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Decay Characteristics of Neutron Excess Aluminum Nuclei

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Abstract

In neutron star mergers, neutron excess nuclei and the r-process are important factors governing the production of heavier nuclear systems. A single-particle model evaluation of aluminum nuclei suggests that the heaviest Z = 13 nucleus will have mass 53 with filling of the $1f_{5/2}$ neutron shell. A = 40 – 53 aluminum isotopes have limited experimental half-life data, but the model predicts beta decay half-lives in the range of 0.534 – 4.62 ms. Based on previous calculations for Z = 9 -12, 20, 26, and 30 systems and comparisons to the ^{40}Al – ^{43}Al calculations summarized in the Japanese Nuclear Data Compilation, the single-particle model results likely overestimate the half-lives of A = 40 – 53 neutron excess aluminum nuclei.

1.0 Introduction

The nucleosynthesis of heavy elements occurs by three basic processes that add protons or neutrons to a nuclear system[1,2]. The p-process adds protons and the s- or slow process and r- or rapid process adds neutrons. Capture of protons by nuclear systems produces predominantly proton-rich nuclei that tend to decay by positron emission and electron capture[1,2]. Neutron capture creates neutron-rich nuclei, and the resulting nuclear systems depend upon the rate of neutron addition and the beta decay rates of the residual nuclei.

In the s-process neutron capture chain, the time between successive neutron captures is sufficiently long for the product nucleus to beta decay to a stable system. Within the r-process, the time between neutron captures is too short to permit decays except for very rapid beta transitions. Therefore, the r-process must occur in an environment that has a high density of neutrons. The s-process typically occurs in red giant stars. The r-process occurs in a variety of astronomical events, including supernovae explosions and stellar mergers.

Binary neutron star or neutron star and stellar-mass black hole mergers can form a massive rotating torus around a spinning black hole[1]. The matter ejected from these structures and from supernovae explosions is an important source of rapid neutron capture (r-process) nucleosynthesis[1]. Fully understanding the r-process requires knowledge of the properties of neutron excess nuclei involved in

creating heavy nuclear systems. Unfortunately, the majority of these neutron excess systems have never been studied[2].

Closing this knowledge gap was a motivation for funding facilities for rare-isotope beams (FRIB) constructed at research facilities located around the world. These facilities are located at RIKEN (Japan)[3,4], GSI (Germany)[5], and Michigan State University (US)[6]. The FRIB facilities enable a new class of experiments to determine the physical properties needed by theoretical models of the structure of unstable neutron excess nuclei. Theoretical studies would complement the forthcoming experiments that will provide critical information on the unstable nuclei that must be understood in order to explain nuclear abundances observed in the universe[2]. In particular, the study of neutron excess systems and their decay properties are significant considerations in understanding the r-process, and its importance in producing the observed elements in the universe.

The study of neutron excess systems is also important for studying nuclear decay properties, nuclear structure under extreme conditions, and nuclear reaction mechanisms. Existing theoretical models have not been extensively applied to many of these neutron excess nuclei.

This paper attempts to partially fill the void by calculating the decay properties of neutron excess systems that are important in nucleosynthesis. These theoretical studies should also assist in planning future experiments associated with neutron excess systems that are far removed from the line of stability.

Neutron excess nuclei that merit study occur throughout the periodic table[2-7] including nuclei in the $Z \leq 32$ range[7]. Although neutron excess nuclei occur throughout the periodic table, this paper focuses on aluminum systems as part of a continuing investigation of neutron excess nuclei that are of potential astrophysical significance[8-14]. Previous publications addressed neutron excess calcium[8], iron[9], fluorine[10], zinc[11], neon[12], sodium[13], and magnesium[14] systems.

The study of light nuclear systems, including aluminum, is important for a comprehensive astrophysical interpretation of nucleosynthesis. For example, Terasawa et al. [15] studied the role of light neutron-rich nuclei during r-process nucleosynthesis in supernovae. Specifically, Ref. 15 noted that light neutron excess systems can significantly affect the heavy-element abundances.

Recent studies emphasize the importance studying aluminum isotopes as well as their astrophysical significance. These studies include both theoretical as well as experimental efforts.

