the flow to the element volume we are studying:

$$\delta Q = T^{ab} \xi_a \xi_b V \lambda \delta \tau \tag{4}$$

T^{ab} is a current tensor of the spacetime substance, defined by equation (4), and ξ_a is the covariant notation of the Cartan's frame X_t. We hereby denote the other frames X_i as $\eta_a^{(i)}$.

It is well-known that the expansion θ of the cross-sectional area of a flow can be obtained by Raychaudhuri's equation. In our scenario, the variation of expansion gives us the variation of the cross-sectional area of the spacetime substance flow, i.e. the 3D hypersurface volume by $\delta V = V \delta \theta$. We need to find the similar for $\delta \lambda$.

Since the volume forms are defined from the frames $\eta_a^{(i)}$ and ξ_a, following the same argument in Wald's 9.2[3], $B_{ab} = \nabla_a \xi_b$ contains the information of the expansion in time and space directions. In the textbook, it is assumed that the flow is along the geodesics, because in the conventional discussion, Einstein's equation thus the metric is given before applying Raychaudhuri's equation, unlike here the metric is a floating unknown tensor to be solved from the spacetime substance volume conservation rule.

Namely, ‘geodesics’ is unknown with unknown metric here, so the spacetime substance flow has to be general enough for any time-like vector field.

Thus, even in the convenient synchronous gauge, in general we still need to decompose B_{ab} into one more term than in the textbook:

$$B_{ab} = -\alpha\xi_a\xi_b + \frac{1}{3}\theta h_{ab} + \sigma_{ab} + \omega_{ab} \tag{5}$$

where $h_{ab} = g_{ab} + \xi_a\xi_b$ is the spacial metric, characterized by $h^{ab}\xi_a = 0$ for the time direction frame ξ_a.

Contracting the above equation with ξ_a, we get $\xi^a\nabla_a\xi_b = \alpha\xi_b$. Further contraction gives us the result $\xi^a\nabla_a(\xi^b\xi_b) = 2\alpha$. This result implies that the connection $\nabla_a g_{bc}$ does not vanish on arbitrary vector fields in our setup. When requiring α to be small, it nicely shows up only as the next-to-the-leading order term in the small value variation along the $\frac{\partial}{\partial\tau}$ flow. In the physical sense, it represents to what extent a bug in the flowing fluid described previously fails to calibrate its metric instantaneously to the flow velocity to cancel the stretching/compression of the scale along the flow.

Because in our initial setup $\mathcal{P}$ is obtained by the vector field straightening, the foliation structure safely available. We can thus further require the flow direction $\frac{\partial}{\partial\tau}$ to be orthogonal to Σ, and this will put constraints on the metric (and connections) through Frobenius’s theorem[3], that the antisymmetric term $\omega_{ab} = 0$ vanishes.

Recall that we chose the gauge $X_t \propto \frac{\partial}{\partial\tau}$. Thus the variation along the spacetime substance flow $F \sim \frac{\partial}{\partial t}$ is proportional to

$$\frac{\delta}{\delta\tau}B_{ab} \propto \xi^c\nabla_c B_{ab} = \xi^c\nabla_c\nabla_a\xi_b \tag{6}$$
$$= \xi^c\nabla_a\nabla_c\xi_b + R_{cab}{}^d\xi^c\xi_d \tag{7}$$
$$= \nabla_a(\xi^c\nabla_c\xi_b) - (\nabla_a\xi^c)(\nabla_c\xi_b) + R_{cab}{}^d\xi^c\xi_d \tag{8}$$
$$= \xi_b\nabla_a\alpha + \alpha B_{ab} - B_a{}^c B_{cb} + R_{cab}{}^d\xi^c\xi_d \tag{9}$$

Contracting equation (6, 9) with h^{ab}, we can get the famous Raychaudhuri’s equation:

$$\xi^a\nabla_a\theta = \alpha\theta - \frac{1}{3}\theta^2 - \sigma^{ab}\sigma_{ab} - R_{ab}\xi^a\xi^b + R_{cabd}\xi^c\xi^d\xi^a\xi^b \tag{10}$$

Because R_{cabd} has antisymmetry, the last term goes to zero.

Contracting equation (6) with $\xi^a\xi^b$,

$$\xi^a\xi^b\xi^c\nabla_c\nabla_a\xi_b = -\xi^a\xi^c(\nabla_c\xi^b)(\nabla_a\xi_b) = -\xi^a\xi^c B_c{}^b B_{ab} \tag{11}$$

which cancels the B^2 term in equation (9), thus

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$$\xi^a\xi^b\xi_b\nabla_a\alpha = -\alpha B_{ab}\xi^a\xi^b \tag{12}$$

$$\xi^a\nabla_a\alpha = -\alpha^2 \tag{13}$$

Equation (10) and (13) are proportional to $\frac{\delta\theta}{\delta\tau}$ and $\frac{\delta\lambda}{\delta\tau}$ with the same factor $\frac{\delta t}{\delta\tau}$, and as the variation of the expansion factor on the frame, they are related to the 3D spacial and 1D time-dimension volume variations through $\delta V = V\delta\theta$ and $\delta\lambda = \lambda\delta\alpha$.

$\delta\alpha = -\alpha^2\delta\tau$ is easy to see from equation (13). For $\delta\theta$, the first term on the right-hand side of equation (10) gives an exponentially diverging or decaying mode. Dropping the second-order terms, we get:

$$\delta\theta = -R_{ab}\xi^a\xi^b\delta\tau \tag{14}$$

Those dropped terms contribute partially to the 'back-reaction' term in Buchert's gauge[4]. A cosmological model, timescape cosmology, that diverged from Buchert's discussion on the backreaction term ressembles the fractal cosmology diverging from the 4D spacetime substance volume conservation theorem in this article in many aspects, and we will discuss them in the section 4.

