Review of: "The Materiality of Numbers"

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Review of Karenleigh A. Overmann's *The Materiality of Numbers*

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Abstract

Karenleigh A. Overmann's book, *The Materiality of Numbers: Emergence and Elaboration from Prehistory to Present*, undertakes what can be described as a "Copernican Revolution" in the way we understand the relationship between numbers and the material devices we use to record and manipulate them. Its main thesis is that, rather than an a priori concept of number stored in the brain being the cause of the external representations of numbers created by humans since prehistory, it was the external material devices for counting and calculation, created in prehistory, that caused the emergence of the mental contents we call "number concepts." She argues that numbers have material roots in the counting and calculation artifacts and techniques found in the archaeological and ethnographic records, challenging the traditional Platonist view that posits numbers as non-created, non-spatiotemporal entities.

In recent years, a new trend has emerged in the philosophy of mathematics, particularly concerning the investigation of the nature of numbers. Research in this area is becoming more interdisciplinary as other fields—such as numerical cognition—are making discoveries and proposing theories that impact the way we understand our experience of numbers and mathematics. Philosophers such as Buijsman (2021), Decock (2021), dos Santos (2021a), De Cruz (2019), Menary (2015), Pantsar (2021), and Zahidi (2021) have been working on incorporating these new findings into the philosophical understanding of the nature of numbers and mathematics. Overmann's book stands as the latest major contribution to the constellation of non-philosophical works—now coming from archaeology—that may have a significant impact on the philosophical perspective on the nature of numbers.

As an archaeologist, Overmann is mainly concerned with explaining how humans adopted numbers as a way to think of quantity in the remote past. Archaeologists deal primarily with the material records left by ancient cultures and, as such, Overmann's interest lies in the material devices used by pre-historic (and contemporary) peoples to record and manipulate numbers, such as notched bones, shells, and handprints believed to be used as tallies. Her interest in the "materiality of numbers," however, is not limited to what are usually seen as material *instantiations* of numbers. For Overmann, such material devices does not simply instantiate previously existing *immaterial* numbers. Rather, she sees numbers as "essentially material in their origin" (Overmann, 2023, p. xxv). In this regard, she is aligned with central aspects of the views of the philosophers mentioned above (in fact, she cites Zahidi (2021) approvingly). As she elaborates and defends this thesis throughout the book—a thesis about the nature of numbers—it is no exaggeration to say that Overmann's book main contribution is philosophical.

The dominant view in philosophy has been that numbers are abstract entities existing outside of space and time and that numerical ideas are *a priori*, i.e., humans are believed to have number concepts independently of their senses and bodies. According to this view, numerical thinking is either innate or learned by purely rational means. As a result, notations, tallies, abacuses, finger-counting techniques and other related material devices are thought of as merely instantiating or externalizing mental contents and processes, not offering anything new. Against this view, Overmann carries out what can be described as a "Copernican Revolution" in the way we understand the relationship between numbers and the material devices (notations included) we use to record and manipulate them. The main thesis of her book is that, rather than an a priori concept of number, stored in the brain, being the cause of the external representations of number we create, it is the external, material devices we use to count and calculate that cause the emergence of the mental contents we call "number concepts." In her words:

> *Historically, archaeologists—along with pretty much everyone else in all the other disciplines—… [have seen] physical devices as external memory storage, something that aids our cognition by offloading our mental content (Overmann, 2023, p. 42).*
>
> *Once the neurocentric notion that numbers are a wholly mental phenomenon is relinquished, the reason why numbers elaborate … becomes relatively straightforward. They are not a latent mental capacity that 'switches on' in response to some (unspecified) external condition. Instead, they are the outcome of recruiting and using material devices for counting (Overmann, 2023, p. 131).*

In other words, she displaces the brain (with its supposedly a priori number concepts) from the center and places the material devices we create and use for counting and calculating at the center. Rather than being the passive recipients of a priori mental contents about numbers, "material forms used for [representing and manipulating numbers] are then considered to precede and inform the resultant numerical concepts and to act as an integral component of numerical thinking" (Overmann, 2023, p. 16).

