Review of: "Mathematics Is Physical"

Erwan Beurier[1]

1 Polytechnique Montréal

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The article entitled “Mathematics is physical” by Biao Wu is an opinion article arguing that mathematics is physical because, 1. quantum mechanics allowed to understand that we could define information in a quantum manner, and 2. because mathematics is performed by researchers, who are finite beings, thus limited by physical space.

I found this article very easy to read. Some repetitions might bother other readers but I find they help reading. I also noted a beautiful quote in the conclusion.

However, I am uncomfortable with the reasonings and arguments the author developed throughout this article. Even if I find the ideas developed by the author very debatable, the rest of this review will not focus on arguing against them, but rather on questioning the arguments given by the author.

The author mentions “Gödel’s incompleteness theorem” in a singular form. I am not sure whether this is standard. It is not clear whether the author is referring to the Gödel’s first incompleteness theorem, or to both incompleteness theorems as if they were only one, but this is a detail.

The author argues several times that Gödel’s first incompleteness theorem (let’s use the singular form) highlights that humans are finite beings only able of finite tasks, and that the finiteness of mathematics proves that mathematics is physical.

After reading the whole article, I still don’t understand the links, on the one hand, between Gödel’s first incompleteness theorem and the finiteness of humans, and on the other hand, between the finiteness of mathematics and the fact that it is physical.

The incompleteness theorems show that 1. there are always statements that can't be proven true or false in arithmetic, and 2. consistency of a theory is one of such statements. The proof does use natural numbers to encode logical statements, but I don't see the link between the finiteness of our atoms and these theorems...

Also, the author repeats several times that the cardinality of proofs is countable, which is true, but that the cardinality of mathematical propositions is uncountable, which is false!

With a proof very similar to that of the author, one can build a proposition$E(f)$ on every function $f\colon \mathrm{R} \to \mathrm{R}$, and the cardinal of R^{R} is strictly greater than R, so that would mean that mathematical propositions have at least the same cardinality as R^{R}?

And we can also build a proposition $E(F)$ on every function $F\colon \mathrm{R}^{\mathrm{R}} \to \mathrm{R}^{\mathrm{R}}$, so mathematical propositions would be at least of the same cardinality as $\mathrm{R}^{\mathrm{R}^{\mathrm{R}^{\mathrm{R}}}}$... By counting all propositions $E(\ldots)$ we end up with at least one proposition per set in the universe (in the sense of ZF).

Model theory gives the answer. The cardinality of propositions depends on the signature of the language, that is, its alphabet, that is, the set of symbols we agree to use to write the language. In typical mathematics, the alphabet is countable (not finite, because we allow all x_n, y_n, α_n, and so on, for n natural number).

Here, the author is using a countable alphabet to compute the cardinality of all proofs, but then, to compute the cardinality of all propositions, they are using an alphabet that includes all real numbers, which makes an uncountable alphabet. So of course, the cardinality of all the propositions one can make with this alphabet is as large as the real numbers.

This "enriched" alphabet has a name in model theory, but I don't remember it.

But, if you don't enrich the alphabet with all real numbers, then the cardinality of all propositions (in a finite or countable alphabet) is countable, and is equal to the cardinality of proofs.

Another proof: every mathematical proposition can be part of a mathematical proof, and we can even see propositions as 1-line proofs. So there is an injection from the propositions to the proofs, so there are “less” propositions than there are proofs, so we can’t have uncountable propositions with countable proofs.

The author argues that physics allowed to make a breakthrough in terms of computer science, due to the rise of quantum information, and thus physics brought a lot to mathematics. First of all, I find it very strange that, despite being exactly within the topic of the article, quantum Turing machines are never mentioned, neither are qbits.

While it is true that quantum information is a breakthrough, based on the arguments of the author about Turing machines, I would argue that quantum information brought a lot to computer science, and not exactly to mathematics (unless the author considers that computation theory, or even computer science, contains all of mathematics, which we may argue if we refer to the Curry-Howard correspondence).

The insight provided by the author is still very interesting. As a mathematician myself, I am aware of the link between physics and mathematics and I find the point of view of the author, about mathematics being dependent on physics, very intriguing. However, the paper would require more revisions to make things clearer and fix the points I raised.