Substituting those variations of the volumes back to equation (3), we get:

$$-\lambda V R_{ab}\xi^a\xi^b\delta\tau - \lambda V\alpha^2\delta\tau + T^{ab}\xi_a\xi_b V\lambda\delta\tau = 0 \tag{15}$$

Note that $-1 = g^{ab}\xi_a\xi_b$ for the time-like frame vector. Although we have noticed that this normalization varies along the $\frac{\partial}{\partial\tau}$ flow, in a short period of time along the flow, i.e. in the scenario of calculating the variation δ, we can still use the normalization of $-1 = g^{ab}\xi_a\xi_b$ as an approximation at the leading order. Thus for arbitrary time-like frame vector ξ_a, the scalar equation (15) gives rise to the covariant tensor equation:

$$T^{ab} = R^{ab} - \alpha^2 g^{ab} \tag{16}$$

It looks like the Einstein equation that we are familiar with, but not exactly. We have made no statement about the spacetime substance current tensor T^{ab} by far, and it needs a little bit of dressing to be connected to the energy-momentum tensor. As a derivation from the skew-symmetry property of general volume forms, Bianchi's identity holds for the Ricci curvature in equation (16) regardless of the slight drifting of the metric $\nabla_a g_{bc} \neq 0$ mentioned before. According to the Bianchi identity, T^{ab} is not conserved on $\mathcal{U}$:

$$\nabla_a T^a{}_b = \nabla_a R^a{}_b - 2\alpha\nabla_b\alpha \tag{18}$$

$$= \nabla_b R - 2\alpha\nabla_b\alpha \tag{19}$$

Rewriting the T^{ab} in equation (16) into $\tilde{T}^{ab}$, where

$$\tilde{T}^{ab} \equiv T^{ab} - \frac{1}{2}Tg^{ab}, \tag{19}$$

we get

$$R^{ab} - \frac{1}{2}Rg^{ab} + \alpha^2 g^{ab} = \tilde{T}^{ab} \tag{20}$$

When $\alpha = 0$, this equation takes the exact form of the Einstein's equation. Up to this point, we have 'derived' the Einstein's equation, at least something taking its form, from only two postulates.

1. **Our physics lives on a 4D pseudo-Riemannian manifold. The flow of 4D spacetime substance can be described by an arbitrary vector field living on this manifold.**
2. **The 4D spacetime substance volume is conserved along its flow.**

Metric, thus the corresponding volume's definition has the basic features as in the differential manifold context, to ensure the general assumptions of a well-behaved physics system, such as smoothness and local Euclidean. Some of the conventional thoughts in vanilla general relativity, such as the absolutely non-drifting metric and geodesics taken for granted before-hand, has to be loosened. Lastly, the perturbative expansion in the above derivation assumes the variation of the volumes, shears, distortions, and curvature are small quantities that can be Taylor expanded.

Now we try to interpret the physics out of the equation (20) whose derivation is highly just geometrical. By comparing the equation (20) with the original Einstein's equation, it seems that $\tilde{T}^{ab}$ takes the role of an energy-momentum tensor. Taking the dual of $\tilde{T}^{ab}$, it goes back to $T^{ab} = \tilde{T}^{ab} - \frac{1}{2}\tilde{T}g^{ab}$. Recall, that the current tensor T^{ab} was originally introduced in this article in equation (4), to describe the in/output of the spacetime substance to the element volume we are studying. Combining all those intriguing hints, we can conclude a new perspetive on the concept of 'matter' that has been standing in the center of physics research in the past thousands of years:

The concept 'Matter' in the physics world is the current of spacetime substance that we subconsciously identify with the energy conservation. Our early infancy (3-5 months) cognitive development of the 'object permanence'[5] automated this process.

One important reason that we can make such a statement is that equation (16) to (20) only takes a rewrite to separate some of the degrees of freedom, and the two equations are mathematically equivalent. Moreover, the reason that such a cognitive strategy is developed so early and so widely among different species (for example, cats[6]) is probably because equation (20) with vanishing $\delta\alpha$ applies in almost every earthly scenario, thus becoming a ubiquitous feature trained out from evolution of the neural systems.

To see why $\delta\alpha$ is negligible in any earthly scenarios but could play a non-negligible cosmological role, we can use the earlier introduced analogy of the spacetime substance flow and a regular fluid flow, for example, the water flowing in a riverbed. The α term regulates the rescaling of the whole frame, thus the metric, due to the fluid cylinder thickness stretching/compressing along the flow. Under the volume conservation law, such an effect is only significant when the cross-sectional area variation is significant. In the flowing river case, that corresponds to flowing from a branch to a mainstream or the inverse. In the flowing spacetime substance case, that corresponds to the scenario where the variation of the expansion θ, which on the first order is proportional to the Ricci curvature R^{ab}, becomes significant. The natural physics environment on the earth is known to be extremely gravitationally weak, in contrast to the strong (in natural units) gravity case that only becomes relevant in astronomical and cosmological discussions.

Back to equation (20), the first interesting feature we can see is that the metric factor $\alpha^2(x)$ is always positive. In the notion here we stressed that $\alpha(x)$ could be an arbitrary function of the 4D spacetime coordinate, because up to this point, all our discussion happens on an element volume on a patch $\mathcal{P}$ on the spacetime manifold $\mathcal{M}$. We will discuss the space and phase space average, which is more relevant for a practical observational universe scenario, in the next section. In any case, one can see the similarity and difference between $\alpha^2(x)g^{ab}$ term and the Einstein's constant term Λg^{ab}: the coefficient for the former is a scalar function, while the later is a constant; Thus, the former spoils the conservation of energy-momentum tensor by $\nabla_a \tilde{T}^a{}_b = 2\alpha\nabla_b\alpha$, while the later nicely respects the conservation of energy-momentum tensor $\nabla_a \tilde{T}^a{}_b = 0$ – one of the reasons for it to be introduced by Einstein originally; Lastly, the former coefficient $\alpha^2(x)$ is always positive, while the later Λ could be positive, zero, or negative, corresponding to de Sitter, flat, and Anti de Sitter universe.

In the past decades, it has been almost certain that our observational universe is de Sitter, i.e. when testing expansion history data in the framework of cosmological constant, we get a positive Λ. So the automatic positivity of the metric term in equation (20) is quite encouraging. But the breakdown of energy-momentum conservation is not so welcoming, although we briefly discussed before how this rule could originate from the cognitive adaption for the earth environment instead of a more fundamental physics rule. In the theory framework in this article, the conservation of energy momentum tensor is only a special case secondary result of equation (20) when α is approximately constant throughout the physics system in the question, and a consequence of contracted Bianchi identity. Thus any physical consequence of the breakdown of energy-momentum conservation is only expected to show up in scenarios like the expansion history of the universe or a strong gravity environment such as near a black hole horizon, where a non-negligible gradient of α presents on either extremely large-scale or strongly curved spacetime patches. The violation of energy momentum conservation in those regimes is in general irrelevant with, and thus does not ruin, the gravitational dynamics of Newtonian systems.

