

On possible formal expressions of *catuṣkoṭi* and *Śūnyatā*

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Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.

Abstract

Buddha *Sakyamuni* famously preached *Śūnyatā* which is recorded in *Prajnaparamita Sutra*. Also during his numerous Dialogical engagements with the contemporary aspirants he used to talk about four distinct possible standpoints to a given state of affair - the famous *catuṣkoṭi*. Most remarkable among the four *koṭies* is the “both” option - A and not-A

This option continued to stimulate discussion for centuries about logical paradox as well as interpretational enigmas entailed. Indian logicians continued to debate whether Buddhist logic *admits* contradiction or is a warning *against* contradiction. Very recently modern logicians - particularly those who are advocating *paraconsistent logic* are getting interested in Buddhist hostage of “both” option. Some of them are even hopeful to find the signature of contradiction in quantum physics. However opinions differ over a wide spectrum of disagreements.

Here we will sketch a brief overview referring finally to category theory and its relative merits compared to the other proposals to formally express *Śūnya* and *catuṣkoṭi*.

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Part I

1. Introduction

Buddha *Sakyamuni* famously preached *Śūnyatā* in *Rājgir* which was recorded in *Prajñapāramita sūtra*. Also his numerous Dialogical engagements with others reveal his ontological standpoints regarding a given state of affair - *catuṣkoṭi*.

Here we will first discuss about the relationship of *catuṣkoṭi* and *Śūnyatā* in Buddhist literature. In section II we will touch upon some modern logical counterparts particularly in relation to *both* option in *catuṣkoṭi*. And finally in section III we will compare category Theory with other proposals to make sense of the relative merit to talk about *Śūnyatā* and *catuṣkoṭi*.

2. Buddha's standpoint in Dialogical engagements

Queen *Mallika* of *Kosha* famously asked the Buddha a few questions regarding *ātman* and *Prithvī* –

1. Is *Prithvī* eternal? 2. Is *Prithvī* non-eternal? 3. Is *Prithvī* finite? 4. Is *Prithvī* Infinite? 5. Is Body different from *ātman*? 6. Whether *Tathāgata* can live still after Death? 7. Whether *Tathāgata* cannot live after Death? 8. Whether He can be in a state both living and non-living? 9. Whether He can take rebirth after Death? ^[1]

Buddha is known to have firmly refused to answer these questions and he declared these questions as irrelevant as they can't help to attain *Nirvāṇa*. The *Buddha* is famously known to have declined to subscribe to any binary framework of response in terms of simple **Yes** and **No**. He asserted that, the wrongly motivated questions should not be answered. (*Digh Nikaye*, Volume 1). However one must take note that, Buddha's responses during most of his dialogical engagements were not about any empirical situation.

What messages can we possibly read from the Buddha's refusal to answer these questions? Was this a strategy to dispense with *Metaphysics* or did he implicitly advocate a separate category for these type of questions which were considered to be *avyakrita* or *unanswerable*?

In course of preaching, Buddha is occasionally found to have spelled about four possibilities. Given any question, Buddha advocated *four possibilities* - *yes*, *No*, *Both*, *Neither* - the celebrated *catuṣkoṭi* (Greek *Tetra* lemma). Formally, given a state of affair *A*, there can be four possibilities like *A*, *not- A*, *A not-A*, *not (A not-A)*.

However, Buddha doesn't seem to have endorsed explicitly in favor of any possibility, but trace of his departure from *common sense ontology* is often claimed to have been endorsed in his advocacy for 'both' option together with the other three *faithful* to common sense - crux of the present day debate (championed notably by Graham Priest^[2]) about whether Buddha implied *contradiction* (*paraconsistency*) as an essential part of our understanding of reality or not revolves around this. Perhaps this is how we can best relate or reconstruct *Buddha's agenda against Metaphysics* with the way Nāgārjuna substantiated later in terms of *self-contradictory nature of all our means of acquiring knowledge* and argued reality (or no-reality) as even *beyond* the framework of four possibilities and hence *catuṣkoṭi binirmukto* — *Śūnya*!