Refs. 16 and 17 illustrate the importance of aluminum isotopes in astrophysical studies. These studies involve Wolf-Rayet and asymptotic giant branch (AGB) stars[16]. Studies of galactic cosmic rays[16] also illustrate the importance of aluminum isotopes in astrophysical applications.

In Wolf-Rayet and AGB stars, the ^{26}Al(p, γ)^{27}Si reaction[16] is expected to govern the destruction of the cosmic γ-ray emitting nucleus ^{26}Al. The rate of this reaction is uncertain. Margerin et al.[16] present a high-resolution inverse kinematic study of the ^{26}Al(d, p)^{27}Al reaction as a method for constraining the strengths of key astrophysical resonances in the ^{26}Al(p, γ)^{27}Si reaction. Ref. 16 presents an example of the importance of aluminum isotopes and their underlying nuclear structure in understanding astrophysical phenomena.

M. H. Israel et al.[17] report new elemental source abundances from carbon to nickel nuclei for galactic cosmic rays, using observations from the Cosmic Ray Isotope Spectrometer (CRIS) on board the National Aeronautics and Space Administration Advanced Composition Explorer spacecraft. Abundances were calculated using CRIS energy spectra at energies below 550 MeV/nucleon from the 1997–98 and 2009–10 solar-minimum periods, as well as from the 2001–03 solar-maximum period[17]. These new results, illustrate the importance of aluminum isotopes in understanding the space radiation environment.

Theoretical studies have investigated a number of aluminum isotopes. For example, shell model calculations have been utilized to investigate the energy levels of aluminum isotopes ^{24}Al and ^{25}Al[18] and ^{28}Al and ^{29}Al[19].

Experimental studies have also investigated aluminum isotopes and their nuclear physics as well as astrophysical importance. Gamma-ray and fast-timing spectroscopy were used to study levels in ^{30}Al populated following the beta decay of ^{30}Mg[20]. Five new transitions and three new levels were observed in ^{30}Al. Ref. 20 continues to emphasize the importance of studying aluminum isotopes.

Ref. 21 studied the nuclear properties of a series of aluminum isotopes from ^{26}Al to ^{31}Al using Collinear Laser Spectroscopy. The magnetic dipole and electric quadrupole moments were determined. These moments are sensitive to the structure of the nucleus and can be used to evaluate the adequacy of the normal shell model. Deviations could suggest a deviation in expected characteristics of the strong force. These theoretical aspects can affect the formation of aluminum and heavier isotopes during nucleosynthesis.

Measurements of neutron capture cross sections of neutron-rich nuclei are essential for understanding nucleosynthesis and stellar evolution processes. These cross sections are relevant for r-process nucleosynthesis, and were investigated in Ref. 22. These systems included ^{34}Al and ^{35}Al.

The $^{27–32}$Al isotopes were studied via high-resolution collinear laser spectroscopy at ISOLDE-CERN[23]. Hyperfine spectra of these aluminum isotopes were measured. These detailed measurements assist in developing comprehensive theoretical models that improve the understanding of nucleosynthesis.

2.0 Calculational Methodology

A variety of models could be applied to the investigation of neutron excess nuclei. These vary in sophistication, but the proposed model utilizes a basic single-particle approach. This is a reasonable first step because there are uncertainties in the nuclear potential that likely are more significant than the limitations introduced by a single-particle approach.

Since the method for calculating single-particle energies in a spherically symmetric potential is well-established only salient features are provided. The model used to describe the particle plus core system represents an application of the standard method of Lukasiak and Sobiczewski[24] and Petrovich et. al.[25]

The binding energy E_{NLSJ} of a particle in the field of a nuclear core is obtained by solving the radial Schrödinger Equation

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) - E_{NLSJ} - V_{LSJ}(r)\right]U_{NLSJ}(r) = 0 (1)$$

where r is the radial coordinate defining the relative motion of the nuclear core and the particle; $V_{LSJ}(r)$ is the model interaction; E_{NLSJ} is the core plus particle binding energy; $U_{NLSJ}(r)$ is the radial wave function; and L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively. The N quantum number is the radial quantum number, and μ is the reduced mass.