In an idealized case, let us imagine what will happen if a spacetime patch $\mathcal{P}$ is not 'small' but can flow to large τ, even asymptotic infinity, in $\frac{\partial}{\partial\tau}$ direction. When approximating $\frac{\delta t}{\delta\tau}$ with a constant, the integral of equation (13) tells us $\alpha \sim \frac{1}{\tau}$, which goes to zero as $\tau \to 0$. Even taking $\frac{\delta t}{\delta\tau}$ variation into account, as long as it does not change sign, the trend of vanishing α with $\tau \to \infty$ would still apply. So it seems that after long-enough time, the congruence of geodesics converges to the spacetime substance flow, as we expected. On the other hand, if we regard the integrated τ as the lifetime of a patch of observational universe, under the approximately constant assumption of $\frac{\delta t}{\delta\tau}$, equation (20) together with $\alpha \sim \frac{1}{\tau}$ suggests that $\sqrt{\Lambda} \sim |\alpha| \sim \frac{1}{\tau}$, just like what we have found out about our own observational universe.

Indeed, if one has not noticed this, the widely cited values of cosmological constant Λ, Hubble constant, universe lifetime, and many other ways of reformulating the first (zeroth) order expansion rate of the universe, are all roughly the same degree of freedom extracted from redshifts and distances data. All the attempts trying to jump out of this box to give an alternative quantitative description of the above physics, for example, the calculation of Λ interpreting it as the vacuum energy in QFT, has been a failure. The argument in this section is another bold and rare endeavor in the literature to reason why $\Lambda \sim H_0$ might not be a coincidence. We will give a completely new, independent estimator of the expansion rate, or approximately Λ, from the time-domain astronomical observation in section 4.

Before we conclude this section, it is worth noting the two postulates here are much motivated by the thermodynamics explanation of the Einstein's equation by Ted Jacobson[7]. Instead of looking into the black hole case where one of the space dimensions is highly compressed, here the subjects are the less-special, well-behaved 4D spacetime submanifolds, and the conservation of energy $dQ = TdS$ in[7] is substituted by the conservation of 4D volume proposition $dU + dQ = 0$. The fundamental arguments are quite the same, that the Einstein's equation is describing how the spacetime distortion is driven by the flow of thermal energy/4D volume current tensor, under the constraint of energy/volume being a conservation law.

III. Cosmological effect

Now that we have Einstein's equation with a positive metric term, we want to see if we can connect it with the observational 'dark energy'.

In the field of observational cosmology, dark energy has been a placeholder for the unexplained fact that we measure an acceleratingly expanding universe around us. The astrophysical objects at distances far enough to be in the 'Hubble flow' run away from us with 'increasing speed'. Such an accelerated expansion reality is fairly homogeneous, and the negative pressure portion of the energy density of the cosmic fluid has an equation of state very close to $w \equiv \frac{p}{\rho} \sim -1$[8][9][10]. Those are about the uncontroversial part of what we know of the observational dark energy so far.

Dark energy has no observed perturbative effects so far, most of the time, it is only discussed on the background level, in the Friedmann equations. We will focus on the background cosmology in this article.

Let us denote the average over the space as $\bar{x}$ and the expectation value over the full phase space as $\langle x \rangle$. The two Friedmann's equations are the time and space components of the Einstein's equation in a space-averaged gauge:

$$\langle \bar{R} \rangle^{ab} - \frac{1}{2} \langle R \rangle g^{ab} + \langle \bar{\alpha}^2 \rangle g^{ab} = \langle \bar{\tilde{T}} \rangle^{ab} \tag{21}$$

Assuming that our observable universe patch has evolved 'long enough' time, that the geodesics are saturated to the averaged mainstream of the spacetime substance on the patch, the leading background order $\langle \bar{\alpha} \rangle$ vanishes. This could be regarded as a default calibration of any small (much below Hubble-scale) observer in the universe, that their frames have always been tuned to the spacetime substance flow on their cosmological (Hubble scale) patch.

Hence the metric term in equation (21) $\langle \bar{\alpha}^2 \rangle g^{ab}$ is effectively $\sigma^2(\bar{\alpha}) g^{ab}$, where the variance of α, $\sigma^2(\bar{\alpha}) = \langle \bar{\alpha}^2 \rangle - \langle \bar{\alpha} \rangle^2$. Comparing equation (21) with the original Einstein's equation with a cosmological constant:

$$R^{ab} - \frac{1}{2} R g^{ab} + \Lambda g^{ab} = \tilde{T}^{ab}, \tag{22}$$

we see that $\sigma^2(\bar{\alpha}) g^{ab} \sim \Lambda g^{ab}$, but with a possibly spacetime-dependent coeffecient over extremely large scales, thus breakdown of energy momentum conservation in the cases discussed before.

Now the mystery remaining to accommodate the theoretical story in the observational reality is the incredibly stable scaling of $\sigma^2(\bar{\alpha})$ with the scale factor a and its homogeneity.

We give a guess on the zeroth-order scaling of the variance of spacetime substance flow speed matching the background order expansion history of the universe out of the physical dimension analysis. In natural unit, we denote the dimension of energy, spacial distance, and time distance as $[\epsilon] = 1, [d] = 1, [t] = 1$. The physical dimension of the Einstein's equation is -2, so is the cosmological constant $[\Lambda] = -2$, and they are consistent with $[\alpha] = -1$ from its definition as the time-derivative of a dimensionless geometric property.

We know that the physical meaning of α is the stretching of time frame along the spacetime substance flow direction, and we discussed how this effect should be roughly the order of the spacetime curvature. Thus, in a matter dominated universe, without resolving the details of the dynamics happening at smaller scales, from physical dimension analysis we deduce that

$$\sigma^2(\bar{\alpha}) \approx d_F \bar{\rho}_m \tag{23}$$

The dimension of the above equation is -2, where d_F should be some dimensionless universal constant that does not care about the detailed spacetime substance dynamics happening on a patch, as we have already operated the phase space average in the calculation of $\sigma(\alpha)$.

A perfect candidate for d_F coefficient is the fractal dimension of the Poission-like distribution of the matter in our universe, which has a measured value of 2.4 as shown in[11]. It is dimensionless, local dynamics-insensitive, and manifests how an isotropic physical quantity is equally partitioned in equivalent physical dimensions.

Next, let us check if this guess made out of physical dimension analysis works quantitatively. In our local universe we have the measured values on $\Omega_\Lambda \approx 0.7$, $\Omega_m \approx 0.3$ and $d_F \approx 2.4$. It so happens that these numbers nicely fit into equation (23) that $0.7 \approx 2.4 \times 0.3.$ [2].