As a competent archaeologist, she does not defend this thesis on purely a priori philosophical bases. Rather, she marshals empirical evidence from various sources to show that, to the best of our knowledge, the number systems humans use have their roots in material devices. Her strategy consists of showing that the way many ancient and contemporary cultures conceive of numbers is highly determined by the properties of the material devices they call into play. Beyond her thesis about the materiality of numbers itself, that is Overmann's second main contribution to philosophy: she gives plenty of empirical support to a thesis about a topic—the nature of numbers—which is traditionally seen as not open to empirical investigation.

One of the most important sources of evidence for the hypothesis that material devices shape numerical thinking is language. As Overmann points out, many languages have terms like "digit" that mean both finger and number, and use words for five and ten as bases for generating higher number words, highlighting the influence of finger-counting on numeracy (Overmann, 2023, p. 24-25, 77). But even before finger-counting was in use, other material devices are likely to have contributed to the emergence of the very first numerical ideas, according to Overmann. It is widely accepted that humans (and many other animals) have the capacity to subitize, i.e., to quickly and accurately recognize quantities up to about three or four. Many cognitive scientists take the existence of subitizing as an indication that number concepts are innate, but Overmann rejects this view (see, e.g., Overmann (2023, p. 50-51)). Overmann's point is that, although the ability to subitize is innate, concepts for one, two, and three are not likely to emerge while individuals are not able to think of the *oneness*, *twoness* or *threeness* of the collections they perceive as separated from the collections themselves. She points out that languages of contemporary small-scale societies in which number words have emerged recently, so that their etymology is still transparent, support this claim. For example, the word the Mundurukú use to refer to two is derived from their word for the arms and the word the Hup use to the same purpose is derived from their word for the eyes. The Xerénte use a similar strategy to refer to three: their word for three is derived from their word for deer footprints (Overmann, 2023, p. 215). (The Mundurukú, the Hup and the Xerénte are Brazilian indigenous peoples.) Overmann calls things like the arms, the eyes and deer footprints "distributed exemplars," that is, "features of the natural or cultural environment that typify a particular subitizable quantity" (Overmann, 2023, p. 151). Thus, the first number concepts might come from the use of distributed exemplars in association with subitizing. "Using the twoness of arms to characterize *twoness* in other pairs, as the Mundurukú are reported to do… exemplifies the psychological construct of abstraction" (Overmann, 2023, p. 343-344). An *a posteriori* abstraction, it must be added.

The abstraction and expression of numbers by reference to distributed exemplars is restricted to three or four, the subitizing limit. Research has demonstrated that humans can estimate larger quantities, even without employing counting techniques or external aids. However, these larger quantities are perceived as fuzzy, and when the difference between them is not significant enough, they are hardly distinguishable in size (Overmann, 2023, Ch. 3). Here is where fingers and toes come in. To overcome the fuzziness of unassisted estimation, most cultures create tallying techniques using fingers and toes. As the etymology of number words in many languages suggests, finger-counting is likely to have emerged *before* verbal counting, since counting words are derived from terms for fingers and gestures. "The expression of quantity by means of gesture precedes its verbal expression precisely because a lack of preexisting words and explicit concepts inhibits only verbal expression, while gesture enables us to illustrate and then perhaps verbalize what is nascent and ineffable" (Overmann, 2023, p. 73). According to Overmann, the invention of finger-counting is responsible for giving numbers some of their most fundamental properties:

> *Our predisposition to use the fingers influences our numbers, in more ways than grouping by fives, tens, and*

