Review of: "A Random Journey Through the Math of Gambling"

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Potential competing interests: No potential competing interests to declare.

The paper discusses (variants of) the ballot problem by connecting it to a discrete random walk and counting patterns of up-down paths / staircase walks. For closed-form formulas, enumeration is approximated by probabilities of suitably adjusted random walks.

My understanding is that the overall intent of this work is to be an educational contribution by giving a reader-friendly introduction to the topic. The topic is interesting, and the author does a good job of making the paper self-contained and the mathematical reasoning elementary. I consider this educational goal and accessibility of math a strong plus.

However, I am concerned about a large body of related work that has not been referred to, including other educational works on the same topic, which somewhat undermines the educational purpose of the article. Below are some results that should be credited or compared to:

1. It is well known that counting non-negative / positive up-down paths can be done with Catalan numbers. In particular, Proposition 2.2 should be credited to combinatorial textbooks or educational surveys, see for example. The paths counted in Equation (5) are also known as Dyck paths.
2. Counting up-down paths is a well-known tool for analysing ballot problems, see [1].
3. Quantitative results obtained by random walks are also well known. For example, Proposition 3.3 is known as the “arcsine law” and appears in lectures and educational materials about ballot problems [3].
4. It might be worth noting that further generalizations are possible. For instance, [2] analyses basketball walks.
5. Results on the ruin problem, such as the version with a biased walk and two barriers, should be credited to the literature.

Summing up, I think that the paper in its current version does not clearly make its contribution. Whether the author offers a more elementary proof, a simulation that has not appeared before, draws a connection that is novel, or does a survey of related work, I believe this should be clearly compared to what other authors have developed in the same context.

[1] Renault, Marc. "Four proofs of the ballot theorem." *Mathematics Magazine* 80, no. 5 (2007): 345-352.

[2] Bettinelli, Jérémie, Éric Fusy, Cécile Mailler, and Lucas Randazzo. "A bijective study of basketball walks." Seminaire Lotharingien de Combinatoire 77, no. Art. B77a (2017): 1-24.

[3] Vishesh Jain, “Lecture notes on statisitcs”, https://jainvishesh.github.io/STATS217_Lecture3.pdf