Equation (23) seems fine for low-redshift area $z <\sim 1$ in the sense that it does not drastically disobey any observational facts. But trouble shows up when we consider not just low redshift, but high redshift expansion history. If equation (23) applies for the spacial averaged matter density regardless of the scale, deep into high redshifts, then we will not get the expansion history confirmed by the current data, especially the CMB power spectrum.

The amendment here is to add constraints on the regime where the proportionality between average matter density and $\sigma^2(\bar{\alpha})$ holds. The variance $\sigma^2(\bar{\alpha})$ only keeps track of the average matter density up to a Hubble scale patch, because the physics beyond this scale are causally disconnected.

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Assume that the distribution of α are uncorrelated beyond Hubble scale, then we can regard the calculation of the variance of $\bar{\alpha}$ in a large region consisting of several Hubble sized patches as carrying out redraws on a same distribution, thus suffers a suppression by factor $1/N$. For example in figure 2, on a shell of $\chi(a)$ away from us, there are number of $N = V_{\text{shell}}/V_{\text{Hubble}}$ causaly disconnected patches that follow roughly the same distribution of $\bar{\alpha}$ in their local Hubble volume [3]. Hence the variance $\sigma^2(\bar{\alpha})$ is suppressed by $1/N$.

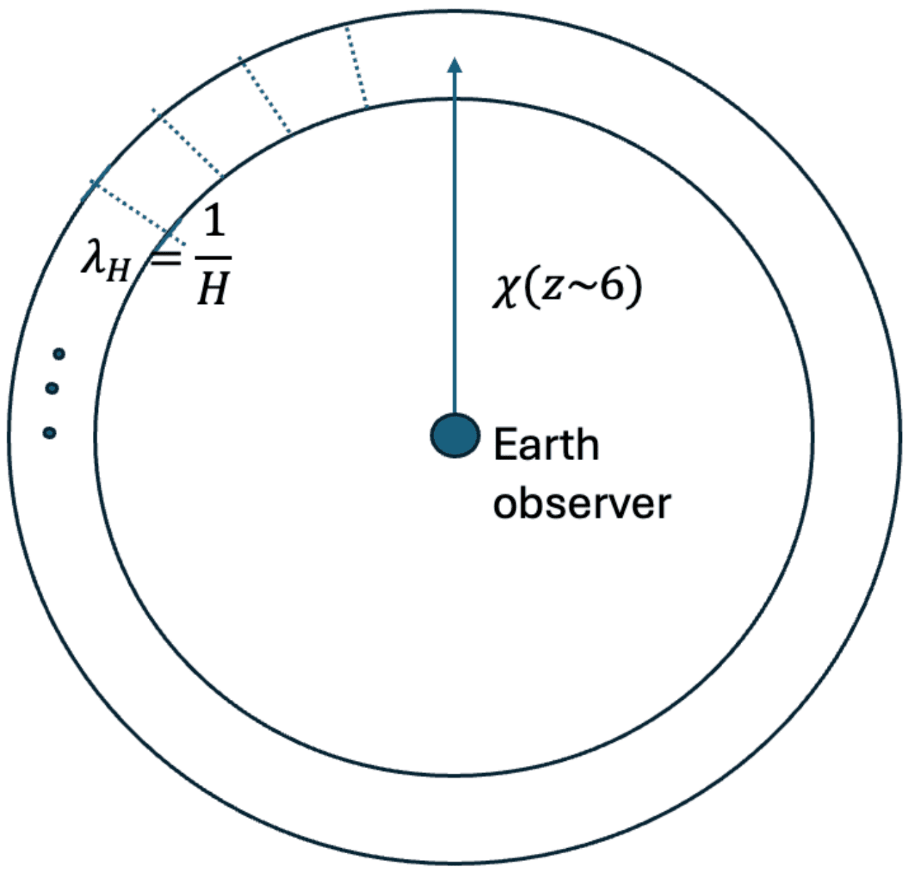

Figure 2. All the currently observable galaxies around certain redshift, say $z \sim 6$, reside on a shell χ away from us. The Hubble horizon $\lambda_H = 1/H$ at that epoch (in our chronicle) is much smaller than us, thus such a shell accommodates many Hubble patches.

In the regime of $\chi(a) >> 1/H(a)$ where the above approximations applies, we have:

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$$\Lambda(a) \equiv \sigma^2(\bar{\alpha}) \approx \frac{\sigma^2(\alpha)}{N} \tag{24}$$

$$\approx \frac{d_F \rho_m(a)}{V_{\text{shell}}(\chi(a))/V_{\text{Hubble}}(a)} \tag{25}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{4\pi\chi^2\lambda_H/(4\pi/3\lambda_H^3)} \tag{26}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{3\chi^2/\lambda_H^2} \tag{27}$$

We can solve for the evolution of dark energy density by taking derivative of the integral equation (27) up to $N = \chi/\lambda_H = 10$, corresponding to a = 0.2 thus redshift z = 4 in a universe with $\Omega_\Lambda = 0.7$. Denoting $X(a) = \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$, from equation (27) we get

$$X'(a) = 2\sqrt{3}\frac{X^{3/2}(a)}{a^{1/2}} - 3\frac{X(a)}{a} - 2E'(a)E^{-1}(a)X(a) \tag{28}$$

where $E(a) = H(a)/H_0 = \sqrt{\Omega_\Lambda(a) + \Omega_m a^{-3}}$, and we used the relationships $\chi = \int_a^1 \frac{1}{a'^2 H(a)} da'$ and $\lambda_H = 1/H$.

On the other hand, in the regime $\chi(a) << \lambda_H$, roughly $z < 0.1$, $\Lambda(a)$ in our Hubble volume is expected to saturate as in equation (23). The intermediate regime $0.1 < z < 4.0$ needs more dedicated modeling, which we leave for future work.

