

Peer Review

Review of: "Restoring Heisenberg-Limited Precision in Non-Markovian Open Quantum Systems via Dynamical Decoupling"

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Paper: Restoring Heisenberg-Limited Precision in NonMarkovian Open Quantum Systems via Dynamical Decoupling

by

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The authors started with the main concepts of quantum metrology, particularly by means of quantum Fisher information in closed quantum systems as well as in open quantum systems. The last concept allowed them to introduce the notion of restoring Heisenberg Scaling in open quantum systems via dynamical decoupling techniques. Therefore, an important theorem is introduced, namely the theorem that showed the necessary and sufficient conditions for dynamically decoupled quantum sensing.

The subject is interesting, but before giving any decision, major revisions are required. The following comments should be considered for the revised version:

1. A list of keywords is required.
2. In the abstract, "Heisenberg Scaling" should be replaced with "Heisenberg scaling".
3. In the abstract, "Dynamical Decoupling Techniques" should be replaced with "dynamical decoupling techniques".

4. In the abstract, you mentioned HS (you mean by it Heisenberg Scaling) but it is not indicated before.
5. In the abstract, the sentence "we show that HS can be achieved irrespective of whether the noise is Markovian or non-Markovian" is not clear.
6. In the introduction, at the end of the first sentence, the authors should add the original reference of Helstrom and other references. The same remark applies to the sentence "...estimate the parameter"".
7. Unfortunately, the authors forgot in many places to use links like: Moreover, However, Indeed,... For example, "By leveraging.." should be "In fact, by leveraging..."
8. In the introduction, at the end of the first paragraph, you mentioned "This represents the ultimate precision limit attainable with quantum resources" What do you mean by this? Do you mean HS in the case of nonclassical resources? You should clarify.
9. In the second paragraph of the introduction, "...challenging due to decoherence" should be "...challenging due to the decoherence phenomenon".
10. In the same paragraph, you mentioned "the precision is typically reduced to the SNL". It is not clear. Do you mean the precision is typically reduced according to the SNL?
11. In the introduction, "and, (iii) using should be "Finally, (iii) by using".
12. Finally, in the last paragraph of the introduction, you mentioned "we outline the general framework of quantum sensing in ideal closed quantum systems". But, in the main text, you have also introduced this for open systems in part B. You should also mention this in the introduction.
13. In section II, join "The ultimate precision" to the previous paragraph using a link.
14. In section II, "the parameter has value ω " should be "the parameter takes the value ω ".
15. In section II, "...state $|\psi_\omega(t)\rangle$, The QFI.." should be "...state $|\psi_\omega(t)\rangle$, the QFI..".
16. In section II, you mentioned "we see that the system achieves HS with respect to time, with the QFI scaling as t^2 ". Is it your result? If not, you should add a reference to this.
17. In Eq.(5), H_{tot} should be $H_{\text{tot}}(\omega)$. The same thing applies in the text below when you mentioned the unitary operator.
18. von Neumann should be von-Neumann.
19. Below Eq.(7), when you mention the Symmetric Logarithmic Derivative operator, add a reference. The same applies when you mention Eq.(8).
20. In Eq.(10), you added a time-dependent control Hamiltonian, namely $H_C(t)$. The idea

is interesting; I read also the appendices. But, still, something is unclear for me. Why and how? Is it correct to add a Hamiltonian (control Hamiltonian) directly to a well-known Hamiltonian (total Hamiltonian of any open system+environment)?

21. Below Eq.(16), you mentioned "the system is effectively decoupled from the environment". Why? Do you mean here that the system does not affect the environment and vice-versa? Does that mean that one cannot take into account the theory of open systems here? If yes, this is not the goal since you are interested in studying open systems.

22. After Eq.(16), you mentioned that the system is effectively decoupled from the environment. However, in theorem 1, in HSE eff, the degrees of freedom of the environment appeared by including the identity operator. Moreover, in the same equation, you have HSE. While you indicated that the system is decoupled from its environment, how is this possible?

The authors should be careful about this point.

23. After Eq.(27), $C_e(t)$ should be given in terms of cosh and sinh or indicate appendix B.

24. In Figs.(1) and (3), you plot Eq.(28), which is the well-known quantum Fisher information of a mixed state. What is its relationship with the theorem? Where is the novelty here?

The authors should be very careful here to logically connect different parts of the paper.

Attachments: available at <https://doi.org/10.32388/M7GJEP>

Declarations

Potential competing interests: No potential competing interests to declare.