Accelerated Motion Towards Relativistic Velocities Described by Newtonian Mechanics

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Abstract

This paper presents a simple geometric derivation of the equations of motion for an object that accelerates from zero speed towards a relativistic velocity. This is not done by using Special Relativity, but with Newtonian Mechanics only. The results are identical to those derived by Special Relativity, with the exception of the expression for time dilation, in case the clock postulate is not applied. The clock postulate is used by Special Relativity, and assumes that the time dilation of an accelerating object at any moment in time depends on its contemporary velocity only. It is shown that the clock postulate is violated in case of continuous longitudinal acceleration, and an alternative expression for accelerated time dilation is presented without the need for the clock postulate.

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Introduction

In the year 1887 the American physicists Albert Michelson and Edward Morley obtained the first experimental results indicating that the perceived speed of light is independent of the motion of the observer with respect to the light source [1]. This observation could not be explained by the then prevalent physical models for waves travelling through a medium of propagation. This initiated a line of research that eventually led to a theory known as 'Special Relativity'[2]. Special Relativity (SR from now on) is still used today to describe the motion of objects with velocities approaching the speed of light, as opposed to Newtonian Mechanics which so far could only be used for velocities much smaller than the speed of light. One of the fundamentals of SR is that no observer can ever see an object reach the speed of light, regardless of how fast and how long it accelerates. As a result, the classical expressions for traveled distance $d(t)$ and perceived velocity $v(t)$ after a time t for an object with initial position 0, initial velocity 0 and acceleration a,

$$d(t) = 1/2at^2 \qquad (1a)$$

$$v(t) = at \qquad (1b)$$

are replaced by alternative expressions[3]:

$$d(t) = \frac{c^2}{a}\left(\sqrt{1 + a^2t^2/c^2} - 1\right) \qquad (2a)$$

$$v(t) = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}} \qquad (2b)$$

where c is the speed of light in vacuum. In addition, the perceived acceleration $a'(t)$ of the object diminishes over time from its initial value a to

$$a'(t) = a\left(\frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}}\right)^3 . \qquad (2c)$$

It can be easily verified that Equation (2b) assures that the velocity $v(t)$ is limited to values smaller than c, regardless the value of at, whereas Equation (2c) assures that the perceived acceleration $a_z(t)$ eventually becomes 0. For $at \ll c$ (for low velocities compared to the speed of light) Equations (2) reduce to the Newtonian expressions given by Equations (1).

SR is based on a number of postulates, one of which is an undeclared assumption made by Einstein in his landmark paper [2], later referred to as the *clock postulate*. The clock postulate states that the Lorentz factor

$$\gamma(t) = \frac{1}{\sqrt{1 - \frac{v^2(t)}{c^2}}} \qquad (3)$$

depends only on the instantaneous value of the velocity $v(t)$ and not on any of its derivatives. In other words, the clock postulate assumes that the value of $\gamma(t)$ in an accelerated frame of reference is identical to its value in a momentarily co-moving inertial frame. In this paper it will be shown that this assumption is incorrect, which leads to an inaccuracy in the familiar expression for time dilation of a continuously accelerated object.

Relativity described by Newtonian Mechanics

We will use Newtonian mechanics to describe the motion of a ‘relativistic rocket’, with the inclusion of time dilation effects. So far these equations have only been derived by SR with the use of Lorentz transformations, but it will be shown that

Newtonian Mechanics suffice by using 3 alternative postulates that lay the foundation for a ‘simplified theory of relativity’:

1. The Earth moves through a 4-dimensional Euclidian space, with uniform velocity equal to the speed of light *c*, in a direction perpendicular to the three orthogonal dimensions *x*, *y* and *z* that set up the 3-dimensional space that we see around us. We will call this the *w*-direction. Despite being an ordinary spatial dimension, this *w*-dimension is ‘hidden’, in the sense that it can not be perceived. All we perceive are the 3 dimensions perpendicular to our velocity vector.
2. At any point in time, matter manifests itself only in the 3-dimensional space that contains it current position and which is perpendicular to its direction of motion through this 4-dimensional space. We call this the ‘space of existence’.
3. Observers can perceive an object only if they are located inside the object’s space of existence.

Note that the postulates underlying SR have been abandoned, so that any known result derived by SR (such as that no solid mass can ever reach the speed of light) does not necessarily apply at this time. Nevertheless, it will be shown that with above postulates, 17th century Newtonian Mechanics can reproduce the relativistic equations of motion knowm from SR with a very simple geometrical method and without the need for a clock postulate as required by SR.

Travelled distance during continuous acceleration

Figure 1 represents the projection of a 4-dimensional space, with a hidden *w*-dimension along the vertical axis and an arbitrary direction of our 3-dimensional world along the horizontal axis, for which we have chosen the *z* direction. At time $t = 0$, an observer (1) and a rocket (2) are both located at point A. According to postulate 1, both move with the speed of light *c* in the *w*-direction, which is perpendicular to the *x*, *y* and *z* directions. At $t = 0$, the engine of the rocket starts to run at constant power, accelerating the rocket into the *z* direction. We will not consider mass loss due to fuel consumption, so that the rocket will maintain a constant acceleration *a* throughout its entire journey. Since this acceleration is perpendicular to the velocity vector of the rocket, it will perform a circular orbit in the *z*-*w* plane according to Newtonian mechanics, with a radius *R* around a center point M. This orbit is indicated in Figure 1 by the dotted circle segment running from point A to point C. From Newtonian Mechanics we know the relationship between the centripetal acceleration *a*, the orbital velocity *v* of an object and the radius of curvature *R* of the resulting circular motion:

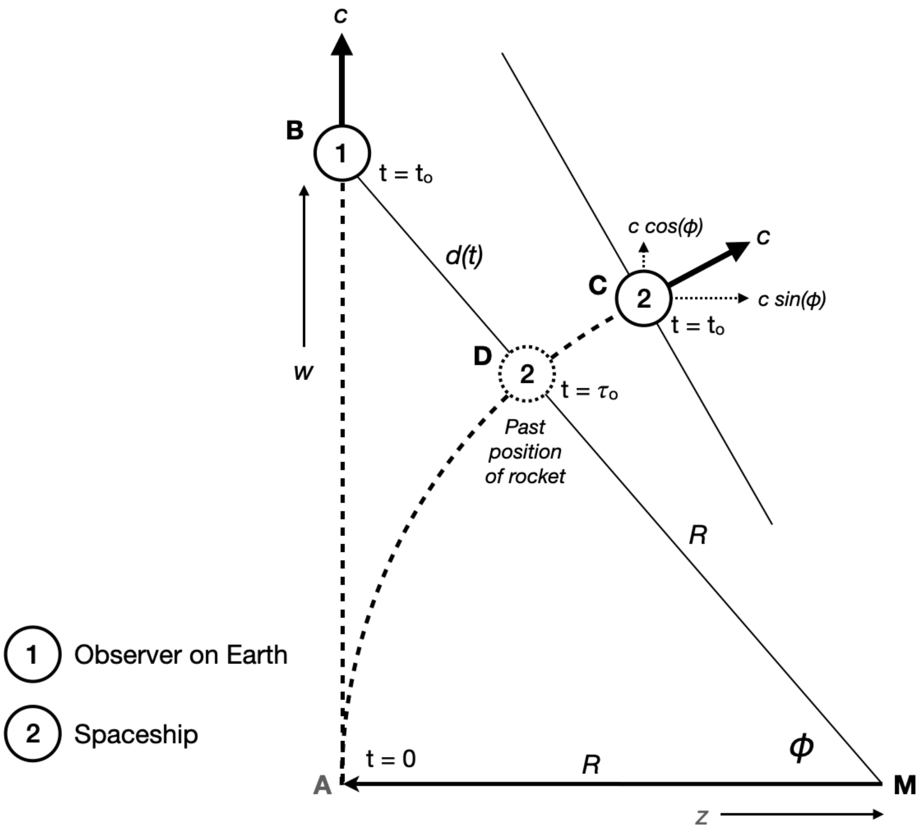

Figure 1. Relativistic rocket (2) leaving Earth (1) at point A with constant acceleration a in the z direction. The situation is shown in the z-w plane at a time t_o after the launch. According to the first postulate, at launch the rocket (2) already moves in the w direction with the speed of light, and because its acceleration is perpendicular to its velocity it will perform a circular motion in the z-w plane. The earth (1) will continue to move along a straight line into the w direction.

$$a = \frac{v^2}{R}. \qquad (4)$$

Because the orbital velocity of the rocket remains unaltered and equal to the speed of light c, we find

$$R = \frac{c^2}{a}. \qquad (5)$$

Figure 1 shows the situation at a time $t_o > 0$. The Earth has moved in a straight line from point A to point B, whereas the rocket has moved the same distance along a circular orbit from point A to point C. However, the observer on Earth will not be able to see the rocket at this point. This is the result of the 2nd postulate, which states that matter manifests itself only in the 3-dimensional space perpendicular to its velocity vector. In Figure 1, a part of this 'space of existence' of the rocket has been indicated by a solid line piece through point C (the total space of existence would have to be represented by an infinitely long line). This space of existence does not contain the position of the observer (point B). As a result, according to the 3rd postulate, the observer will not be able to see the rocket. However, Figure 1 also shows that the observer is located inside the space of existence of the rocket at an earlier time $_o$, when it was at point D. As a result, at time t_o the observer will see the rocket at position D, at an earlier time $_o$, and at a distance *d (t)* as indicated in Figure 1. At this point one might argue that, at $t = t_0$, the observer cannot see the rocket at an earlier time $_o$, because the rocket will no longer be there. Keep in mind, however, that the concept of time in this study is not the same as how we experience time in our daily life. In this study, an observer can see an object when he is located inside the space of existence of that object, regardless the corresponding time stamps. At non-relativistic velocities, the spaces of existence of the observer and the rocket, or of two observers, will (nearly) coincide and the values of t_o and $_o$ will be (nearly) identical. This describes the experience in our daily life: two observers always seem to see each other at the same time t_o and seem to share the same past at an earlier time t_o - t. However, in the analysis presented in this paper each observer manifests himself at various times, whereas it is perfectly possible for both observers to see the other observer at an earlier time $_o$, provided that the space of existence of one observer intersects with the current position of the other. The exact value of $_o$ will be derived later in this paper. We will now first derive the expression for *d (t),* which can be done with simple trigonometry. See Figure 1. At time *t* the distance AB amounts to *ct*, so that, using the Pythagorean theorem we immediately find

$$d(t) = \sqrt{R^2 + c^2t^2} - R = R(\sqrt{1 + c^2t^2/R^2} - 1). \qquad (6)$$

Substitution of Equation (5) leads to

$$d(t) = \frac{c^2}{a}(\sqrt{1 + a^2t^2/c^2} - 1). \qquad (7)$$

This expression is identical to the one derived by SR[3].

Perceived velocity and acceleration

The velocity v_z of the rocket as observed from Earth is obtained simply by taking the time derivative of Equation (7):

$$v_z(t) = \frac{1}{2}\frac{c^2}{a}\frac{1}{(\sqrt{1 + a^2t^2/c^2}}\cdot\frac{2a^2t}{c^2} = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2t^2} + \frac{1}{c^2}}} \qquad (8)$$

whereas differentiating once more reveals the expression for the perceived acceleration a_z:

$$a_z(t) = -\frac{1}{2}\left(\frac{1}{a^2t^2} + \frac{1}{c^2}\right)^{-3/2}\cdot\frac{-2}{a^2t^3} = \left(\frac{1}{\sqrt{\frac{1}{a^2t^2} + \frac{1}{c^2}}}\right)^3\cdot\frac{1}{a^2t^3} = \left(\frac{1}{\frac{1}{at}\sqrt{1 + \frac{a^2t^2}{c^2}}}\right)^3\cdot\frac{1}{a^2t^3} = a\left(\frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}}\right)^3. \qquad (9)$$