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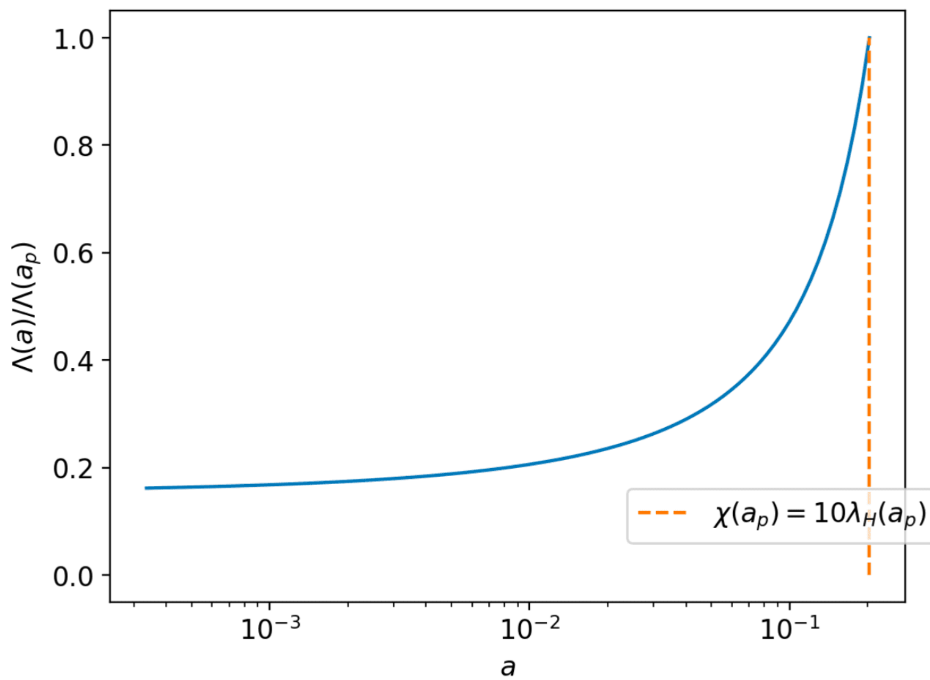

Figure 3. Dark energy density, or the variance of time-flow $\sigma^2(\bar{\alpha})$ as a function of scale factor a. In the regime where far-field approximation holds, $\chi >> \lambda_H$, dark energy density is suppressed by $1/N$ factor towards high redshift, as required by the CMB observation.

Figure 3 shows the density of dark energy evolution in the far field $z > 4$ by solving the ordinary differential equation (28). Reassuringly, it is not scaling up as a^{-3} with the matter density, instead decreasing to a smaller platform, thus agreeing with the observation that the dark energy was subdominant in the early universe. The decreasing rate varies with the initial guess of $\Lambda(a_p)$, but the trend stays stable with reasonable trial values that confine $X(a_p)$ between 0 and 1. In a sentence, it seems regardless of the initial/boundary fraction of dark energy at far-field redshift a_p, a general case is that the $1/N$ suppression dominates thus diluting the metric term $\sigma^2(\bar{\alpha})g^{ab}$ out when we look out toward smaller scale factor a.

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IV. Discussions

A. Fractal Cosmology and Comparison with Other Cosmological Models

One implication of the explanation of the negative pressure dominated universe using the variance of the spacetime substance flow is, that our 'currently' acceleratedly expanding universe is not special in space and time of the universe. Anthropic principle is not needed in the picture implied by the theorems in section 2, which we hereby name as 'Fractal Cosmology', because at any redshift, an observer living well below the Hubble horizon scale would see an acceleratedly expanding universe around them. Inside a smaller Hubble patch at higher redshift, resides another acceleratedly expanding universe, and when the residents in that Hubble patch look outwards in a universe centered on themselves, they would see a similar 'history' of the evolving universe as us. When we received signals from their cosmological patch, we only took random snapshots on what have happened and what would happen on that patch of spacetime with no chronicle. Regardless of all the exotic theorems in this article, one should think twice on the statement of 'we are reconstructing the history of our universe by tracing down to those high-redshift objects', because those objects that we are observing now are light-like connected to us in spacetime, not time-like. That means none of those objects that we are observing today will evolve to the current same-time hypersurface in any frame transformation. Apparently, an absolute chronicle of the universe since 'The Big Bang' loses its meaning in this picture, when radially tracing far enough along light-like curves every observer reaches their own 'Big Bangs', and the scale factor could play the role of time coordinate for any local (below Hubble size) observer in the universe. Forward-time, or the flow direction of the spacetime substance, might be a concept as trivial as downward-direction of the universe. When zooming out to large-enough scale, the universe manifests itself as a series of indefinitely unfolding self-similar structures at hierarchical scales, only pivoted at a local observer's scale when a chronicle story needs to be told.

The idea of such a fractal cosmology conceptually inherits some genes of the steady state theory of cosmology, in that they both imply that the universe does not have a global beginning or ending, and looks quite the same at any epoch. However, unlike the steady state theory, which stresses the unchanging of the universe over **time**, the fractal cosmology stresses the self-similarity of the universe over **spacetime**, which is a more radical application of the postulates of relativity. On the other hand, one might have noticed that fractal cosmology echoes many aspects of the conformal cyclic cosmology (CCC) [12][13], especially the smooth joint of the conformal infinity of one patch and the Big Bang of another. The additional spice in fractal cosmology comparing to CCC is its suggestion of the cosmological patch-wise dissynchronization, which leads to potentially more timely observational tests that will be discussed in section 4.2.

The property of $\sigma^2(\bar{\alpha}) \sim \Lambda$ tracking the value of background average matter density in equation (23) is similar to the proposal in ever-present Lambda models[14][15]. However, like we have discussed, such a positive correlation between these two physical quantities has to be valid only in a bound region or it will heavily violate the expansion history suggested by real data at high redshift. We gave a possible amendment to this problem in section 3.

Lastly, Timescape Cosmology[16] might be the cosmological model that shares the most common points with fractal cosmology in the current literature. It explicitly introduced the concept of 'volume-averaged time' and 'lapse', which is the multiplicative difference between the voids and walls area time frames. In its current development, it seems that the observational tests of the model still focuses on its effects on the expansion history, and a recent supernovae Hubble diagram data analysis did present a concrete constraint on its major parameter void fraction[17]. In the last subsection 4.2.4 of this article, we will discuss what kind of time-domain signals could provide another venue to probe the physics that differ this type of cosmology from other candidates.

We also want to note that, the Buchert's average gauge[4], the theoretical basis of the timescape cosmology, resembles the patch average view in this article in many ways, and their backreaction term might correspond to the dropped-off higher order geometrical variations in section 2.