The method of searching for E_{NLSJ} is provided by Brown, Gunn, and Gould[26], and the methodology of Ref. 27 is utilized to obtain a converged solution. Refs. 8 - 13 and 25 provide a more complete description of the model, its numerical solution, and further definition of the individual terms appearing in Eq. 1.

3.0 Nuclear Interaction

Nuclear stability with respect to alpha decay, beta decay, positron decay, and electron capture is addressed using the method previously published by the author and coworkers[8-14, 25] that is similar to the approach of Ref. 28. The single-particle level spectrum is generated using a Woods-Saxon potential. Parameters of the potential are obtained from a fit to the single-particle energy levels in ^{209}Pb and ^{209}Bi performed by Rost[29]. The central potential strength of the Rost interaction[29] has a standard form and can be explicitly defined as

$$V_0 = 51.6\left[1 \pm 0.73\frac{N-Z}{A}\right] (2)$$

where the upper (lower) sign applies to protons (neutrons). The remaining parameters were held constant and are given by Rost[29]: r_0 = 1.262 (1.295) fm, r_{so} = 0.908 (1.194) fm, a = 0.70 (0.70) fm, and γ = 17.5 (28.2) for protons (neutrons) [25,29]. The spin-orbit interaction strength V_{so} is related to γ by the relationship[29]:

$$V_{so} = \frac{\gamma V_0}{180} (3)$$

The scaling relationships of Eqs. 2 and 3 yield reasonable fits to observed single-particles levels in ^{120}Sn and ^{138}Ba. The pairing correction term of Blomqvist and Wahlborn[30] is used in the calculations presented herein. The pairing correction improves the predicted energies of occupied levels in ^{120}Sn, ^{138}Ba, and ^{208}Pb[25].

When applied to specific nuclei, this methodology requires modification. For example, Ray and Hodgson[31] note that ^{40}Ca and ^{48}Ca require different potentials to properly fit their single-particle level structure. Schwierz, Wiedenhöver, and Volya[32] also investigated ^{40}Ca and ^{48}Ca and noted that a proper fit to the single-particle levels required a different potential for each energy level. Difficulties in the selection

of an appropriate potential is an additional motivation for the utilization of a single-particle model and was noted in studies of neutron excess calcium[8], iron[9], fluorine[10], zinc[11], neon[12], sodium[13], and magnesium[14] nuclei. Similar issues also apply to aluminum systems.

In view of the results of Refs. 31 and 32, the following modification is made to obtain the aluminum potential strength (V_A):

$$V_A = 51.6\lambda\left[1 \pm 0.73\ \frac{N-Z}{A}\right][1 \pm a(A)]MeV(4)$$

where λ is a potential strength multiplier that is selected to ensure consistency with available data, and a(A) is a constant that is introduced to account for the variations in potential strength with A[31,32]. In previous excess neutron nuclei calculations for calcium[8], iron[9], and zinc[11], a value of λ = 1.0 was utilized. A λ value of 1.5 for fluorine[10], neon[12], sodium[13], and magnesium[14] was determined by the available experimental data[33-35]. Given the proximity to the A = 9 - 12 systems, a value of λ = 1.5 is also utilized for aluminum. Since the paper's primary purpose is investigation of the neutron excess nuclei, determining a common a(A) value for the heaviest aluminum systems is desirable.

The heaviest mass A = 13 isotope[33-35] suggested experimentally is ^{39}Al. Given the expected order of energy levels, ^{39}Al would have a $1f_{7/2}$ neutron single-particle level structure. Isotopes heavier than ^{39}Al would require filling of the $1f_{7/2}$ and the more weakly bound $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutron single-particle levels. The possibility of bound aluminum isotopes with A ≥ 40 is addressed in subsequent discussion. Calculations incorporated into the Japanese nuclear data compilation[35] provide calculated half-lives for ^{40}Al, ^{41}Al, ^{42}Al and ^{43}Al.

4.0 Calculation of Half-Lives

Using Eq. 4, single-particle levels are calculated for A ≥ 22 aluminum isotopes. A ≥ 22 aluminum nuclei were evaluated for stability with respect to alpha decay, beta decay, positron decay, and electron capture. These calculations were performed to ensure that the nuclear structure contained no interloping states or structural defects, and that any decay modes in conflict with data were identified.

