Review of: "Results in Cone Metric Spaces and Related Fixed Point Theorems for Contractive Type Mappings With Application"

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Potential competing interests: No potential competing interests to declare.

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\documentclass[12pt]{article}
\usepackage[utf8]{inputenc}
\usepackage[table]{xcolor}
\usepackage{lmodern}
\usepackage[T1]{fontenc}
\usepackage[english]{babel}
\usepackage{graphicx}
\usepackage{amsmath,color}
\usepackage{amssymb}
\usepackage{hyperref}
\usepackage{caption}  % Hyperlink to the figure rather than its title.
\usepackage{geometry}
\usepackage{enumerate}
\usepackage{mathtools}
\usepackage{amscd}
\usepackage{geometry}
\geometry{
    a4paper,
    total={170mm,257mm},
    left=15mm,
    top=15mm,
}

\begin{document}
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\begin{center} \Large \textbf{Second report } \end{center}

\subsection*{Main issues}

\begin{enumerate}

\item \textbf{On pages 3, 4, 8, and 9 in the inequalities (3.1)-(3.2) and (4.1)-(4.2)}, the terms

$$\frac{d(Tx,x)d(Ty,y)}{d(x,y)} \quad \text{and} \quad \frac{d(Tx,x)d(Ty,y)}{d(x,y)+d(Tx,y)+d(Ty,x)}$$

are not defined (a division is not defined) since $d(x,y)$ and $d(x,y)+d(Tx,y)+d(Ty,x)$ belong to a Banach space, and therefore they are not real numbers (not necessarily)!

Therefore, I wonder how you checked the inequality (3.1) in example 3.9, since, as I just mentioned, the division is not defined!

\textbf{Consequence:} The rest of the article makes no sense.

\textbf{Remark:} In Definition 4.1 and Definition 4.2, the inequalities (4.1)-(4.2) are verified \textbf{only for all }$x,y\in B\left(x_{0},c\right) $ \textbf{and not for all} $x,y\in X$. Otherwise, Definition 3.1 and Definition 3.2 would be similar to Definitions 4.1 and Definition 4.2, respectively.

\item \textbf{On page 5, in the line before the inequality (3.6)}, i.e.

$$\leq a_1\left[d(Tx_n,x_n)+d(Tx^*,x^*)\right]+a_2\frac{d(Tx_n,x_n)d(Tx^*,x^*)}{d(x_n,x^*)}+a_3\frac{d(Tx_n,x_n)d(Tx^*,x^*)}{d(x_n,x^*)+d(Tx_n,x^*)+d(Tx^*,x_n)}+d(x_{n+1},x^*)$$

$$\textcolor{red}{d(Tx^*,x^*)} \leq \frac{1}{1-a_1}\left[a_1 d(Tx_n,x_n)+d(x_{n+1},x^*)\right]$$

Can you please explain how you got the last inequality (colored red) from the previous one? Please, can you write the details for the calculations?

\item In the same way, \textbf{on page 7, at the bottom}, you write the following estimate

$$\textcolor{red}{d(Tx^*, x^*) \le \frac{1}{1-a_1}\left([a_1 d(Tx_n, x_n) + d(x_{n+1}, x^*)]\right)}$$

Can you please explain how you got the last inequality (colored red) from the previous one?

\item \textbf{On page 9, in the line before the inequality (3.6)}, i.e.

$$d(x_0, Tx) \le (1-(a_1+a_2+a_3))c + a_1[d(Tx_0, x_0) + d(Tx, x)] + a_2 \frac{d(Tx_0, x_0)d(Tx, x)}{d(x_0, x)} + a_3 \frac{d(Tx_0, x_0)d(Tx, x)}{d(x_0, x) + d(Tx_0, x) + d(Tx, x_0)}$$
$$\le \textcolor{red}{(1-(a_1+a_2+a_3))c}$$

Can you please explain how you got the last inequality (colored red) from the previous one?

\item \textbf{On page 10, in the lines before Corollary 4.5}, i.e.

$$d(x_0, Tx) \le (1-(a_1+a_2+a_3))c + a_1[d(Tx_0, x) + d(Tx, x_0)] + a_2 \frac{d(Tx_0, x_0)d(Tx, x)}{d(x_0, x)} + a_3 \frac{d(Tx_0, x_0)d(Tx, x)}{d(x_0, x) + d(Tx_0, x) + d(Tx, x_0)}$$
$$\le \textcolor{red}{(1-(a_1+a_2+a_3))c + a_1 2d(x_0, x)} \le (1-(a_1+a_2+a_3))c + 2a_1 c$$
$$\textcolor{blue}{= (1-(a_1+a_2+a_3))c}$$

Can you please explain how you got the estimates (colored red) from the previous one?