This is interesting to note that though the appearance of Zero as a numerical digit is difficult to trace to its historical origin, its appearance in *grammar* and other *proto-philosophical* literature as a semantic category is quite evident.

But *Śūnya* does not logically follow from *catuṣkoṭi*. Neither the "both" option within the framework of *catuṣkoṭi* is applicable for our everyday reality we can have common sense access. Nāgārjuna must have had a tough time to relate the *Buddha's* teaching of *Śūnya* with *catuṣkoṭi* — *catuṣkoṭi binirmukto* — *Śūnya*

Nāgārjuna (2th CE), in fact, raised serious doubt about the *possibility to acquire knowledge of reality* in its pristine form. His exposition provided pioneering motivation for *Asaṅga* and *Vasubandhu* during 5th CE to take care of what we describe in present terminology as Buddhist epistemology, and this was put in a reasonably solid ground not much later by *Dignāga* in *Pramāṇa-samuchchaya*.

Ever since, the scholarly Dialog in Indian tradition was added with a new dimension. The early manual of debate or non-Buddhist *Nyaya* was generically within the scope of jurisdiction of metaphysics outlined in *Prasthānatraya*, but Dignāga can be credited to have initiated a new level of rigor in his *Nyayamukha* and *Pramāṇa-samuchchaya* that can be claimed to have marked a new beginning of apparent independence of logic from its primordial metaphysical embryo.

Part II

3. Can the concept of ‘both’ be captured within the framework of logic of vagueness?

Let us get more into the “*both*” option. Is it possible to express “both” within an adequate semantic framework? “Both” can be understood as something of ill-defined membership of a Set where the bivalence condition is not fulfilled.

In fact “both” can be arguably understood as instance of *borderline cases*. In recent literature of philosophical logic *borderline cases* are described in terms of the concept of vagueness. Vague predicates are those which can't be assigned any sharp Truth value. But the question remains that whether vagueness is purely linguistic or ontic. Is it a part of essential reality or something to do only with our linguistic representation of the world? Indeed there are numerous vague predicates like *young, tall, fair, bald* ... and like in our everyday language which can't be made further precise to be described in terms of sharp bivalence.

This situation can be schematically represented as

$$\forall x G(x) \rightarrow G(x+1) \rightarrow G(x+2) \rightarrow \dots$$

Frege famously held that a vague predicate with a fuzzy boundary of application don't have a precise logical sense. Unfortunately any natural language is full of vagueness. Can the “both” option be captured adequately within the framework of vagueness?

3.1. Vague predicates and their extension

As we know that the standard Fregean predicates of first order logic are strictly bivalent in the sense that their denotation indicates a bipartition in domain D into two disjoint Subsets - the extension of D denoted by *Ext(P)* and its complement relative to D. The objects of the domain D which belong to *Ext(P)* are characterized by property ascribed by P, while those which belong to the complement of P lack that property.

Vague predicates, by definition, can't be characterized by this kind of sharp bipartition of domains. D accommodates objects which belong neither to *Ext(P)* nor to its complement - they inhabit *both*!

They are considered to be *vague* in the sense of hosting generic semantic indecision about whether they have the considered property or not (see Terricabras and Trillas, 1989)

But the question remains that whether this kind of formal artifacts can capture the meaning of “both” in Buddhist logical

sense. In fact there are debates among the modern logicians about the meaning of vagueness as a feature of linguistic expression in question of the ascribed property of predicate P in our example, while the objects concerned in the domain are supposed to be well defined.

As Putnam used to put it, “On the metaphysical realist view there are vague concepts, vague ways of thinking, but not vague objects.” (Putnam 1983)

But in spite of this kind of skepticism on behalf of many philosophers about *real* ontic vagueness, substantial portion of recent literature on quantum foundational debate is devoted to make sense of quantum objects as *vague* in *ontic* sense (see French and Krause, 2006).