The decay modes and half-lives of 53 ≥ A ≥ 22 aluminum isotopes are summarized in Table 1, and compared to available data[33-35] and calculations incorporated in the Japanese data compilation[35]. The alpha decay energies are calculated using the relationship based on Ref. 36

$$Q_\alpha = 28.3MeV - 2S_n - 2S_p(5)$$

where S_n and S_p are the binding energies of the last occupied neutron and proton single-particle levels, respectively. Alpha decay half-lives can be estimated from Q_α using standard relationships[24]. Fortunately, no alpha decay modes occurred in the Table 1 summary of 53 ≥ A ≥ 22 aluminum isotope decay properties.

The beta decay half-lives are determined following the log ft methodology of Wong[36]. Allowed (first forbidden) transition half-lives were derived using the values of log ft = 5 (8). Given the uncertainties in the

calculated level energies, second and higher order forbidden transitions were not determined. Positron and electron capture half-lives were determined following the approach of Ref. 24.

5.0 Model Issues

Spherical single-particle energy level calculations produce reasonable results for alpha, beta, positron, and electron capture transitions[8-14, 28-32]. However, these calculations are not expected to accurately model the very short-lived proton decay mode of ^{21}Al[33-35]. Since ^{21}Al is far removed from the neutron excess aluminum isotopes of interest in this paper, this system is not addressed. In addition, very heavy aluminum isotopes have the potential to decay via neutron emission modes. However, these decays have not been observed in aluminum[33-35]. The single-particle model is not the best approach for neutron emission calculations, and these decay modes are not included in this paper. Therefore, the results for the heaviest neutron excess aluminum nuclei only include the alpha decay, beta decay, positron decay, and electron capture modes. Except as noted previously, the single-particle model should provide reasonable results for the systems considered in the paper.

6.0 Results and Discussion

Using Eq. 4, the a(A) value was varied in increments of 0.0001 to assess the applicability of the proposed model to predict the decay properties of most $53 \geq A \geq 22$ aluminum isotopes. In view of uncertainties in the model and associated interaction, a smaller increment was not deemed to be justified for most aluminum systems. However, for nuclei that have half-lives that deviate from stability trends in neighboring systems, a smaller increment was utilized. For example, a(A) was adjusted in increments of 0.00001 for the stable ^{27}Al system.

The issues associated with fitting all calcium, iron, fluorine, zinc, neon, sodium and magnesium nuclei with a single potential[31,32] were noted in Refs. 8-14. These considerations are also applicable to the aluminum systems considered in this paper.

Table 1 summarizes the complete set of $53 \geq A \geq 22$ aluminum isotopes considered in this paper. The lighter $53 \geq A \geq 22$ aluminum isotopes fill the $1d_{5/2}$ (^{22}Al – ^{27}Al), $2s_{1/2}$ (^{28}Al – ^{29}Al), $1d_{3/2}$ (^{30}Al – ^{33}Al), and $1f_{7/2}$ (^{34}Al – ^{39}Al) neutron single-particle levels. These systems are the heaviest aluminum systems noted in Ref. 33 – 35 that have been observed experimentally. ^{34}Al – ^{39}Al partially fill the $1f_{7/2}$ neutron single-particle level. Given the extrapolation used in formulating the single-particle potential of Eq. 4, the results become more uncertain due to the paucity of data for A>39 aluminum isotopes. The heavier $53 \geq A \geq 22$ aluminum isotopes, that complete the $1f_{7/2}$ and fill the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutron single-particle levels, are also summarized in Table 1. These systems represent the heaviest possible neutron excess systems that would occur in the Z=13 system.