> *twenties. For example, using the hand in counting also makes numbers discrete. Remember that the upper limit of the subitizing range, about three or four, is approximate or "fuzzy," and that the first nonsubitizable quantity to emerge is five, as expressed and represented by the hand. It is difficult for about three or four to remain fuzzy when the numbers one through five are represented on adjacent fingers; in fact, representing them on adjacent fingers makes them visually and thus conceptually discrete. Using the hand in counting is thought to influence numbers toward stable order, something in which the dedicated neural network for ordinality likely plays a part. Using the hand also makes numbers linearized. This is because finger-counting leverages the topographical organization of our sensorimotor cortices, a layout that is inherently linear as a function of the way an organism grows, from head to tail, from its DNA blueprint. Numbers also acquire linearity from the way the hand is used in counting. Not infrequently, finger-counting proceeds in a rough line across the hand, say, from thumb to little finger. Our visual interaction with the material properties of the hand, plus the linear organization of the neurological substrate, influence the expression of numbers with the fingers toward linearity, and this influences the associated concepts of numbers toward linearity as well (Overmann, 2023, p. 87).*

Discreteness, stable order and linearity are properties we usually think of as pertaining to numbers themselves, independently of any human factors. Overmann, however, proposes that numbers have these properties *because* our bodies have certain properties, such as the discreteness and regular ordering of our fingers. This clearly exemplifies the Copernican Revolution that characterizes Overmann's approach: numbers' properties are derived from our bodies and interactions with the outer environment, rather than being a priori properties of previously defined entities. She is not alone in defending this thesis. The linguist Heike Wiese (not cited by Overmann, though) has defended a similar view, associating the emergence of properties such as stable order with finger-counting (Wiese, 2007).

The body and other material devices continue to shape our numbers as new tools are created to represent and manipulate them. Especially with respect to written notations, Overmann claims that they add another distinctive characteristic to numbers, namely, "entitivity," i.e., the idea that numbers are self-existing entities, similar in this regard to physical objects. "Numbers that are not written—seven Mesopotamian tokens, seven Yoruba cowries, seven Polynesian coconuts, seven Inka knots—are collections of discrete objects. In contrast, a handwritten number like the cuneiform seven … becomes recognized as an object in its own right through the neurological reorganizations associated with handwriting and literacy" (Overmann, 2023, p. 333). As she explains it, the fusiform gyrus is a part of the temporal lobe of the brain whose evolutionary function is to recognize objects and faces. Learning to read (both words and written numerals) involves training the fusiform gyrus to recognize written characters, making use of its evolutionary ability to recognize objects and faces. Overmann suggests that the reuse of the brain structures devoted to recognize objects to recognize numerals contributes to numbers being perceived as objects in their own right.

In sum, by tracing the roots of our concept of number to characteristics of our brain, body and man-made tools such as written notations, Overmann strengthens the thesis that numbers themselves come from our brain, body and man-made tools (in contrast to the widely held belief that things such as fingers and notations simply express what numbers are a priori, in the mind or in a Platonic heaven).

At this point, many philosophers sympathetic to more traditional views may object to Overmann's conclusion, by pointing out that there is a key difference between the origins of *our concept of number*—which may well be rooted in our brain, body and material devices—and *numbers in themselves*—whose properties are proved, in mathematics, to be independent of any human traits. In her defence, Overmann can affirm that blurring the distinction between number and our concept of number is the central point of her book. As she puts it,

> *This perspective is new. It has not been tried in at least the 2500 years since Plato pronounced that numbers were mental constructs—eternal but intangible and invisible concepts existing somewhere external to the human minds that somehow still managed to contact and apprehend them. In explaining this new perspective on numerical origin and elaboration, this book draws its conclusions from a different set of assumptions about numbers, their material component, and the nature of the human mind (Overmann, 2023, p. xxv).*

The different set of assumptions she mentions is the *Material Engagement Theory* (MET), developed by her fellow archaeologist Lambros Malafouris (Malafouris, 2013). MET endorses the premises of 4E Cognition, that is, the view that cognition is embodied, enacted, embedded and extended. MET and 4E Cognition challenge the view that cognition is entirely located within the brain and that thinking is separate from the body and the environment. Rather, the brain, the body, and the environment are seen as working together to bring about the phenomenon we call cognition. By rejecting the notion that cognition is brain-centered, MET and other 4E Cognition frameworks deny that beliefs and knowledge can originate from within the brain solely, let alone from a disembodied, a priori cognition or Platonic realm. Accordingly, the conflation Overmann makes between humans' concept of number and numbers in themselves seems less misplaced in this context than for Platonist minds. Even so, this conflation has very odd consequences that are unlikely to be accepted by most philosophers and mathematicians as well, be they Platonist or not.