Expressions (5) and (6) can be further simplified. In Figure 1, it can be seen that the velocity of the rocket can be decomposed into a *z*-component and a *w*-component, $v_z = c \sin(\phi)$ and $v_w = c \cos(\phi)$, so that

$$\cos(\phi) = \sqrt{1 - \sin^2(\phi)} = \sqrt{1 - \frac{v_z^2}{c^2}} = \frac{1}{\gamma} \qquad (10)$$

where is the Lorentz factor from SR. The angle ϕ changes as the rocket accelerates, and can be written as (see Figure 1)

$$\phi(t) = \tan^{-1}(ct/R), \qquad (11)$$

so that after substitution of Equation (5)

$$\phi(t) = \tan^{-1}(at/c). \qquad (12)$$

Combining Equations (10) and (12) gives

$$\frac{1}{\gamma} = cos\,(\tan^{-1}(at/c)). \qquad (13)$$

By using the relationship

$$cos\,(\tan^{-1}(x) = \frac{1}{\sqrt{1+x^2}} \qquad (14)$$

this can be written as

$$\frac{1}{\gamma} = \frac{1}{\sqrt{1+\frac{a^2t^2}{c^2}}}. \qquad (15)$$

After substitution of (15) into Equations (8) and (9), we find:

$$v_z(t) = \frac{at}{\gamma} \qquad (16)$$

$$a_z(t) = \frac{a}{\gamma^3}. \qquad (17)$$

Equations (13) and (14) are identical to the expressions for the rocket's velocity and acceleration as derived by SR[3].

Time dilation

Now we will compare the distances travelled through the 4-dimensional space by the observer on Earth and by the rocket at the time of its observation. Figure 2 shows the situation at time t_o when the Earth (1) has moved a distance $T = ct_o$ in the 'hidden' w-direction, and the rocket (2) has performed a section of a circular orbit with length ct_o and radius R = c^2/a. As explained before, the observer on Earth will not be able to see the rocket at its position at $t = t_o$. Instead, it will see the rocket at an earlier position at time $t = \ _o$ (indicated by the dotted circle), when the rocket had performed a section of a circular orbit with total length S. By definition:

$$\phi = \frac{S}{R} \qquad (18)$$

and also

$$\phi = \tan^{-1}\left(\frac{T}{R}\right) = \tan^{-1}\left(\frac{ct_o}{R}\right) \qquad (19)$$

so that, with Equations (18) and (5)

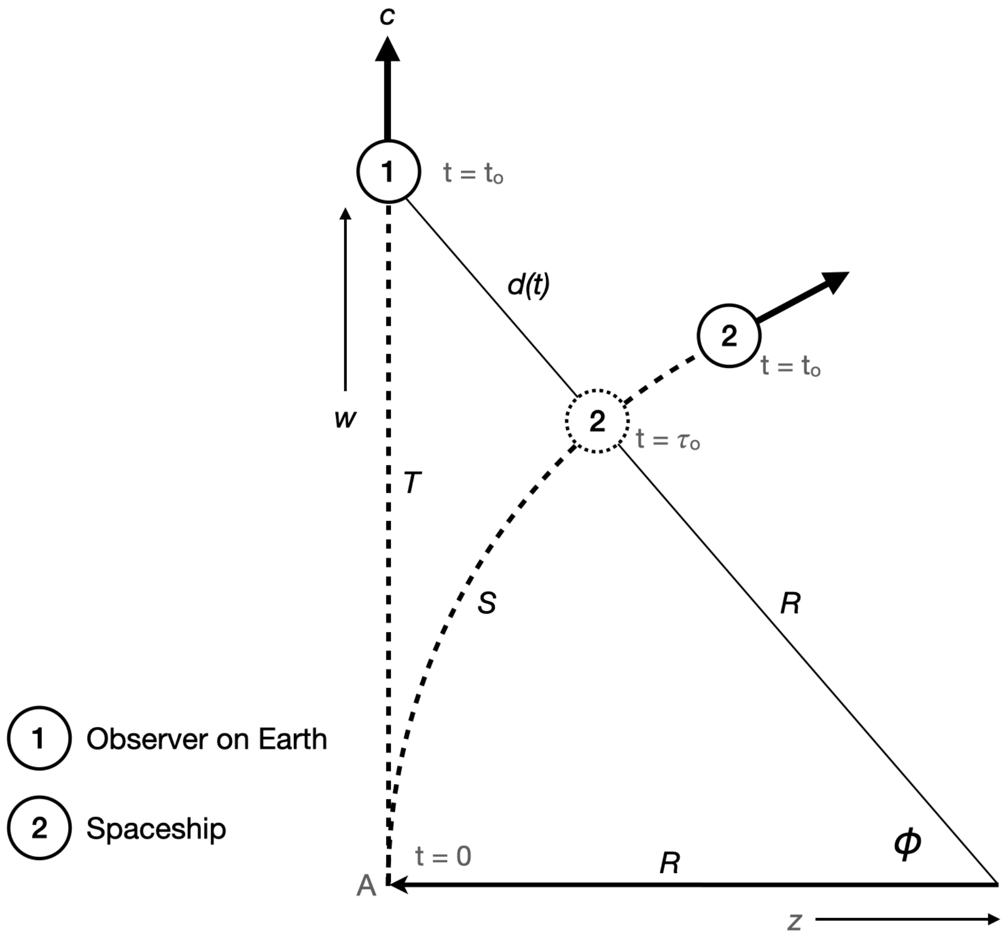

Figure 2. Time dilation as experienced by observer on Earth (1) when seeing the rocket at its past position (2, dotted circle).

$$S = R\tan^{-1}\left(\frac{ct_o}{R}\right) = \frac{c^2}{a}\tan^{-1}\left(\frac{at_o}{c}\right). \qquad (20)$$