B. Observational Tests

So far, most of the beyond standard cosmological models focus on their observational implications in the background level expansion history and perturbations on the density field, for example large-scale structure and CMB power spectrum. The danger of following such conventions in the analysis of beyond ΛCDM cosmology is that, we are playing with too few degrees of freedom in the data with too wide theoretical possibilities. Specifically, in recent discussions on the Hubble tension, most of the new models are just translating the same set of degrees of freedom contained in the Hubble diagram into different fancy-named theoretical parameters. A bigger problem is that, in the maze of transformation on the same set of numbers, we could lose track of the real input and output information of a theory, and make circular argument like the one recently spotted for the 'Hubble cut-off' in holographic dark energy model[18].

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So we will try to present a discussion on different, independent aspects in observations to scrutinize what kind of signals could be implied by the fractal cosmology model proposed in this article. In general, fractal cosmology unavoidably introduces variation in the Hubble parameter correlated with local overdensity and redshift. Besides, the most characteristic observational implication of fractal cosmology could be the dissynchronization of the observer-dependent cosmological time between different Hubble-sized patches.

1. The Positive Correlation Between Hubble Rate Variation and the Matter Overdensity

It is expected in ΛCDM that the perturbation in Hubble rate is negatively correlated with the perturbation of matter overdensity. It can be physically understood as a result of the standard gravitational theory, in the way that the mass particles in a void would be sucked away from the void center by the relative overdensity towards the outbound. As a result, the observational Hubble rate $H = \frac{\langle v \rangle}{\langle d \rangle}$ measured from the center of the void exceeds the overall average. There are multiple ways to semi-analytically derive such a negative correlation relationship at linear perturbation level of the standard Einstein gravity (with or without cosmological constant)[19], and such relationship has been validated by N-body simulation experiments repetitively in literature[20].

Hence, it is important to stress an unusual implication of equation (23), that the metric term, thus the main contributor to the accelerated expansion of the current universe, would be **positively** correlation with the regional averaged matter density in fractal cosmology. It is obviously different from the standard cosmology prediction described in the first paragraph.

Even though the negative correlation between Hubble perturbation and matter overdensity has been a consensus among the cosmologists, especially the N-body simulation experts, the observation confirmation of this statement in our real universe has been a blank space in the past decades. It is by no means an easy task to obtain trust-worthy measurements on the Hubble rate centered on a distant location, not to mention to reconstruct the matter overdensity field at a required precision to test this relationship. The Hubble rate centered on ourselves, the earth, has only came to the precision $\sim 10\%$ not so long ago.

Recently, a real data analysis on the Hubble rate variation and regional matter density variation has been carried out for the first time in the literature, using the density field reconstruction from BOSS DR12 and Supernovae Hubble rate measurement from Pantheon[21]. They have surprisingly found a **positive** correlation between local Hubble rate and the reconstructed matter density field. While such a counter-

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intuitive result still awaits for cross-validation from independent groups analysis, the model proposed in this article provides one of the possible theoretical explanations for such an unexpected phenomenon if it proves more concrete in the future.

2. Redshift Evolution of the Dark Energy-like Term

Note that the conventional expansion history data analysis involving any tracer and the combination of the reforms of their redshifts and distances is only directly probing the density-redshift dependence of a cosmic fluid. Its constraint on the equation of state w of the fluid is obtained from the assumption of energy-momentum conservation (Fluid Euler equation), which we have discussed why is fairly safely loosened in the story presented in this article. Since the Friedmann equation conventionally used to derive the prediction on the redshift-distance relationship is based on the tt component of the Einstein's equation, we can directly borrow the discussion on the metric term thus its tt component in section 3, and deduce three features of the fractal cosmology when considering under the framework of dynamical dark energy, which is the mainstream language in the field:

1. At low redshift, the metric term behaves much like the cosmological constant term.
2. When going to higher redshift, at some point, the analysis shown in figure 3 suggests that the phantom point ($w_{\rm de} = -1$) will be crossed. Here by 'phantom behavior', we mean that the 'dark energy' density will appear to be increasing with scale factor.
3. Combining the first two implications, when analyzing fractal cosmology in the w0wa cosmology framework, negative w_a is likely preferred.

The above perspectives agree with the recent results from DESI[22].

3. High-redshift Astronomical Objects with a History Longer Than Our Universe Lifetime

An important implication of the fractal cosmology is that the lifetime of the observational universe centered on a civilization living in a galaxy at, say $z = 6$, could be longer than 1Gyr, which is the number of the 'universe lifetime' at $z = 6$ calculated in our time frame assuming a ΛCDM cosmology. As mentioned in section 4.1, in the perspective of fractal cosmology, the light signals that we receive nowadays from high redshift objects are likely non-chronicle snapshots drawn from the whole history of their Hubble bubble, namely their observatinoal universe around their Hubble scale. The history of such a high redshift event could be longer than the universe lifetime calculated as in our time frame, because

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those astronomical events governed by baryonic physics follow the proper time of their local atomic clocks.

As a result, this effect gives longer accretion time for those high-redshift supermassive black holes, whose overabundance and overweight have been a concerning confusion in recent high redshift observations[23]. The discovery of many $> 10^9 M_\odot$ supermassive black holes (SMBH) above redshift $z > 6$ forcing astrophysicists to look for exotic mechanisms to allow super-Eddington accretion of the black holes, where Eddington limit is the accretion rate at which the radiation pressure force cancels the gravity. Even with a relatively heavy black hole seed $\sim 100 M_\odot$, the Eddington limit accretion needs at least ~ 0.8 Gyr to form a SMBH $\sim 10^9 M_\odot$, and the universe lifetime at redshift 6 based on Big Bang theory is just about enough. Many cosmological approaches to the problem rearrange the expansion history of ΛCDM; However, in the picture proposed by this article, an observer-dependent origin thus the lifetime of the universe could be an alternative cure.

Similar to the unexpected over development of the supermassive black holes, astrophysicists might find some of the high redshift galaxies behave older than theory predictions. In recent and upcoming high-redshift astrophysical surveys like JWST[24][25], those kinds of puzzling early-universe but highly-evolved galaxies have already been found, with arguably inconclusive significance. How galaxies have formed their stars and quenched their star formation at the stage so early of the universe have already triggered a wide discussion[26][27][28].

Although now still troubled by systematics and selection effects, those high-redshift galaxy and SMBH properties, especially the charts on their ages as theoretically predicted by astronomical and baryonic physics, will be crucial to test the implications pointed out in this article.

On the other hand, futuristic astronomical events could also be observed at high redshift as predicted by fractal cosmology. In the high redshift observations, we could potentially uncover the past **and future** of our own patch of the observational universe.

