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Peer Review

Review of: "Inequalities Revisited"

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This paper presents a groundbreaking formalism for interpreting mathematical inequalities through a geometrical lens. The core idea originates from information theory, particularly from the study of non-Shannon-type entropy inequalities, but the author extends this formalism to universally quantified inequalities across various branches of mathematics.

The work begins with the AM-GM inequality, showing that its validity can be completely characterized by a specific region in $\mathbb{R}^2$. The author then explores Markov's inequality in probability theory, providing an analogous geometric treatment that defines a region representing all valid pairs of expected value and threshold exceedance probability. Similarly, the Cauchy-Schwarz inequality in inner product spaces is examined through the achievable region of inner product and norm values, demonstrating that the inequality precisely characterizes the boundary of this region when the vector space dimension is at least two.

A significant portion of the paper is dedicated to reviewing the developments in entropy inequalities since the 1990s. The author recounts the emergence of non-Shannon-type inequalities and discusses how these insights have impacted several fields such as group theory, matrix analysis, Kolmogorov complexity, and quantum information. The paper includes detailed discussions on entropy cones, the verification of linear inequalities using software (e.g., ITIP, AITIP), and the distinction between Shannon-type and non-Shannon-type inequalities.

The final sections establish connections between entropy inequalities and network coding theory, conditional independence, and positive semi-definite matrices. The author argues that the same geometric formalism could be used to discover new inequalities in different areas of mathematics.

The paper introduces a unifying geometric formalism that can describe and analyze inequalities from various branches of mathematics, originating from entropy theory.

It rigorously applies this framework to classical inequalities (AM-GM, Markov, and Cauchy-Schwarz),

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showing that each defines a sharp boundary of achievable pairs or triples of quantities. The author provides new proofs and interpretations of these inequalities using this geometric model and characterizes their ``tightness'' and completeness.

The work bridges the gap between traditional mathematical inequalities and modern information-theoretic entropy constraints, including discussions on the nature of entropy cones and the distinction between Shannon-type and non-Shannon-type inequalities.

The paper appears original and builds upon the author's prior foundational work in entropy theory. The paper opens new theoretical pathways and generalizes classical results in a meaningful way. It will be of significant interest to readers in information theory, inequality theory, and applied mathematics.

Declarations

Potential competing interests: No potential competing interests to declare.

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