The rocket has a constant orbital velocity equal to c, so that the time it took for the rocket to perform the circular segment with length S amounts to $_o = S/c$ and, therefore, after substitution of Equation (20)

$$\tau_o = \frac{c}{a}\tan^{-1}\left(\frac{at_o}{c}\right). \qquad (21)$$

The third postulate states that, at time $t = t_o$, the observer on Earth will see the rocket at time $t = \tau_o$. Since $\tau_o < t_o$, this will be perceived by the observer as a ‘time dilation’ because it will appear as if the time aboard the rocket has been running slower. Note that expression (21) is different from the familiar time dilation expression for a relativistic rocket, which has the following form[3]:

$$\tau_o = \frac{c}{a}\sinh^{-1}\left(\frac{at_o}{c}\right). \qquad (22)$$

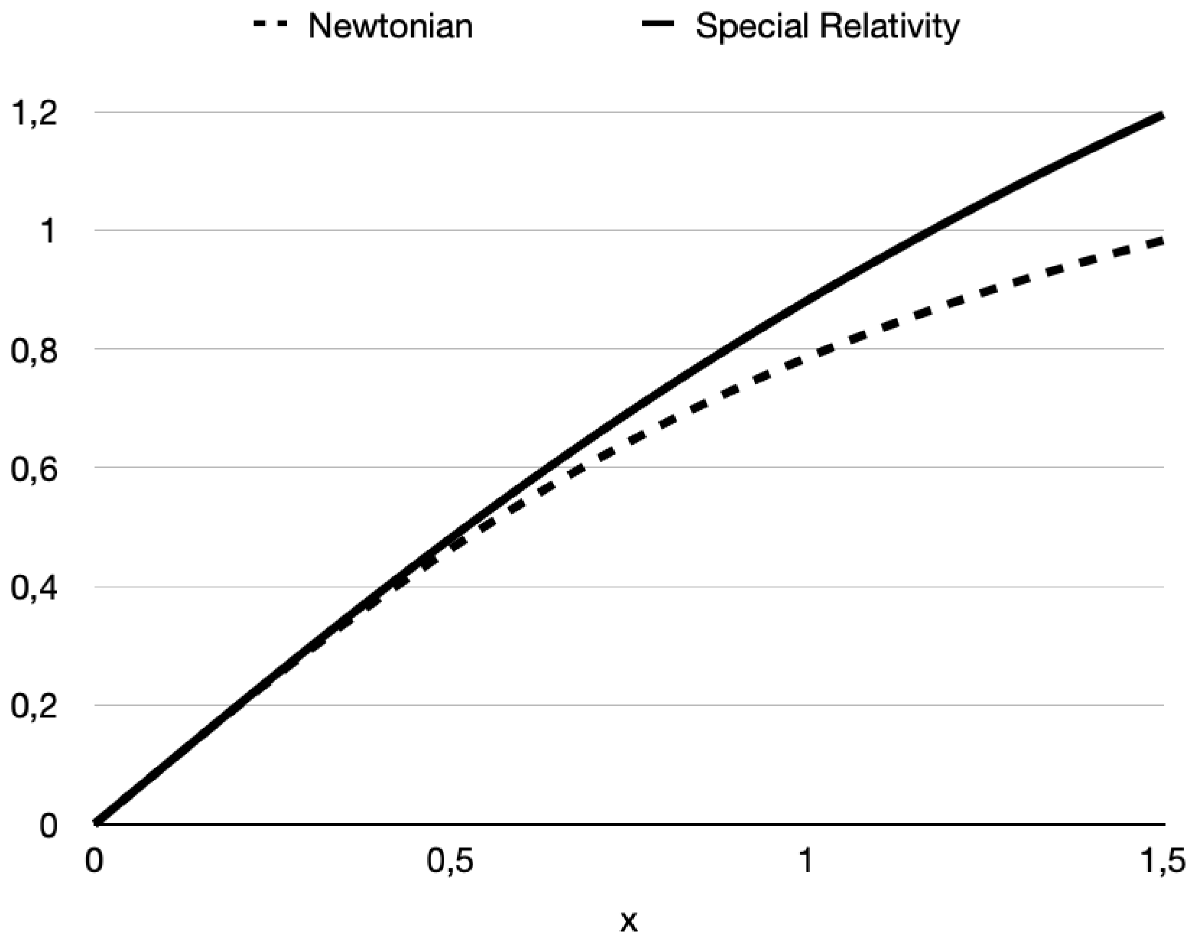

Figure 3. the functions $\tan^{-1}(x)$ (dotted line) and $\sinh^{-1}(x)$ (solid line) corresponding to the expressions for time dilation of a continuously accelerated object according to Newtonian Mechanics, as proposed in this paper (Equation 21), and the SR with use of the clock postulate (Equation 22).

The cause of this discrepancy will be discussed in the next paragraph. Here we will demonstrate that expression (21) makes, at least intuitively, perfect sense. Taking the inverse of Equation (22) yields

$$t_o = \frac{c}{a} tan\left(\frac{a\tau_o}{c}\right). \qquad (23)$$

Equation (23) contains a singularity when a_o/c approaches the value of /2, for which t_o will become infinitely large. This happens at time

$$\tau_o = \frac{\pi c}{2a}. \qquad (24)$$

At this point the Equations of motion (7), (8) and (17) reduce to:

$$d(t) = ct \qquad (25)$$

$$v_z(t) = c \qquad (26)$$

$$a_z(t) = 0 \qquad (27)$$

so that the observer on Earth will eventually see the rocket move away with (almost) the speed of light, and with (almost) zero acceleration. The angle ϕ in Figure 2 will then be equal to

$$\phi(\tau_o) = \frac{S(\tau_o)}{R} = c\frac{\pi R}{2cR} = \frac{\pi}{2}. \qquad (28)$$

Indeed, it can be seen in Figure 2 that t_o will become infinitely large when ϕ approaches the value /2. This means that the observer will have to wait an infinitely long time before he can see the rocket reach the speed of light. In other words, in any finite amount of time the observer will never see the rocket reach the speed of light, regardless its acceleration a. This finding is in perfect agreement with SR and all experimental observations so far.