One of the most essential and widely recognized characteristics of numbers is their *objectivity*. Even those who reject a Platonist account of the nature of numbers acknowledge that arithmetic is objective. In fact, it has become customary to see objectivity in mathematics as possibly independent of the existence of mathematical objects. As Dummett (1973, p. 508) remarks, "what is important is not the existence of mathematical objects, but the objectivity of mathematical statements." By blurring the distinction between number and our concept of number, Overmann fails at this point: objectivity is lost.

The problem can be traced back to Overmann's "working definition of number," presented in her first chapter. She defines numbers as follows: "Numbers are *concepts* of discrete quantity, arranged in magnitude order, with relations between them, and operations that manipulate the relations" (Overmann, 2023, p. 6, emphasis added). Key here is what she takes to be a concept. Although she does not explicitly defines what concepts are, her use of the word suggests that she thinks of concepts as mental entities. For example, she speaks of "concept-generating stimuli" (Overmann, 2023, p. xvii) and "the developmental acquisition of number concepts" (Overmann, 2023, p. 23). But numbers cannot be mental entities. Psychologism about mathematical entities is widely held as effectively debunked by Frege back in the nineteenth century. The claim that numbers are mental entities is incompatible with their mathematical properties. For example, in mathematics there exists only one of each number, but if numbers were mental entities, there could be at least one of each number per numerate person (Frege, 1960, §27, p 37-38). And, what is worse, "my" numbers could be different from "yours," since mental contents usually differ across minds (Frege, 1997, p. 336). In fact, in Overmann's account this is the case—although not across minds, but cultures. For example, in a passage where she compares "Western numbers" with numbers from other cultures, she writes:

> *Not all numbers have attributes like the meshwork of potential relations—for example, two being the square root of four and the difference between 1,245,762 and both 1,245,760 and 1,245,764—that characterize Western numbers (Overmann, 2023, p. 5).*

In a more specific comparison, Overmann explains the method the Oksapmin (a people from Papua New Guinea known by their body-counting system) use to subtract seven from sixteen. Instead of relying on the fact that 16 = 7 + 9, as most people trained in Western-style mathematics would do, the Oksapmin perform a body-counting procedure Overmann describes as "double-enumeration." How this procedure works is not important for us here, beyond the fact that it does not make use of the numerical relationship 16 = 7 + 9. From this difference, Overmann (2023, p. 227) observes:

> *Oksapmin double-enumeration has important implications. One is that the need to develop such a strategy shows that numbers do not emerge preloaded with relations. That is, the Oksapmin numbers seven, nine, and sixteen are not related beyond their order in the counting sequence: Sixteen is not nine more than seven or seven more than nine, nor is it three more than thirteen or any of the other relations we might think of. Rather, it is the number between fifteen and seventeen and has no other relations.*

She concludes that Oksapmin numbers "lack the elaborated relations that characterize Western numbers (as for example, *sixteen* is *seven* more than *nine*)" (Overmann, 2023, p. 226).

These passages contrast with others where Overmann acknowledges that numbers are fairly uniform across cultures. "Despite all this variability between cultural traditions and within any particular tradition over time, numbers are astonishingly similar" (Overmann, 2023, p. 21). She also acknowledges that such uniformity bolsters Platonism: "Since we all discover the same things and agree that they are the same things, there is a very real sense in which numbers are 'out there' somewhere, waiting for us to discover them" (Overmann, 2023, p. 11). In fact, preserving objectivity is the primary advantage of philosophical accounts such as Platonism, and that is why many philosophers endorse realism about mathematics. The Platonist would say that numbers have always the same properties and relationships, although some cultures do not know or explore some of them, as it seems to be the case of the Oksapmin with regard to 16 = 7 + 9.