Part III

4. How Category Theory take care of this situation

It is recently claimed that Category theoretic constructions are reminiscent of the Buddhist concept of *Śūnya* (Sisir Roy and Venkat Rayudu, 2020) – “*the universal mapping property definition of mathematical objects*”; Roy and Rayudu proposes “.. where objects of a universe of discourse are defined not in terms of their contents , but in terms of their relations to all objects of the universe is reminiscent of *Śūnyatā*.”

Stated plainly, what something ‘*is*’ - according to Category Theory, is not due to what it possesses intrinsically; but by virtue of its relation to *ALL* objects of its given category. Of course the domain of discourse is restricted by the defining category itself.

In fact Buddhist philosophy is generically about *negative existential* propositions - No Self (*anātman*), impermanence (*Anitya*), No intrinsic nature (*Śūnyatā* / *niḥsvabhāva*)...

So the basic issue is to avoid unwarranted metaphysical commitments to non-existent objects/ entities. [And] needless to say that, the domain of entities for the ancient logicians were not strictly bi-partitioned in Cartesian dualistic terms as it is in practice today. So the technical assignment of the Buddhist logicians was more or less well defined - to take care of the “burden” of objective reference to something non-existent. Logic of *empty subject term* was Buddhist response to this assignment, which is well known to have been systematized after Dignāga during the 6th century. Technically speaking, the logical counterparts of category theoretic strategy can be recognized to be close to this Logic of Empty Subject Term.

Systematic treatment of these categories of existence can be said to have started with Dignāga during the 6th CE onwards. In his earlier work, *Nyayamukha*, Dignāga is explicitly about the Negative existential propositions

"[Question]: But suppose that we are to prove (not a property but) a subject (*dharmā*, property-possessor) to be existent or non-existent. For [Thesis] Primordial matter (*pradhāna*) exists.

[Reason] Because we see that the various individuals possess a (similar) general characteristic.

While some others [i.e., the Buddhists] argue:

[Thesis] [Primordial matter] does not exist.

[Reason] Because there is non- apprehension of it. (*na santi pradhanadayo nupalabdheh*)

How to explain this?

[Answer][As for the first inference,] the thesis should be formulated as “The various individuals certainly possess one and the same cause [i.e., primordial matter],” but they do not prove [directly the existence of] primordial matter [i.e., the subject]; hence, there is no error [of proving the subject of the thesis with the reason].

[As for the second inference,] when they argue that [primordial matter] does not exist

[because of non- apprehension], “non-apprehension” is a property of the imagined concept

[i.e., primordial matter] (*kalpita- syanupalabdhir dharmah*); hence, there is also no error of

[Proving] the subject of the thesis [with the reason]

Here Dignāga deals with two of the four *koties* - “primordial matter exists” and “primordial matter does not exist”.

Reflection of *Saṅkhya* is visible here as *Saṅkhya* accept primordial term as a first cause in their cosmological scheme. However as the Buddhists do not accept any first cause, “primordial matter” for them is an *empty term*.

Later Indian and Tibetan Buddhist tradition combined the approach to problem of empty subject term with the theory of *Apoha* (*exclusion*).^[3]

The problem of empty subject term is well known to have formed one of the core concerns of modern analytical philosophy as well as Buddhist philosophy right from the beginning.

Rayudu and Roy also claim that “the objective logic of perception , with percepts modeled as a category of two sequential processes (sensation followed by interpretation) and with its truth value object of four truth values , is reminiscent of Buddhist logic of *catuṣkoṭi*.” (Sisir Roy and Venkat Rayudu, 2020)

Boundary of an object A is claimed to be a sensible physical counterpart of “both” (A and not-A) [Lawvere 1991, 2003]. But does it really capture the intention of what “both” was meant for in Buddhist literature? In fact a boundary is perceived not in isolation from what it is a part, but integrally. Can this be singled out to cite as an instance of perceiving “both”!