Lorentz contraction, mass increase and the invariability of the speed of light

The physical model presented in this paper also provides a straightforward interpretation of Lorentz length contraction and mass increase. Figure 4 shows a schematic of 2 spaceships in the 4-dimensional space shown in the *zw*-plane. Each spaceship has a rest length L_o while both move relatively to each other with a velocity equal to $c \sin(\phi)$, similar to the situation depicted in Figure 1.

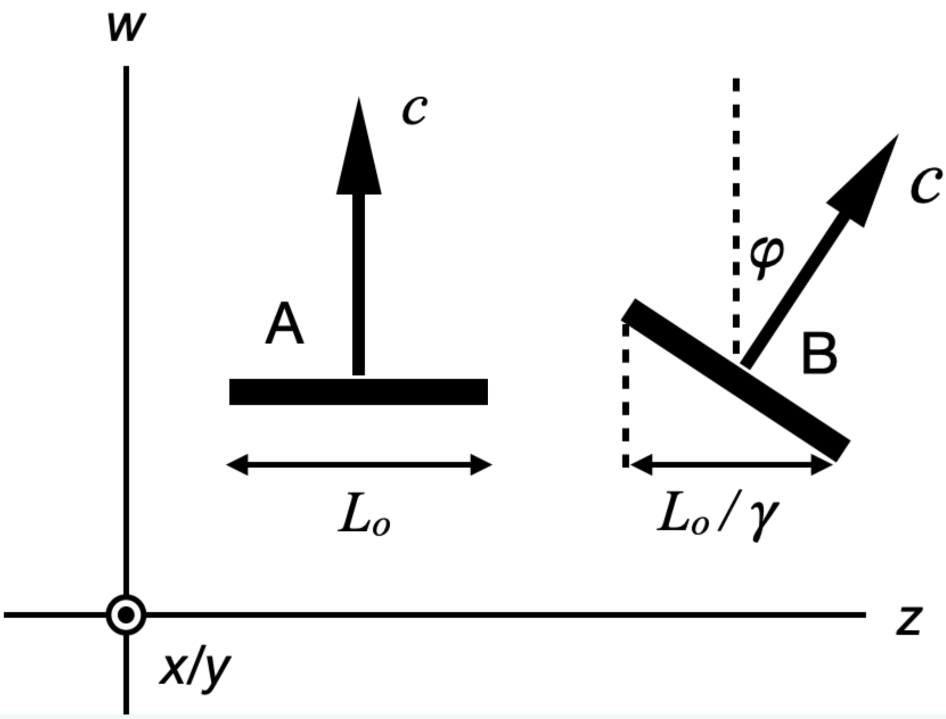

Figure 4. Schematic of two observers A and B in a 4-dimensional space with orthogonal dimensions x, y, z, and w, shown in the zw-plane. Both spaceships travel at the speed of light c and have the same rest length L_o. Their relative velocity amounts to v = c cos(ϕ). Observer A cannot perceive the w dimension, and consequently perceives the other space ship as an orthogonal projection on his own space of existence xyz with Lorentz contracted length Lo/ . This observation is mutually symmetric for both observers.

The observer in A, not able to distinguish the *w*-dimension according to postulate 1, will perceive the length of the other spaceship equal to $L_o \cos(\phi)$, i.e., as the space ship's orthogonal projection onto the xyz space. With equation (10) we find that this perceived length is

$$L = L_o/\gamma \qquad (29)$$

It can be easily understood that this effect is symmetric: the observer in B will also see the length of spaceship A equal to L_o / , identical to the results of SR.

An identical reasoning explains why both observers will perceive the mass of the other to increase by a factor of . If the observer in A applies a force *F* in the z-direction to the space ship in B, the effective force as experienced by the space ship B (that is, the force that will contribute to its centripetal acceleration), will be equal to $F \cos(\phi) = F/$. This reduced effective force will be interpreted by the observer in A as a mass increase by a factor of . This can be seen from Newton's second law $a = F/m$, which in this particular case takes the form

$$a = F/\gamma m_o \qquad (30)$$

where m_o is the rest mass of the space ship. The reduced effective force $F/$ (in stead of F) will be interpreted by the observer in A as an increased effective mass m_o (in stead of m_o). Once again, it is straightforward to see that that that this experience will be mutually symmetrical for both observers in A and B.

Even the invariability of the speed of light can be easily explained by the concepts presented in this paper. Imagine a light source emitting a light wave with a hyper-spherical wave front in a 4-dimensional space, that is, a wave front described as

$$x^2 + y^2 + z^2 + w^2 = (ct)^2, \qquad (31)$$

which would correspond to the solution of a 4-dimensional version of Maxwell's Equations. According to postulate 1, an observer will perceive this wave front as a 3-dimensional projection on its own space of existence, regardless its own velocity with respect to the light source. Consequently, this projection will always take the shape

$$(x_o - p)^2 + (y_o - q)^2 + (z_o - r)^2 = (ct)^2 \qquad (32)$$

where x_o, y_o and z_o represent three orthogonal dimensions perpendicular to the observer's velocity vector, whereas the location of the light source perceived by the observer is (p, q, r). Equation (32) shows that the observer will always see the wave front approach with velocity c, regardless his own velocity with respect to the light source. Interestingly, the same observation would be made in case the light wave would use a medium of propagation, such as a luminiferous aether, which would provide an alternative explanation for the observations made by Michelson and Morley [1].

Violation of the Clock Postulate

The clock postulate states that an accelerating clock, at any point in time, compared to a clock that is stationary with respect to an observer, always has its timing slowed down by a factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \qquad (33)$$

In other words, the time dilation factor depends only on the instantaneous velocity$v$ of the clock on a given moment, and not on any of its derivatives, such as acceleration.SR makes use of the clock postulate in order to predict how accelerating objects behave [2], however, we have never known if the clock postulate is true. So far it has just been a postulate, which cannot be proven by definition. In this paragraph we will demonstrate that an accelerating clock violates the clock postulate, which explains the discrepancy between Equations (21) and (22).