Overmann does not explain Oksapmin's case in these terms probably because distinguishing what the Oksapmin know about numbers from what numbers are in themselves seems to imply, at least at first sight, commitment with the existence of an a priori concept of number, which she rejects. However, fortunately for anti-Platonists and other philosophers who may sympathize with Overmann's approach but are unwilling to give up the objectivity of numbers, there is a simple way of reconciling Overmann's account with objectivity. As mentioned above, the problem is that her definition of number is psychologistic, equating number with mental number concepts. In a sense, her definition is still too neurocentric. It is because the Oksapmin (supposedly) do not entertain in their mind the thought that "sixteen is seven more than nine" that she concludes that the Oksapmin numbers "lack the elaborated relations that characterize Western numbers." My suggestion is that a definition of number in line with Overmann's thesis about the materiality of numbers should shift the focus from the mind to the outer environment; more specifically, to the actions people carry out when counting and calculating.

Despite the great variability in tallying, counting, and calculation techniques across cultures—they differ not only in the materials they use, but also the numerical relationships they exploit, as Overmann points out—the rules that govern the basic procedure behind all of these techniques are uniform. Overmann calls attention to the wide, "occasionally astonishing," cross-cultural variability in finger-counting techniques across the world (Overmann, 2023, p. xxii, 71), but she herself recognizes that all these techniques share a common core structure: the fingers act as a *reference set*, used to instantiate the quantity of the enumerated objects or *target set* (Overmann, 2023, p. 77), and the quantity of fingers that compose the reference set is determined by one-to-one correspondence. For each object in the target set, one finger is added to the reference set. "Putting sets into correspondence in this manner creates the opportunity to recognize that they share cardinality" (Overmann, 2023, p. 91-92), and in this way the fingers can be used to refer to the cardinality of the target set.

This core structure is shared not only by finger-counting techniques, but also by other tallying and counting techniques employing a variety of materials, such as body parts, shells, mud stripes, and even words. For example, in the Oksapmin body-counting system, body parts make up the reference set, and each part is paired with one item of the target set. In verbal counting, an initial segment of the sequence of counting words acts as the reference set, and each word is paired with an element of the target collection. Importantly, this shared process is rule-based and its rules are deterministic, which gives the process objective properties of its own, regardless of what people following the rules think or know about it.

For example, although the Oksapmin do not make use of or do not know the relationship 16 = 7 + 9, this relationship is still present in their body-counting system. The following is a true description of how their system works: counting up to sixteen involves passing through the right forearm exactly when one counts seven and stopping at the left ear (which represents sixteen), with nine positions between the right forearm and the left ear. That is, even if the person performing the body-counting does not realize that when she reaches the right-forearm step of the process she has counted seven items, nor that there are nine intermediate steps between the right forearm and the left ear, these are still true facts about the process she performs. This means that the relationship 16 = 7 + 9 "is out there," objectively, in the process, even if no Oksapmin has ever noticed it.

A definition of number that focuses on the external, observable process performed for counting, rather than the mental contents of the performers, is more likely to account for the objectivity of numbers. Furthermore, as far as this process is cross-culturally constant,[1] such a definition is more likely to account for the cross-cultural uniformity of numbers, and can explain the "very real sense in which numbers are 'out there' somewhere, waiting for us to discover them" (Overmann, 2023, p. 11). It is the properties of the counting *process* that are out there, waiting for us to discover them. Sfard (2008) and dos Santos (2021b; in press) have proposed accounts of the nature of numbers centered on the idea that numbers are reifications of the cross-culturally constant elements of the counting process. I highly recommend Sfard's and dos Santos's work for those who sympathize with Overmann's approach but do not want to give up the objectivity of numbers.

