Unflavored Excitations of Nucleon-Nucleon States in Terms of a First-Order Mass Formula

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Abstract

The possibility that hadrons could exist with structures beyond conventional qq or qqq quark configurations was noted by Gell-Mann[1]. Clement et *al.*[2] provide an example of these exotic hadrons in the unflavored dibaryon sector as illustrated through both elastic nucleon-nucleon (NN) scattering and NN-induced pion production.

Clement et *al.*[2] address three specific unflavored excitations of the nucleon: (1) X(2148) $J^{\pi}=2^{+}$, X(2140) $J^{\pi}=1^{+}$, and d^{*}(2380) $J^{\pi}=3^{+}$. These states are addressed in terms of a first-order hexaquark mass formula as proton (uud) and neutron (ddu) baryon clusters. The first-order hexaquark model predicts a state with a mass of 2180 MeV/c^2 with a J^{π} = 0^{+} or 1^{+} spin assignment. The model mass result is within 2% of the X(2140) $J=1^{+}$ state.

1.0 Introduction

The possibility that hadrons could exist with structures beyond conventional qq or qqq quark configurations was noted by Gell-Mann[1]. An example of these exotic hadrons has been observed in the unflavored dibaryon sector[2]. These states can be investigated through both elastic nucleon-nucleon (NN) scattering and NN-induced pion production. The dibaryon states decay into products that usually contain unflavored excitations of the nucleon.

Clement et *al.*[2] address narrow resonances near NN particle thresholds. These states are presumed to be weakly bound systems having a molecular character. In particular, Clement et *al.*[2] note that the following NN states fall into the unflavored excitations of the nucleon: (1) X(2148) $J=2^{+}$, X(2140) $J=1^{+}$, and d^{*}(2380) $J=3^{+}$. Ref. 2 also notes the existence of possible ΔN P wave states. However, these states are outside the scope of the first-order mass formula, discussed in Section 2.0, because the model assumes L = 0 between the baryon clusters. The hexaquark is modeled as a neutron plus proton configuration.

Previous theoretical studies investigated hexaquark candidates based primarily on combinations of baryon structures. Theoretical studies include a (1) chiral constituent quark model[3], (2) baryon-antibaryon resonance model using a chromomagnetic interaction[4], (3) dispersion relation technique[5], (4) string model using the strong coupling regime of quantum chromodynamics[6], and (5) coupled channels resonating group method[7].

The unflavored NN states will be investigated in terms of the applicability of a first-order hexaquark mass formula. This mass formula approach has been successfully applied to tetraquark[8-20], pentaquark[21-24], and hexaquark structures[25-27].

2.0 Model Formulation

Zel'dovich and Sakharov[28,29] proposed a semiempirical mass formula that provides a prediction of mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These mass formulas are used as the basis for deriving a first-order hexaquark mass formula. In particular, the model utilized in this paper assumes the hexaquark is partitioned into two baryon clusters with the interaction between the clusters providing a minimal contribution to the mass. In addition, zero angular momentum is assumed to exist between the clusters.

The baryon (b) mass (M) formula of Refs. 28 and 29 is:

$$M_b = \delta_b + m_1 + m_2 + m_3 + Z (1a)$$

$$Z = \frac{b_b}{3}\left[\frac{m_0^2}{m_1 m_2}\sigma_1 \cdot \sigma_2 + \frac{m_0^2}{m_1 m_3}\sigma_1 \cdot \sigma_3 + \frac{m_0^2}{m_2 m_3}\sigma_2 \cdot \sigma_3\right] (1b)$$

where the m_i labels the three baryon quarks (i = 1, 2, and 3) and δ_b and b_b are 230 MeV and 615 MeV, respectively[29]. In Eq. (1), m_1 , m_2, and m_3 are the mass of the first, second, and third quark comprising the baryon, m_0 is the average mass of first generation quarks[30,31], and the σ_i (i = 1, 2, and 3) are the spin vectors for the quarks incorporated into the baryon.

For a particle with a total baryon spin 1/2, the following prescription is used if the baryon (comprised of three quarks q_1, q_2, and q_3) contains two identical quarks[29] q_2, and q_3

$\sigma_2 \cdot \sigma_3 = 1/4$ (2)

$\sigma_1 \cdot \sigma_2 = \sigma_1 \cdot \sigma_3 = -1/2$ (3)

For completeness, the reader should note that $\sigma_i \cdot \sigma_j$ has the value +1/4 for a J= 3/2 baryon. In addition, these basic $\sigma_i \cdot \sigma_j$ relationships must be modified if the baryon contains three different quarks. The methodology is detailed and described in Ref. 29.

In formulating the hexaquark mass formula, effective quark masses provided by Griffiths[30] are utilized. The effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. These masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3 e), u(+2/3 e)], [s(-1/3 e), c(+2/3 e)], and [b(-1/3 e), t(+2/3 e)][31]. The three generations are specified by the square brackets and the quark charges (in terms of the unit charge e) are given within parentheses.