We will start with a traditional derivation of the time dilation as experienced by the observer on Earth as he watches the accelerating rocket fly away. Making use of the clock postulate and Equation (15) we get

$$_o = \int_0^{t_0} \frac{1}{\gamma} dt = \int_0^{t_0} \frac{1}{\sqrt{1 + \frac{a^2t^2}{c^2}}} dt \qquad (34)$$

Using

$$\int \frac{1}{\sqrt{1+t^2}} dt = \ln\left(|t + \sqrt{1+t^2}|\right) + C, \qquad (35)$$

we find

$$_o = \frac{c}{a} ln\left(\frac{a}{c} t_o + \sqrt{1 + \frac{a^2 t_o^2}{c^2}}\right) = \frac{c}{a} \sinh^{-1}\left(\frac{a t_o}{c}\right), \qquad (36)$$

which is the inverse of the hyperbolic motion equation from SR:

$$t_o = \frac{c}{a} sinh\left(\frac{a\tau_o}{c}\right). \qquad (37)$$

In the previous paragraph it was already noted that Equation (37) is not in agreement with the analysis presented in this paper, which obtained a different expression for the observer's time given by Equation (23):

$$t_o = \frac{c}{a} tan\left(\frac{a\tau_o}{c}\right).$$

This is remarkable, because the simple geometric construction shown before in this paper accurately reproduced the traditional equations of motion for a relativistic rocket as derived by SR. Closer inspection of Equation (37) shows, that in the limit of very long time spans and/or very high acceleration, it takes the form

$$\lim_{a, \tau_0 \to \infty} t_0 = \frac{c}{2a} e^{\frac{a\tau_o}{c}} \qquad (38)$$

In other words, the time that passes for the observer given any time period $_o$ aboard the rocket only grows exponentially, not asymptotically, as the speed of the rocket continues to increase. In the remainder of this paragraph, we will demonstrate that the traditional expression for time dilation, Equation (37), is not accurate because of a violation of the clock postulate. See Figure 5.

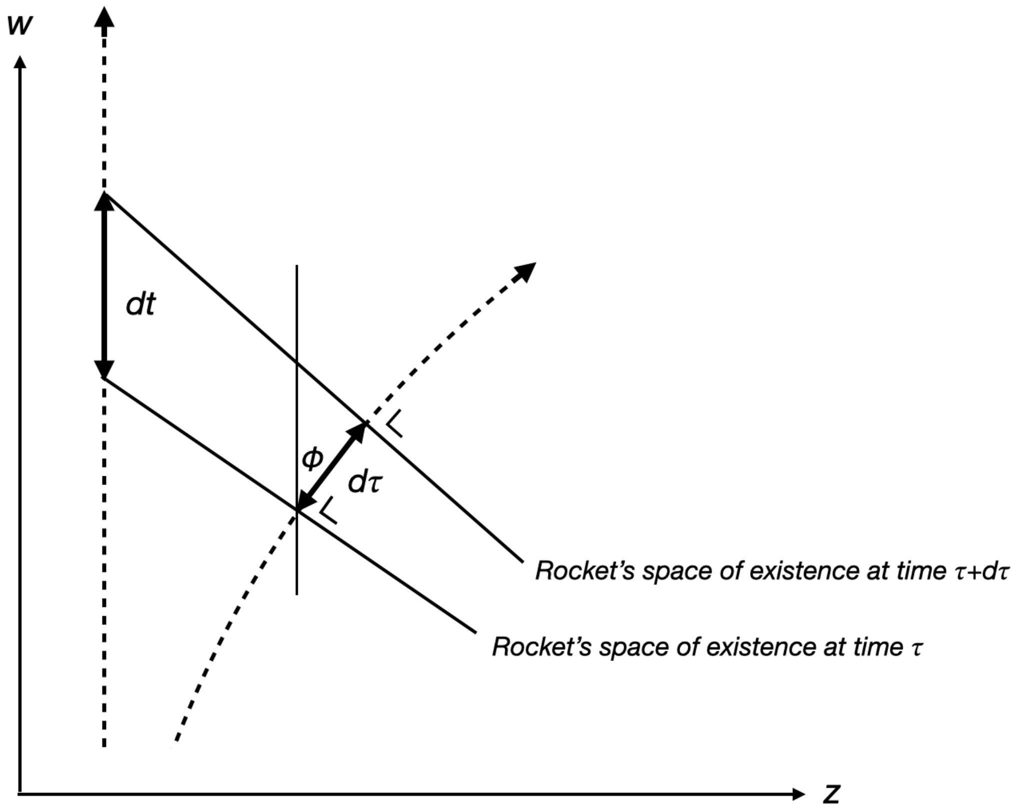

Figure 5. Violation of the clock postulate shown in the z-w plane in the case of an accelerating rocket (curved trajectory) observed by a stationary observer (vertical trajectory). Because the rocket's spaces of existence at successive moments in time are never parallel to each other, the relationship d = cos( ) dt = 1/ dt is not valid. The error gets worse for larger values of , that is, for acceleration towards higher velocities.