Table 1

Calculated Single-Particle and Experimental Decay Properties of Aluminum Systems with 22 ≤ A ≤ 53

Nuclide	a(A)	Half-Life (Decay Mode)[a,b]	
		Experiment	This Work
^{22}Al	+0.0205	90 ms (β^+)[a]	90.0 ms (β^+)[c]
^{23}Al	+0.0455	0.45 s (β^+)[a]	0.451 s (β^+)[c]
^{24}Al	+0.0483	2.07 s (β^+)[a]	2.07 s (β^+)[c]
^{25}Al	+0.0363	7.17 s (β^+)[a]	7.17 s (β^+)[c]
^{26}Al	+0.06227	7.1x10^5 yr (β^+)[a]	1.15x10^5 yr (β^+)[c] 1.23x10^3 yr (EC)
^{27}Al	+0.03000	stable[a]	stable
^{28}Al	+0.0298	2.25 min (β^-)[a]	2.23 min (β^-)[d]
^{29}Al	-0.0018	6.5 min (β^-)[a]	6.46 min (β^-)[d]
^{30}Al	-0.0371	3.68 (β^-)[a]	3.68 s (β^-)[e]
^{31}Al	-0.0235	0.64 s (β^-)[a]	0.641 s (β^-)[e]
^{32}Al	+0.0643	32 ms (β^-)[a]	32.0 ms (β^-)[e]
^{33}Al	+0.0324	41.7 ms (β^-)[a]	41.7 ms (β^-)[e]
^{34}Al	+0.0013	56 ms (β^-)[a]	56.0 ms (β^-)[e]
^{35}Al	+0.0025	38 ms (β^-)[a]	38.0 ms (β^-)[e]
^{36}Al	-0.0502	90 ms (β^-)[a]	90.0 ms (β^-)[e]
^{37}Al	+0.0405	11 ms (β^-)[a]	11.0 ms (β^-)[e]

Table 1 (Continued)

Calculated Single-Particle and Experimental Decay Properties of Aluminum Systems with 22 ≤ A ≤ 53

Nuclide	a(A)	Half-Life (Decay Mode)[a,b]	
		Experiment	This Work
^{38}Al	+0.0474	8 ms (β^-)[a]	8.00 ms (β^-)[e]
^{39}Al	+0.0337	8 ms (β^-)[a]	8.00 ms (β^-)[e]
^{40}Al	+0.0612	f, g	4.62 ms (β^-)[e]
^{41}Al	+0.0681	f, h	3.64 ms (β^-)[e]
^{42}Al	+0.0750	f ,i	2.92 ms (β^-)[e]
^{43}Al	+0.0819	f, ,j	2.37 ms (β^-)[e]
^{44}Al	+0.0888	f	1.96 ms (β^-)[e]
^{45}Al	+0.0957	f	1.63 ms (β^-)[e]
^{46}Al	+0.1026	f	1.38 ms (β^-)[e]
^{47}Al	+0.1095	f	1.18 ms (β^-)[e]
^{48}Al	+0.1164	f	1.02 ms (β^-)[e]
^{49}Al	+0.1233	f	0.881 ms (β^-)[e]
^{50}Al	+0.1302	f	0.771 ms (β^-)[e]
^{51}Al	+0.1371	f	0.678 ms (β^-)[e]
^{52}Al	+0.1440	f	0.600 ms (β^-)[e]
^{53}Al	+0.1509	f	0.534 ms (β^-)[e]

Table 1 (Continued)

Calculated Single-Particle and Experimental Decay Properties of Aluminum Systems with 22 ≤ A ≤ 53

Nuclide	a(A)	Half-Life (Decay Mode)[a,b]	
		Experiment	This Work

[a]Ref. 33.
[b]Ref. 34.
[c]Allowed $1d_{5/2}(p)$ to $1d_{5/2}(n)$ positron decay transition.
[d]Allowed $1d_{5/2}(n)$ to $1d_{5/2}(p)$ beta decay transition.
[e]Allowed $1d_{3/2}(n)$ to $1d_{5/2}(p)$ beta decay transition.
[f] No data provided in Ref. 33 - 35.
[g] The Japanese data compilation[35] notes a calculated value of 3.74 ms for ^{40}Al.
[h] The Japanese data compilation[35] notes a calculated value of 2.20 ms for ^{41}Al.
[i] The Japanese data compilation[35] notes a calculated value of 1.46 ms for ^{42}Al.
[j] The Japanese data compilation[35] notes a calculated value of 0.989 ms for ^{43}Al.