4. Astronomical Event Time Duration Variance Introduced by Dissynchronized Cosmological Clocks

An important new physics that distinguishes the fractal cosmology from the standard cosmology is the dissynchronization of the cosmological time between Hubble scale patches (Hubble bubbles). The standard cosmology implicitly assumes a global time frame regardless of the scales or coordinates in

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spacetime. However, in the picture of fractal cosmology, we have discussed how a Hubble sized patch at any position in the universe could harbor its own cosmological scale evolution history of the local universe. Intuitively, we would expect that the flow of time of the astronomical events happening on a cosmological patch would be anchored to the cosmological time on that patch. Again, one can think of a galaxy or a similar sized astronomical object in the spacetime substance bulk of a Hubble sized patch as a leaf flowing in a river. Those smaller sized objects could have peculiar spacetime substance flow, but on the leading order they should follow its local Hubble patch spacetime substance flow.

In practical observations, we are already equipped with the instruments that can study the objects at high redshifts with $z > 1$, i.e. objects deep into the Hubble flow and on other Hubble patches. If the dissynchronization between Hubble patches really exists, then it will be reflected in the time-domain signal of those high redshift astronomical events.

For example, let us consider the light curve of a supernova. Recently, time dilation has been observed in the supernovae light curves[29]. In the standard cosmology where the cosmological clock is synchronized throughout the whole universe, the time duration of the supernovae light curve is predicted to have a time dilation of $1 + z = \frac{T(z)}{T_0}$, out of a similar argument for the redshifts in textbooks. For the up to $z \sim 1$ sample in DES paper mentioned above, it seems that this prediction is confirmed by the observations.

There are caveats when one draws parallels between the redshift of photons and the time dilation of a macroscopic event. The former does not probe the local time frame, as the emission of photon as a microscopic quanta is instantaneous. The redshift of a photon can be derived from the (spacial) scale factor growth thus the stretching of the photon wavelength without any information needed on the local time frame. On the other hand, beginning and end of a supernovae event are time-like separated events and are sensitive to the dissynchroniazation of the local time frame for sure.

Let us start our discussion from the standard cosmology case. $1 + z = \frac{T(z)}{T_0}$ holds exactly when the clock of us, the observers, ticks at the same speed as the proper time of the source astronomical object. Taking into account the random fluctuation of the time duration of supernovae light curves due to different environments and other unknown astronomical reasons, measurements on the supernovae light curve time duration at redshift z could be denoted by:

$$T(z) = (1 + z)(\bar{T}_0 + \sigma_{\text{int}}) \tag{29}$$

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where $\bar{T}_0$ the average pivot value of the supernovae light curve time duration near $z = 0$, and $\sigma_{\rm int}$ denotes a normal-distributed uncertainty due to the intrinsic scattering.

If there is any dissynchronization between our, the observer's Hubble patch, and the source galaxy's Hubble patch that needs to be modeled, we can use a factor $\gamma \equiv \frac{d\tau_o}{d\tau_s}$ called lapse to quantify it. Here we borrowed the name lapse from the timescape cosmology[16], which was originally introduced to describe the multiplicative factor between volume-averaged clocks of walls and voids. Following the argument on how the clock of an astronomical event should primarily anchor to the clock of the cosmological patch it resides on in the first paragraph of this section, the measurement on the time duration will be dressed by this lapse factor $T(z)- > \gamma T(z)$.

We have no means to find the absolute value of time lapse from observation for a single event. It would be degenerate with the intrinsic scattering of the time duration.

We do, however, have the possibility to statistically test if such non-trivial (non-unity) lapse exists or not. With the characteristic time determined by the astronomical physics captured by the local average measured value $\bar{T}_0$, we assume that the uncertainty around this pivot value could be decomposed into two independent uncertainties, due to the intrinsic scattering cosmological clock dissynchronization:

$$T(z) = (1+z)(\bar{T}_0 + \sqrt{\sigma_{\rm int}^2 + \bar{T}_0 \sigma^2(\gamma)}) \tag{30}$$

where $\sigma(\gamma)$ is the variance of γ, and we used the fact $\bar{T}_0$ on cosmological scale is a small time duration that can substitute dt_s in the definition of lapse $\gamma \equiv \frac{d\tau_o}{d\tau_s}$.

Now we look into the modeling of $\sigma(\gamma)$. Recall one of the most important result in the second section of this article, $\Lambda \sim \sigma^2(\bar{\alpha})$, and when the saturation of geodesic congruence to the spacetime substance flow happens, α can be effectively interpreted as the acceleration of a patch. Assuming a free fall motion, thus a parabolic trajectory of any distant Hubble patch with respect to our Hubble patch, the relationship between the acceleration and the time lapse of a distant patch becomes

$$\gamma_s = \chi_s \alpha_s \tag{31}$$

When writing down this relationship, we are putting the patch in which a distant galaxy resides in a Rindler coordinate, and treating the earth observer as stationary. Then the time transformation between a Rindler proper time and the stationary observer time gives the above result, and χ_s is the comoving distance of the source patch[4]. Thus,

$$\sigma(\bar{\gamma}_s) = \chi_s \sigma(\bar{\alpha}) \tag{32}$$

And the variance of the supernovae light curve duration time modulated by the $(1+z)$ factor and its local average pivot value is:

$$\sigma^2\left(t_c\right) = \tilde{\sigma}_{\text{int}}^2 + \chi^2(z)\sigma^2(\bar{\alpha}) \tag{33}$$

where $t_c \equiv \frac{T(z)}{\bar{T}_0(1+z)}$ is the fractional characteristic time duration of supernovae light curve modulated by the $1+z$ factor, and $\tilde{\sigma}_{\text{int}} = \sigma_{\text{int}}/\bar{T}_0$ is the fractional intrinsic scattering.

Equation (33) has already shown a very obvious difference between the signal predicted by fractal cosmology and standard cosmology: The variance of the observable t_c would have redshift dependence in fractal cosmology, as a result of the cosmological clock dissynchronization, while in the standard cosmology $\sigma(t_c)$ will only have a redshift-independent intrinsic scattering term.

A fake-data illustration of the difference between the standard cosmology and the fractal cosmology prediction on the overplotted supernovae light curves data points grouped by redshift is presented in figure 4. This figure cannot be read quantitatively, as it is only designed to schematically show what kind of mode could potentially distinguish the two models: the light curves will spread in a wider range on the time axis with growing redshift in fractal cosmology, while this effect is not expected to be as drastic in the standard cosmology.