As before, the observer's trajectory in the *z*-*w* plane is represented by a dotted vertical line, and the trajectory of the accelerating rocket by a circular segment. Two successive spaces of existence for the rocket are shown, each perpendicular to its curved trajectory, intersecting the trajectory of the stationary observer. It is immediately seen that the time intervals *dt* and *d* are not related by *dt* = *d* ⁄cos ( ) = *d* . As long as the rocket accelerates, its trajectory in the *zw*-plane will be curved and as a result, its successive spaces of existence are never parallel, not even for infinitely small values of *dt* and *d* . This violates the clock postulate, which explicitly assumes that the rocket's successive spaces of existence would (at least pairwise) be parallel, i.e., identical to a momentarily co-moving inertial frame. This is never the case, and it can be seen in Figure 5 that, in reality, *dt* is always larger than *d* , so that the value of t_o as expressed by Equation (37) will always be too small, more so for acceleration toward higher velocities. This is in agreement with the result shown in Figure 3. The correct expression for t_o can be derived by simple trigonometry from Figure 1, as shown before, and is given by Equation (23):

$$t_o = \frac{c}{a} tan\left(\frac{a\tau_o}{c}\right).$$

However, note that under the assumption of the clock postulate, the relationship *dt* = *d* would be restored, as can be seen from Figure 5, so that Equation (21) would take the shape of the familiar expression from the Theory of Relativity shown in Equation (22). In this case the expression $dt = d$ simply describes time dilation experienced during constant velocity, in which the circular segment shown in Figure 5 will take the shape of a straight line piece.

The Expanding 4D Universe

So far it has been shown that, with a set of alternative postulates, Newtonian Mechanics can reproduce the equations of motion for objects accelerating towards relativistic velocities, including Lorentz contraction, time dilation, mass increase and the invariability of the speed of light. One might argue that one of the alternative postulates, postulate 1, violates the scientific consensus that no solid object can ever reach the speed of light. However, this is a conclusion derived from SR, and since we explicitly abandoned SR for the model presented in this paper (albeit temporarily), postulate 1 is perfectly acceptable. This is confirmed by the conclusion derived from our alternative model, which states that no observer can ever see an object reach the speed of light in any finite amount of time, as shown earlier in this paper. This conclusion is in perfect agreement with SR as well as with countless experimental results.

However, it seems appropriate to put postulate 1 into a context, and hypothesise why the Earth would be moving through a 4-dimensional space with the speed of light. In essence this can be explained by a 4-dimensional version of the classical Big Bang theory. Imagine all matter of the universe concentrated in a single point in a 4-dimensional Euclidian space. Again, this is a space of 4 orthogonal spatial dimensions, w, x, y, and z, not space-time as known from SR. At the birth of the Universe, a Big Bang initiated a symmetrical expansion of matter into all 4 orthogonal dimensions. As a result, all matter in the Universe got distributed over the surface of a 4-dimensional hypersphere, described by:

$$w^2 + x^2 + y^2 + z^2 = c^2 t_b^2 \tag{39}$$

where c is the speed of light and t_b the time since the Big Bang. See Figure 6.

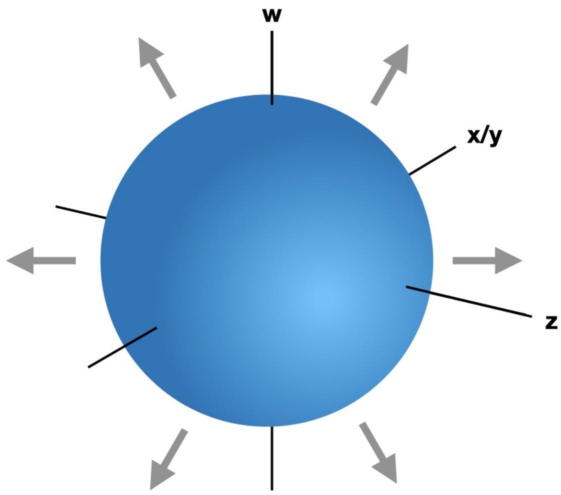

Figure 6. Schematic of the 4-dimensional expanding Universe, showing a hypersphere in a 4-dimensional Euclidian space, expanding into 4 orthogonal spatial dimensions. All matter in the Universe is concentrated over the surface of the 4-dimensional hypersphere that started to expand with the speed of light c at the moment of the Big Bang. The radius of the sphere is therefore equal to $R = ct_b$, where t_b is the time in seconds since the occurrence of the Big Bang.

Figure 7 illustrates what happens when an observer on earth (in point A) looks at a far away celestial object (in point B), shown in the *z*-*w* plane. Both the observer and the celestial object are moving at velocity *c* through the 4-dimensional universe, however, because of the large distance between them there is is a significant angle between their velocity vectors. According to postulate 1, the observer in point A can only see the 3-dimensional space extending over x, y and z, indicated by the horizontal dotted line.