The neutron excess systems summarized in Table 1 were based on an evaluation of alpha, beta, electron capture, and positron decay modes. Other decay modes that could possibly occur in neutron

excess systems (e.g., n and 2n) are not readily evaluated using a single particle model, and were not evaluated. The results of Table 1 must be viewed with this limitation. However, since the neutron and proton decay modes tend to be much shorter than the alpha, beta, electron capture, and positron decay modes[33-35], the model results provide upper bounds on the half-lives of neutron excess aluminum isotopes.

6.1 39 ≥ A ≥ 22 Aluminum Isotopes with Experimental Half-Life Data

^{22}Al – ^{27}Al systems were best fit with a(A) values between 0.0205 and 0.06227 with an average value of about 0.0405. The ^{22}Al – ^{27}Al nuclei fill the $1d_{5/2}$ neutron shell. ^{28}Al – ^{29}Al fill the $2s_{1/2}$ neutron shell and are best fit with a(A) values of 0.0298 and -0.0018, respectively with an average value of 0.014.

^{30}Al – ^{33}Al systems were best fit with a(A) values between -0.0371 and 0.0643 with an average value of about 0.009. The ^{30}Al – ^{33}Al nuclei fill the $1d_{3/2}$ neutron shell.

The heaviest known aluminum neutron excess systems (i.e., ^{34}Al – ^{39}Al) partially fill the $1f_{7/2}$ neutron shell. There is no experimental half-life data for A > 39 aluminum systems.

The ^{34}Al – ^{39}Al systems were best fit with a(A) values between -0.0502 and 0.0474, with an average value of about 0.013. ^{40}Al and ^{41}Al complete the $1f_{7/2}$ neutron shell. The a(A) values for A > 39 aluminum systems are determined by extrapolation utilizing the available isotope data focusing on ^{37}Al and ^{38}Al.

The a(A) values for ^{40}Al – ^{53}Al systems is based on the decreasing lifetime trends noted in the study of magnesium excess neutron systems[14]. Using the ^{37}Al and ^{38}Al values, a linear extrapolation was utilized to obtain the a(A) values for ^{40}Al – ^{53}Al. The derived values are listed in Table 1. These values are consistent with previous calculations[8-14].

The Table 1 aluminum isotope extrapolation follows the beta decay half-life trends noted in Refs. 33 – 35. These extrapolated values are also consistent with the limiting values noted for calcium (0.090)[8], iron (0.115)[9], fluorine (0.115)[10], zinc (0.095)[11], neon (0.119)[12], and sodium (0.160)[13], and magnesium (0.090)[14].

Table 1 lists the half-life of the limiting decay transition (i.e., the transition that has the shortest decay half-life). For example, ^{30}Al has four beta decay transitions that are possible within the scope of the aforementioned single-particle model (i.e., allowed $1d_{5/2}$(n) to $1d_{5/2}$(p) [4.05 h], allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) [3.68 s], allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) [10.5 h], and allowed $1d_{3/2}$(n) to $1d_{3/2}$(p) [4.20 h]). For ^{30}Al, the limiting beta decay mode is the allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) [3.68 s] transition.

With the exception of ^{26}Al, the model predicts the correct decay mode for the known 39 ≥ A ≥ 22 aluminum nuclei[33-35]. The results for the known systems summarized in Table 1 suggest that the model predictions of the neutron excess aluminum systems are reasonably credible.

For nuclei filling the $1d_{5/2}$ neutron shell, model predictions for ^{22}Al, ^{23}Al, ^{24}Al, ^{25}Al and ^{27}Al are within about 1% of the experimental positron decay half-lives[33]. ^{26}Al is predicted by the model to decay through an electron capture transition, but the data[33] suggests that the decay mode is a positron decay transition. As noted in Table 1, the model does predict an allowed $1d_{5/2}$(p) to $1d_{5/2}$(n) positron decay transition, but it

is about a factor of 6 smaller than the experimental positron decay half-life. Other than ^{26}Al, the model predicts the correct decay mode for known aluminum isotopes[33-35].

The $2s_{1/2}$ systems, ^{28}Al and ^{29}Al, are within 1% of their respective experimental beta decay half-lives[33]. Both ^{28}Al and ^{29}Al decay by an allowed $1d_{5/2}$(n) to $1d_{5/2}$(p) beta decay transition.