Moreover, it is obvious that the last inequality (colored blue) does not imply $d(x_{0},Tx) \le c $. Indeed, we could have $a_{2}+a_{3}-a_{1}<0$, even if the condition $2a_{1}+a_{2}+a_{3}<1$ is fulfilled. In this case, $1-\left(a_{1}+a_{2}+a_{3}\right) c>c$, and obviously we cannot claim that $Tx \in B(x_{0},c)..

\item \textbf{On page 11, in the first line of the proof of Theorem 5.3}, you set $x^{*}\in Fix (T^n)$. I guess that n \textbf{is a fixed integer} such that $n\ge 2$. In other words, you assumed that x^{*} is a fixed point for \textbf{some} mapping T^{n} (i.e., \textbf{for some given} integer n). Otherwise, if x^{*} is assumed to be a fixed point of every mapping T^{n}, then necessarily x^{*} is also a fixed point for T, and therefore Theorem 5.1 is useless!

Clarifying this fact, \textbf{you should notice that you cannot take} $n\longrightarrow \infty$ in (5.3), since n is a fixed integer. %he sequence of inequalities in (5.2) is wrong}. Indeed,

\item The same remark may be done for the proof of Theorem 5.4.

\end{enumerate}

\subsection*{Some typos}

\begin{itemize}

\item[a)]The reference [1] in Example 5 is not correct. Please fix this.

\item[b)]In Example 5, replace $d(x,y)=\left\lbrace (x,y)\in E\textbar x,y>0\right\rbrace $ by $\textcolor{red}{P}=\left\lbrace (x,y)\in E\textbar x,y>0\right\rbrace $.

\item[c)] The first line in the proofs of Theorem 5.3 and Theorem 5.4, replace $x^{*}\in F(T^{n})$ by $x^{*}\in \textcolor{red}{Fix}(T^{n})$.

\item[d)]In the proof of Theorem 5.3, you have to replace

$$a_3 \frac{d(T^n x^*, T^{n-1}x^*)d(T^{n+1}x^*, T^n x^*)}{d(T^{n-1}x^*, T^n x^*) + d(T^{n-1}x^*, T^{n+1}x^*) + d(T^{n-1}x^*, T^n x^*)}$$

by

$a_{3} \dfrac{d(T^{n}x^{*},T^{n-1}x^{*})d(T^{n+1}x^{*},T^{n}x^{*})}{d(T^{n-1}x^{*},T^{n}x^{*})+d(T^{n-1}x^{*},T^{n+1}x^{*})+d(T^{\textcolor{red}{n}}x^{*},T^{n}x^{*})}$

\item[e)] In the proof of Theorem 5.4, you have to replace

$$\begin{aligned} d(Tx^*, x^*) &= d(T(T^{n-1}x^*), T(T^n x^*)) \\ &\leq a_1\left[d(T^{n-1}x^*, T^{n+1}x^*) + d(T^{n-1}x^*, T^n x^*)\right] + a_2 \frac{d(T^n x^*, T^{n-1}x^*)d(T^{n+1}x^*, T^n x^*)}{d(T^{n-1}x^*, T^n x^*)} \\ &\quad + a_3 \frac{d(T^n x^*, T^{n-1}x^*)d(T^{n+1}x^*, T^n x^*)}{d(T^{n-1}x^*, T^n x^*) + d(T^{n-1}x^*, T^{n+1}x^*) + d(T^{n-1}x^*, T^n x^*)} \end{aligned}$$

by

$$\begin{aligned} d(Tx^*, x^*) &= d(T(T^{n-1}x^*), T(T^n x^*)) \\ &\leq a_1\left[d(T^{n-1}x^*, T^{n+1}x^*) + d(T^{\textcolor{red}{redn}}x^*, T^n x^*)\right] + a_2 \frac{d(T^n x^*, T^{n-1}x^*)d(T^{n+1}x^*, T^n x^*)}{d(T^{n-1}x^*, T^n x^*)} \\ &\quad + a_3 \frac{d(T^n x^*, T^{n-1}x^*)d(T^{n+1}x^*, T^n x^*)}{d(T^{n-1}x^*, T^n x^*) + d(T^{n-1}x^*, T^{n+1}x^*) + d(T^{\textcolor{red}{redn}}x^*, T^n x^*)} \end{aligned}$$

We popint out tht teh expression $\alpha =$

\item[f)] In section 5, the numbering is not correct. Indeed, Theorem 5.3 and Theorem 5.4 should be

numbered, respectively, Theorem 5.2 and Theorem 5.3.

\item[h)] In the same way, in section 2 (Preliminaries) the numbering should be by section for the sake of consistency with the other sections. For example, Definition 1 and Definition 2 should be numbered as follows: Definition 1.1, Definition 1.2.

\end{itemize}

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