The weak coupling structure is incorporated to minimize model complexity, which is consistent with an initial first-order formulation. In addition, the hexaquark mass formula is assumed to have the following form:

$$\mathbf{M = M_b(1) + M_b(2) + \Phi \ (4)}$$

where $\mathbf{\Phi}$ defines the interaction between the baryon clusters, and $\mathbf{M_b(i)}$ represents Eq. 1 for the ith cluster (i=1,2). Within the scope of this mass formula, the baryon-baryon cluster interaction is assumed to be sufficiently small, relative to the cluster masses, to be ignored. Accordingly, Eq.4 represents a quasimolecular six quark system whose basic character is a weakly bound baryon-baryon system.

The mass relationships of Eqs. 1 and 4 do not predict the total angular momentum of the final hexaquark state, but do permit primitive spin coupling to be specified for the individual baryon clusters. In addition, the angular momentum between the clusters is assumed to be zero. Specific angular momentum assignments based on the first-order mass formula for the hexaquark state is defined in terms of the coupling of (udd and uud) neutron plus proton states.

$$\mathbf{J^{\pi}\ (n + p) = 1/2^{+} + 0 + 1/2^{+}\ (5)}$$

Eq. 5 suggests possible 0^+ and 1^+ spin assignments for the modeled n + p hexaquark.

3.0 Weak Coupling Approximation

The weak coupling approximation is based on fundamental quantum chromodynamics (QCD) arguments[32,33]. Recent one boson exchange as well as lattice gauge calculations provide numerical simulation results that support these QCD arguments[32,33].

3.1 Fundamental Quantum Chromodynamics Arguments

The quark charges are related to the number of colors (N_c) incorporated into the QCD formulation[32,33]. For example, the first generation quark charges within $SU(N_c)$ are:

$$Q_d = \frac{1}{2}\left(\frac{1}{N_c} - 1\right) (6)$$

$$Q_u = \frac{1}{2}\left(\frac{1}{N_c} + 1\right) (7)$$

For conventional QCD using 3 colors, the expected d and u electric charges are obtained. The importance of QCD expansions involving $1/N_c$ is outlined in this section to illustrate the weak coupling assumption.

A key assumption of the first-order mass formula of Eq. 4 is weak coupling between the two clusters. In particular, the model utilized in this paper assumes the hexaquark is partitioned into two clusters with the interaction between the clusters providing a minimal contribution to the hexaquark mass. Within the scope of this mass formula, the cluster-cluster interaction in Eqs. 1 and 4 is assumed to be weak, and sufficiently small to be ignored.

This assumption is justified because QCD can be investigated as an expansion in $1/N_c$[34,35]. The large N_c limit reduces to a field theory of weakly interacting meson-like objects. The physical situation with $N_c = 3$ retains many of the characteristics of the $N_c \rightarrow \infty$ limit, and further justifies the weak coupling approximation.

The $1/N_c$ expansion[36,37] is well accepted in elementary particle physics and leads to the Okubo-Zweig-Iizuka (OZI) rule[36-38] and the Skyrme model[39,40]. In fact, in the $1/N_c$ expansion, QCD is reduced to a weakly interacting meson theory, and the meson-meson and associated cluster-cluster interactions are regarded to be small[34,35]. This situation is also a characteristic of Eqs. 1 and 4.

4.0 Results and Discussion

Using Eqs. 1 and 4, the first-order model predicts a state with a mass of 2180 MeV/c^2 with a $J^\pi = 0^+$ or 1^+ spin assignment. The model mass result is within 2% of the X(2140) $J^\pi=1^+$ state. The X(2148) $J^\pi=2^+$ and $d^*(2380)$ $J^\pi=3^+$ states noted by Clement et *al.*[2] are within the uncertainty of the first-order mass value. However, the spin and parity assignments do not match the model predictions.

5.0 Model Uncertainties and Weaknesses

Although the first-order mass approach provides reasonable results in the description of tetraquark[8-20], pentaquark[21-24], and hexaquark structures[25-27], it has a number of uncertainties and weaknesses. These include:

1. The quark masses are model dependent and can assume a range of values[30,31]

2. The cluster-cluster interaction is not definitively known. Although it is small relative to the combined cluster masses forming a tetraquark, pentaquark, or hexaquark, its value has not been well established. However, fundamental QCD arguments, one boson exchange calculations, QCD Calcuations[41,42], and quark model calculations[43] suggest the cluster-cluster interaction is negligible relative to the individual cluster masses.

3. The angular momentum coupling is primitive. Hexaquark angular momentum is defined by the individual cluster J^π values, but zero angular momentum is assumed between the clusters. This approach allows a range of J^π values, but does not define a definitive angular momentum configuration.

4. Only quark-quark effects are allowed in Eq. 1. There is no consideration of higher order or gluon influenced terms.

5.0 Conclusions

The first-order model predicts state with a mass of 2180 MeV/c^2 with a $J^\pi = 0^+$ or 1^+ spin assignment. This model result is within 2% of the mass of the X(2140) $J^{\pi}=1^{+}$ state. The X(2148) $J^{\pi}=2^{+}$ and d^{*}(2380) $J^{\pi}=3^{+}$ states noted by Clement et *al.*[2] are within the uncertainty of the first-order mass value. However, the spin and parity assignments do not match the model predictions.

The predicted masses of these states provide a crucial test of the validity of the proposed first-order mass hexaquark formula and its weak coupling structure. Moreover, this mass formula provides a general framework to calculate hexaquark masses.

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