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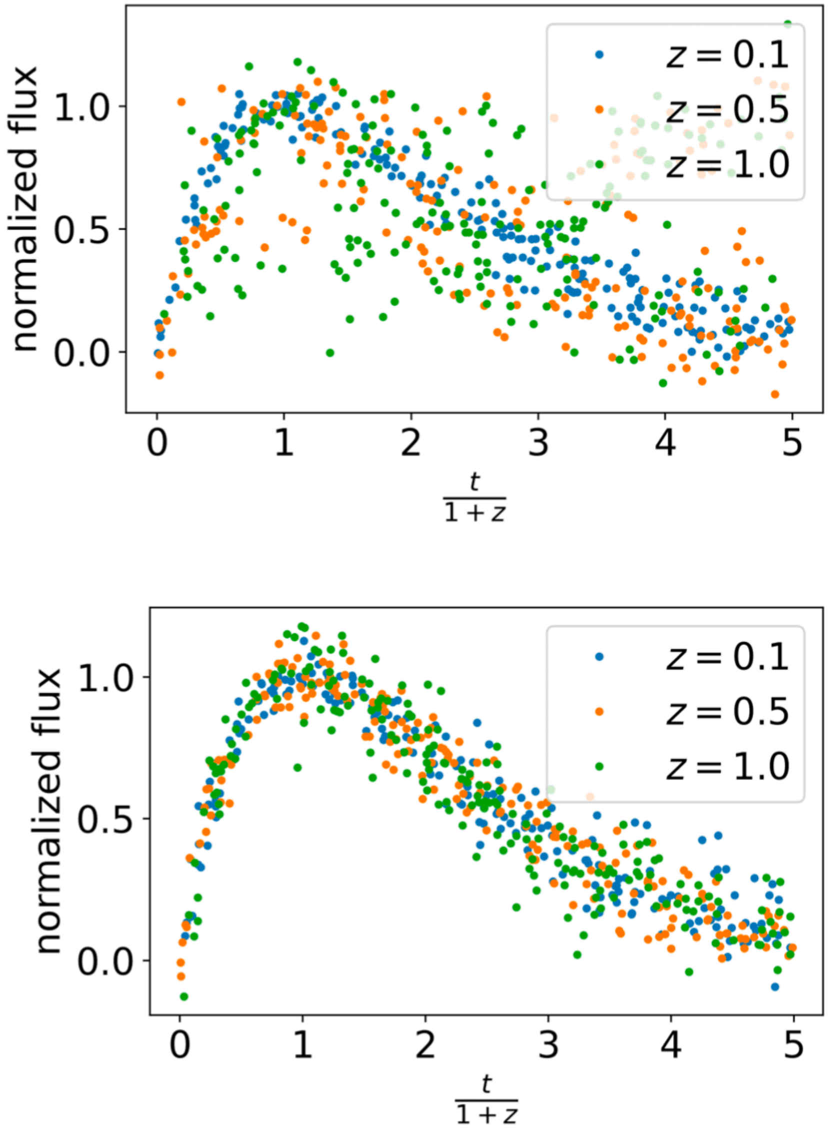

Figure 4. Fake supernovae light curves samples behavior predicted in fractal cosmology (upper panel) and standard cosmology (lower panel). In the upper panel, the lifetimes of the ten fake light curves in a redshift bin are subject to a growing uncertainty described in equation (33), while the lower panel only has a linearly growing uncertainty on the normalized flux mimicking the growing uncertainty on fainter objects and a redshift-independent scattering of the light curve lifetime.

Most intriguingly, equation (33) gives an observable estimation of the metric term coefficient $\sigma^2(\bar{\alpha})$ thus a completely new quantitative prediction of the accelerated expansion rate of our observational universe.

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Specifically, it suggests that if fractal cosmology is a successful cosmological model, at medium redshift ($0.1 <\sim z <\sim 2$), **the variance of the dimensionless characteristic time $t_c \equiv \frac{T(z)}{\bar{T}_0(1+z)}$ could be linearly fitted by the comoving distance square χ^2. The slope is predicted to take approximately the value of the cosmological constant Λ as defined in ΛCDM model, and the constant term accounts for the intrinsic scattering of the time duration T of a type of the astronomical events with characteristic time scale determined by its physics.**

From the observation obtained in[29], a trend of higher redshift supernovae have more scattered light curve can be seen. However, such a trend could be due to fainter luminosity, and larger observational uncertainty on the flux. The observational uncertainty on the variance $\sigma^2(t_c)$ need to be treated carefully when carrying out the linear fitting proposed here, so it is not clear whether the sample size of ~ 1000 in[29] is large enough to carry out the alternative estimation on $\sigma^2(\bar{\alpha})$ presented above before further experimenting on the real data.

Another factor worth noting is the possible redshift-drifting of the intrinsic scattering term $\tilde{\sigma}^2_{\mathrm{int}}$ due to some unresolved environment evolution for supernovae (or other astronomical events) throughout the expansion history. Even if such extra redshift-dependent mode does exist in $\sigma(t_c)$, it is not very possible for such mode to be completely degenerate with the $\chi^2(z)$ mode with a slope $\sim \sigma^2(\bar{\alpha})$. However, it could be a major contributor to the systematics on the $\sigma^2(\bar{\alpha})$ estimator proposed here.

This article hereby raises the analysis described in this section on the lifetime variance of any astronomical event with a characteristic time as a challenge to any group working on time-domain astronomy.

Footnotes

[1] It might reminds one of ether, but the substance of spacetime is not quite the same. Ether flow is a vector field still living on 3D manifold, and the one extra dimension is essential here to save the whole story from the failure of ether. Any measure of time, thus the velocity in physics sense, cannot be made independently from the spacetime substance flow, which is very different from the assumption in the ether (thought) experiments where time can be measured regardless of the dynamics of this medium.

[2] An even more intriguing number game could be that the fraction $\Omega_b : \Omega_m : \Omega_\Lambda$ is well captured by $1 : d_F^2 : d_F^3$.

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[3] Each patch could have a slightly fluctuating $\bar{\alpha}_i$ away from zero, but one can validate by a quick calculation that as long as the distribution around each center value has the same shape, the scaling of $\sigma(\bar{\alpha}) \sim 1/N$ on averaged value stays the same.

[4] In the discussion here, we decomposed the motion of a distant galaxy into the motion of its cosmological patch plus its motion in that patch, and assumed that the latter introduces negligible dissynchronization effect while the former is the focus.

Statements and Declarations

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