As we mentioned before, the four *kotis* preached by the Buddha were actually traditionally interpreted to mean that all means of cognition involves *contradiction* (yes, even the other simple looking three *kotis* also!) and hence are not reliable to capture the true picture of reality. One should get beyond this; hence *Śūnya* is described as something categorically different or beyond four *koties* - “*catuṣkoṭi binirmukto ...*”. *Śūnya* in traditional Buddhist literature actually transcends all linguistic ways of articulation except by metaphorical oxymoron which inspired the Zen monks later to write oxymoronic

kōans.^[4]

So the levels meant by *catuṣkoṭi* and *Śūnya*, traditionally seen, are categorically different by definition and hence refused to be captured within the *same* formal framework. So it seems a bit hurried to translate the traditional Buddhist intension within the formal framework of *Category theory* at least the way it is. This is getting in the issue through *awrong* end. Objective logic of perception can't provide any special status of “both” option which it deserves.

Though the full story of formal counterparts is yet to be developed in adequate details, *logic of vagueness seems* to be a better candidate to talk about “both”, let alone *Śūnya*. We leave further details for another occasion.

Acknowledgement

I am thankful to Professor Mihir Kumar Chakraborty and Professor Umashankar Vyas for valuable discussion.

Notes and References

[1] *Digh Nikaye*, 1st Volume, *Majjim Nikaye*, 1st Volume, *Sanjukt Nikaye*, 3rd Volume, *Anguttarta Nikaye*, 2nd Volume

[2] Graham Priest, *The logic of the catuṣkoṭi*, Comparative Philosophy Volume 1, No. 2, (2010)

Also see : Robinson, R. (1956), “Some logical Aspects of Nāgārjuna's System” *Philosophy East and West* 6: 291- 308

Tillemans, T. (1999), “Is Buddhist Logic Non-classical or Deviant?”, Ch. 9 of *Scripture, Logic, Language: Essays in Dharmakīrti and his Tibetan successors* (Boston, MA: Wisdom Publication)

[3] Taken from, *Empty Subject Terms in Buddhist Logic: Dignāga and his Chinese Commentators*, Zhihua Yao, *Journal of Indian Philosophy* (2009)37: 383-398

[4] Experts say that it is not possible to characterize what Zen is! But Zen, in one way or another, is a battle to get beyond or transcend the dualism inherent in all our linguistic expressions. Our thoughts admit some built-in conceptual division of the world into categories! This is inherent and can't be avoided in our normal psychological process of thinking.

Koans are admittedly linguistic expression of attempts to get beyond language! This is oxymoron, but this is precisely how the *Śūnya* in Buddha's original teaching got expression as a part of spiritual practice also.

The 13th century monk *Mumon* (“No-gate”) is known to have composed forty-eight *koans*! I understand these all as expression of *Śūnya*!

koan:

- Shuzan held out his short staff and said: “If you call this a short staff, you oppose its reality. If you do not call it a short staff, you ignore the fact. Now what do you wish to call it?”

Mumon's commentary:

- If you call this a short staff, you oppose its reality. If you do not call it a short staff, you ignore the fact. It cannot be expressed with words and it cannot be expressed without words. Now say quickly what it is. (From *Godel, Escher, Bach an Eternal Golden Braid*; Douglas R. Hofstadter, Vintage Books, New York, 1989)

For a general overview of Buddhist Logic :

- Tsong ka pa (2006), *Oceans of Reasoning*: a great commentary on Nagarjuna's *Mulamadhyamakakarika*, trans. N. Samten and J. Garfield (New York, NY: Oxford University Press)
- Scherbatsky, Th. (1993), *Buddhist Logic*, Vol 1 (Delhi, Motilal Banarsidass)