Putting the counting process at the center also sheds light on another controversy Overmann discusses in her book. Throughout the book, there is a tension with linguists, who have theorized about the emergence of numbers on purely linguistic bases, dismissing the role of material devices. Undoubtedly, Overmann is accurate in noting that "if the only resource we had for computing was mental and the only form we had to manipulate thereby was verbal, we would not have gotten very far with even simple arithmetic" (Overmann, 2023, p. 124)—the amount of evidence she brings together to highlight the importance of material devices is conclusive in this regard. However, it must be noticed that the counting process can be performed with reference sets consisting of either material or verbal items. Thus, at least in principle, numbers could have emerged if a reference set of words were available. There is also the possibility that language and material devices contributed together to the emergence of numbers (dos Santos, 2021a). Actually, the tension between linguists and Overmann may be less pronounced than it seems at first sight. For example, the linguist Heike Wiese arrives at the same conclusion as Overmann regarding the temporal precedence of material devices over verbal counting. Wiese writes:

> *We can think of the first stage in the development of numerical thinking as the invention of tallies: simple iconic representations for cardinalities. In order to indicate how many elements a certain set has, for instance in order to indicate how many deer one has killed in a hunt, one might use a set of pebbles or a set of notches as representations for the deer (Wiese, 2003, p. 133).*

This is in complete agreement with Overmann's account, even though Wiese emphasizes the linguistic contributions to the emergence of number. In sum, the controversy concerning the linguistic or material origins of numbers is not as deep as it initially seems.

Relatedly, the connection between children's current number learning and the pre-history of numbers becomes evident by focusing on the counting process. Overmann views studies in developmental psychology on how children acquire numerical competence as "not reasonably significant to numerical origins" (Overmann, 2023, p. 40), because their results come from cultures where number systems are already available, whereas she is interested in the very origins of these systems. However, in doing so, Overmann misses the opportunity to utilize developmental data as additional support for her thesis. Studies in developmental psychology have highlighted the importance of the counting procedure to the acquisition of numerical competence. Children start learning to count without knowing the meaning of the counting words nor the purpose of the counting process (Wynn, 1990). They simply recite number words by rote, as mere verbal labels that are paired with objects during a repetitive, rule-governed process. It is from this initially meaningless experience with the counting procedure that they acquire number concepts. This result reinforces the importance of "materiality"—here broadly understood as encompassing the performance of actions using one's body in the outer environment—for the acquisition of numerical competence, even for children growing up in societies where counting has become a mostly verbal activity. The essential fact is that we cannot develop the ability to count mentally without first passing through an initial stage where we must physically engage with the outer environment, which involves pointing to and manipulating the objects we are counting. Furthermore, given the procedural similarity between counting with a verbal reference set and counting with material reference sets, as pointed out above, we can suppose that the emergence of numerical concepts in the mind of the pioneers of numerical thinking followed the same pattern: they started pairing elements to compare set sizes, without having what we call number concepts, and the execution of these one-to-one correspondences eventually engendered number concepts in their minds (dos Santos, 2021a). In sum, the literature on developmental psychology confirms the indispensability of external actions and engagement with material objects for the acquisition (or emergence) of number concepts, both presently and in the past.

As it turns out, there is more convergence between Overmann's ideas and the fields of philosophy, linguistics, and developmental psychology, than what one might suppose from a cursory reading of her book. Philosophers, in particular, might be inclined to reject Overmann's thesis due to her somewhat inadequate treatment of the objectivity of numbers. I hope this brief comment will help foster a more positive reception of Overmann's work within the community of philosophers of mathematics. One of its most crucial shortcomings, the loss of objectivity in numbers, can be effectively addressed by drawing upon ideas already present in the philosophical literature, as I have pointed out. Her book represents a substantial contribution to an emerging and increasingly unified approach towards understanding numbers in philosophy, linguistics, cognitive sciences, and archaeology. While this perspective is yet to become the consensus view in these disciplines, it holds great promise.

Footnotes

[1] It is worth mentioning that there are significant differences in the rules that govern simple tallies, such as those carried out with indistinguishable items like beans, and body or verbal counting (dos Santos, 2023). These differences, though, are not relevant here.

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