^{30}Al, ^{31}Al, ^{32}Al, and ^{33}Al fill the $1d_{3/2}$ neutron shell. The ^{30}Al, ^{31}Al, ^{32}Al, and ^{33}Al systems decay by an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition, and their beta decay half-lives are within 0.2% of the measured values[33].

The $1f_{7/2}$ systems, ^{34}Al – ^{39}Al, are in agreement with their respective experimental beta decay half-lives[33]. These systems decay by an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition. These are the heaviest aluminum nuclides that have measured decay half-life values and beta decay transition information[33].

6.2 53 ≥ A ≥ 40 Aluminum Isotopes without Experimental Half-Life Data

As noted in the previous section, the a(A) values were derived from a fit based on ^{38}Al and ^{39}Al that are the heaviest aluminum isotopes measured experimentally[33]. The derived a(A) values are consistent with the beta decay half-life trends noted in Refs. 8 – 14 and 33 - 35. a(A) values for 53 ≥ A ≥ 40 aluminum systems are provided in Table 1..

Table 1 also summarizes calculated single-particle decay properties of aluminum systems with 53 ≥ A ≥ 40. Although experimental data for 53 ≥ A ≥ 40 aluminum systems are not available[33-35], these are nuclei of interest in astrophysical applications[1-17].

The existence of 53 ≥ A ≥ 40 aluminum systems as predicted by the proposed model is dependent on the characteristics of the interaction of Eq. 4. Although the existence of some of these systems may be an artifact of the model interaction, their study is of critical importance in understanding the role of neutron excess aluminum systems in nucleosynthesis.

The ^{40}Al and ^{41}Al systems complete filling the $1f_{7/2}$ neutron single-particle energy level. These systems have beta decay half-life values that decrease from 4.62 to 3.64 ms. Although no data is available for the ^{40}Al and ^{41}Al systems, the calculated beta decay half-life for thesesystems is consistent with the calculations of Ref. 35. The ^{40}Al and ^{41}Al systems decay through an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition.

The ^{42}Al – ^{45}Al systems fill the $2p_{3/2}$ neutron shell. These systems also decay through an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition. The ^{42}Al – ^{45}Al beta decay half-lives decrease from 2.92 to 1.63 ms, respectively. Japanese Data Compilation[35] calculations for ^{42}Al and ^{43}Al are also consistent with the model results.

The ^{46}Al and ^{47}Al systems fill the $2p_{1/2}$ neutron shell. In a similar manner, these systems decay through an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition. The ^{46}Al and ^{47}Al half-lives are 1.38 and 1.18 ms, respectively.

$1f_{5/2}$ is the last bound neutron shell in aluminum. ^{48}Al – ^{53}Al systems fill the $1f_{5/2}$ neutron shell. These

systems decay through an allowed $1d_{3/2}$(n) to $1d_{5/2}$(p) beta decay transition. The ^{48}Al – ^{53}Al beta decay half-lives decrease from 1.02 to 0.534 ms.

No aluminum isotopes with A > 53 are predicted by the model. This occurs because the $1f_{5/2}$ neutron single-particle level is the last bound neutron state, and only 40 neutrons are bound in aluminum systems. However, in view of the model potential uncertainties, the calculated properties of the heaviest aluminum systems summarized in Table 1 are not definitive.

The predicted A = 40 – 53 aluminum isotopes have no experimental half-life data, but the model predicts beta decay half-lives in the range of 0.534 – 4.62 ms. Based on calculations in Z = 9 - 12, 20, 26, and 30 systems[8-14] and calculations summarized in the Japanese Data Compilation[35], these results likely overestimate the beta decay half-lives of these neutron excess aluminum nuclei. The model results are also likely to be an overestimate of the half-lives because the single-particle level calculations do not evaluate the short-lived neutron decay modes in the A = 40 – 53 aluminum nuclei.

7.0 Conclusions

Single-particle level calculations suggest that neutron excess aluminum isotopes terminate with ^{53}Al and filling of the $1f_{5/2}$ neutron single-particle level. The $40 \leq A \leq 53$ aluminum systems have predicted beta decay half-lives in the 0.534 – 4.62 ms range, and likely overestimate the actual half-life values.

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