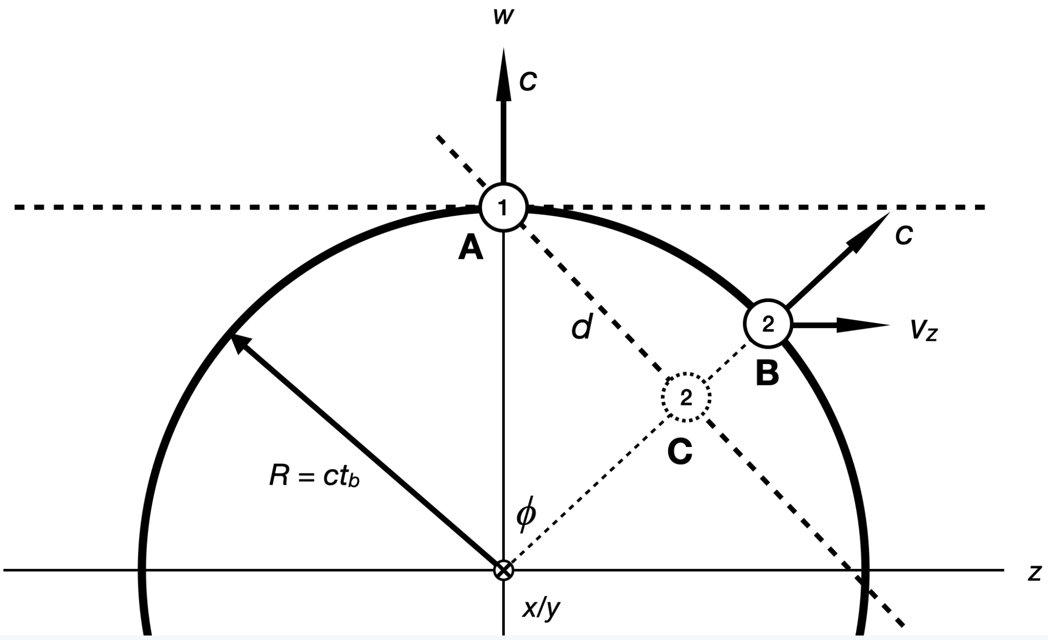

Figure 7. An observer (1) on Earth in point A watching a far away celestial object (2) in point B. Both the observer and the celestial object are located at the outer shell of an expanding 4-dimensional hypersphere. The observer is located inside the space of existence of the celestial object at an earlier time (dotted circle), when it was in point C. The observer will therefore measure a distance $d = ct_b \sin(\phi)$. At the same time, since the observer in A cannot discern the w-dimension, the perceived velocity of the celestial object is the projection of its actual velocity on the z-axis, $v_z = c \sin(\phi)$.

The observer in point A is located inside the space of existence of the celestial object when it was at position C, and, according to postulate 2, will therefore perceive the celestial object at a distance

$$d = R\sin(\phi) = ct_b \sin(\phi)\ , \qquad (40)$$

where t_b is the time since passed since the Big Bang. Since the observer in point A is not able to see the *w* dimension, the velocity of the celestial object is perceived as the projection of its actual velocity projected on the *xyz*-space, so that the observer on Earth sees the celestial object recede with an apparent velocity

$$v_z = c\sin(\phi) \qquad (41)$$

From equations (40) and (41) it directly follows that the ratio of velocity and distance of celestial objects amounts to:

$$H_o = v_z/d = 1/t_b \qquad (42)$$

which is the reciproke of the age of the Universe, also known as Hubble's constant. We see that a simple '4D Big Bang' model not only elegantly describes the relationship between distance and receding velocity of celestial objects, but also provides a rationale why the Earth would move through a 4-dimensional Euclidian space with the velocity of light, as stated by postulate 1. As a matter of fact, the Earth is located at the edge of the 4-dimensional expanding Universe, however, because of the collapsed dimension in its direction of motion (the *w*-direction in Figure 7), it is perceived to be in the center of a 3-dimensional expanding Universe with dimensions *x*, *y* and *z*.

Discussion and Conclusions

Using an alternative set of postulates than those used by SR, we applied Newtonian Mechanics to derive the relativistic equations of motion towards relativistic velocities, in a very simple manner. The resulting equations for travelled distance, perceived velocity and perceived acceleration were identical to those traditionally obtained by SR with the use of Lorentz transformations to an accelerated frame of reference. Also the concepts of Lorentz length contraction, mass increase and the invariability of the speed of light were explained with simple geometrical methods. Interestingly, the explanation for the invariability of the speed of light would even be valid in case electromagnetic radiation would be propagating through a luminiferous ether. This provides an alternative explanation for the observations by Michelson and Morley in 1887, and suggests that a luminiferous ether may still exist, contrary to current scientific consensus. Clearly, the latter hypothesis could have intriguing implications for new research opportunities. In addition, the alternative postulates required for the theory presented in this paper may create new insights in the way our universe works, leading to new areas of research, both experimental and theoretical.

The results obtained by Newtonian Mechanics were identical to those obtained with traditional SR, with one exception: the expression for time dilation during continuous longitudinal acceleration from zero speed towards a velocity close to the speed of light, in case the clock postulate is abandoned. This discrepancy was attributed to a violation of the clock postulate during longitudual acceleration. In fact, when the clock postulate was applied, the same results as obtained by SR were found. This discrepancy, according to the author's knowledge, has not yet been falsified experimentally, because it would require a measurement of time dilation during the period(s) of longitudinal acceleration only, explicitly excluding time dilation during constant velocity. This might be an opportunity for future research. The experiment by Bailey *et al.* [4] does not qualify, because it only considered centripetal acceleration and not longitudinal acceleration as discussed in the this paper.

We emphasize once more that, in case the clock postulate is applied, all results derived by Newtonian Mechanics are in perfect agreement with traditional SR (see Table 1).

		Special Relativity	Newtonian Mechanics
Travelled distance during acceleration		$d(t) = \frac{c^2}{a}(\sqrt{1 + a^2 t^2 / c^2} - 1)$	Identical
Perceived velocity during acceleration		$v_z(t) = \frac{a\,t}{\gamma}$	Identical
Perceived acceleration		$a_z(t) = \frac{a}{\gamma^3}$	Identical
Lorentz contraction at constant velocity		$L = L_o / \gamma$	Identical
Mass increase at constant velocity		$m = \gamma m_o$	Identical
Time dilation at constant velocity		$\tau = 1/\gamma\ t$	Identical
Time dilation during longitudinal acceleration	*With Clock Postulate*	$\tau = \frac{c}{a}\sinh^{-1}\left(\frac{a\,t}{c}\right)$	Identical
	Without Clock Postulate	*N/A*	$\tau_o = \frac{c}{a}\tan^{-1}\left(\frac{a\,t_o}{c}\right)$

Table 1. Comparison of the expressions for various physical quantities derived by the traditional Theory of Relativity, and the expressions derived in this paper by using Newtonian Mechanics and an alternative set of postulates. All results are identical, but Newtonian Mechanics also provide an expression for time dilation during longitudinal acceleration without the use of the Clock Postulate.

Obviously, much more research would be required to provide a mathematically consistent formulation that completely covers SR and all of its implementations and adjacencies in contemporary physics, including consistency with the flat spacetime limit of general relativity (which is currently a topic of the author's ongoing research). However, the elegant simplicity of the presented model, the suggestion that Relativity is not complementary to Newtonian Mechanics but that both formalisms are in essence one and the same, the alternative postulates which might provide new insights into the actual nature of our universe, the suspected violation of the clock postulate during longitudinal acceleration, and last but not least the possibility that a luminiferous aether might exist after all, are considered interesting enough for a communication within the scientific community, and hopefully inspires further research.

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