

Research Article

Inequality Emerges from Networks

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Economists conventionally attribute inequality in employee compensation to differences in the marginal productivities of workers. However, it is possible that inequality arises from an entirely different source – the network structure of the organizations to which the employees belong. We offer an extremely simple network model that accounts for the degree of inequality observed in modern economies.

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1. Introduction

Economists have begun to recognize the significance of networks (modeling connectedness, relationships, information flows, etc.) for market outcomes^{[1][2]}. Other social scientists, particularly sociologists, also have long understood the importance of networks. The formal study of network models can be traced back to Graicunas'^[3] work on the span of control, and subsequent sociological classics include Wasserman and Faust (1994), Watts and Strogatz^[4] and White^[5]. Most of the economic literature has focused on how individual rational agents improve their well-being by forming or dissolving links to other agents. The individual-centric approach yields many fruitful results. One example is the recent demonstration that “six degrees of separation” is the natural outcome of a network situation in which the individuals face a tradeoff between the cost of making connections and their desire to increase their centrality in the network^[6].

We examine networks in a different way. Instead of considering how individual members of a network might act strategically to improve their situations, we focus on how an organization facing market pressures arranges its structure to accomplish particular tasks. We abstract away from all personal characteristics of the members of the organization and their motivations and focus only on the

organization's effectiveness in carrying out its tasks. The remarkable result is that the network structure alone produces considerable economic inequality, entirely independent of the characteristics or abilities of the agents making up the organization.

2. The Basic Model

We begin by modeling the firm as a directed graph, with numbered vertices representing the employees. The production process of the firm consists of two tasks: (1) An associative task, in which each member of the organization is assigned a number and the firm's task is to compute the sum, and (2) an adoption task, in which all members of the organization have to adopt an innovation that is first discovered by one member. The associative task is akin to the adding-up performed by the stylized networks in^[7]. Other versions of the adoption task have been examined in^{[8][9]}.

Our versions of these tasks are set up to be as simple as possible. Obviously, real-world firms perform many varied and complex tasks, but our goal is to illustrate the importance of network structure in the most basic setting. For the associative task, the firm's goal is to add the numbers given to the agents and pass the sum to Vertex #1. Information flows in the direction of the edges of the directed graph. The summation is carried out as follows:

- For each vertex, find the shortest path from that vertex to Vertex #1. These paths may be distinct, or they may overlap.
- When a vertex receives a number from another vertex (by way of a directed edge), it incurs a processing cost equal to $f(V_{in})$, where V_{in} is the vertex's in-degree and f is an increasing function.
- The total cost incurred by the vertex is then $f(V_{in}) \times W$, where W is the "workload," the number of times the vertex processes an input.

If there were no penalty for a vertex's having multiple edges feeding into it, the optimal structure for performing the associative task would simply be a star, with each vertex connected directly to Vertex#1. If not for the workload factor, the optimal firm would just be a single string leading to Vertex#1 incorporating all the vertices. Absent the adoption task, the structure described above leads to tree-like graphs.

The adoption task is even simpler. An innovation is introduced at a particular vertex. The task is to spread the innovation to each of the firm's other vertices. The process of diffusion is akin to the adoption task,

but in reverse – the innovation is passed from the vertex where it first appears through the entire organization.

- The innovation is assumed to first be taken up by a single vertex. Then the shortest path from this vertex to each other vertex is found.
- The benefit a vertex acquires by adopting the innovation is an increasing function of the number of edges feeding information into the vertex (its in-degree). The more vertices passing on information about the innovation, the easier it is for the vertex to adopt it. This benefit is a negative cost.
- There is no workload factor for the adoption task. Once a vertex has adopted the innovation, it incurs no additional cost when other diffusion paths go through it.

Another version of the adoption task might have the benefits increasing with the number of diffusion paths running through the vertex from the initial adopter to the other vertices. This could be thought of as a strengthening of the benefit the vertex receives from adoption because of its taking advantage of the experience of other adopters. The results are qualitatively similar in either version of the model, so we do not report the results of the second form here.

Note the simplifications: There is no discounting associated with the time it takes for the tasks to be accomplished. The cost incurred by a vertex in adding numbers does not depend on the numbers being added. The individuals making up the firm do not behave strategically. All the employees are identical except for their position in the firm's network. We abstract from any changes in the sizes of the firms.

3. Evolution

Efficient firms minimize the total vertex costs of the associative task plus the benefits of the adoption task. However, firms in the real world are never perfectly optimized. In addition to computational complexity (DeCanio 2001) and principal-agent frictions^[10], market and regulatory conditions constantly change. Nevertheless, market competition creates selection pressure. This is similar to selection pressure in biological evolution. All that is required is “the element of environmental adoption by the economic system of a posteriori most appropriate action according to the criterion of ‘realized positive profits’” (^[11]; see also ^[12]). In our model, this takes the form of cost minimization subject to the firm's successful completion of its task(s).

We implement a simple model of “evolution” of lower-cost network structures. The firm's total cost is obtained by adding up all the costs incurred by the individual vertices. Starting with a population of n

randomly generated connected graphs (connected so that the firms are capable of actually performing the tasks), we create a population of $3n$ firms: the original n , another n by randomly adding an edge to each firm, and a final n by randomly subtracting an edge from each firm. From these $3n$ firms (minus those that are dropped because they cannot complete the tasks), we select the n lowest-cost firms and repeat the process. Each repetition is a “generation.” This trimming of the least efficient $2/3$ of the randomly modified firms is sufficient to result in populations with stable values of the firms after 500 generations, at least for firms the size we modeled. This results in a population of low-cost firms exhibiting only a few distinct structures.

This “evolutionary” process normally will not lead to an optimal structure. In fact, it is evident that the evolutionary process finds only local cost minima. Different initial populations will evolve to slightly different evolved populations. In Mathematica^[13], the software we used for all calculations, an initial random “seed” can be set, enabling replication of the results. Varying the seed produces different initial conditions. We experimented with different initial seeds and found that after 500 generations, the variation in average final total costs is quite small relative to the average initial costs and is very much smaller than the gains in efficiency resulting from the competitive winnowing of inefficient firms.

We also implement a version of the model in which the firms exhibit a fixed underlying structure, and evolution adds edges that improve performance. A fixed-structure approach is taken by Stark et al.^[14], but in the context of exploring the evolution of cooperation. In the next section, we first present the results for initial populations of unstructured graphs, then show how the results change when an underlying structure is maintained.

4. Compensation of Employees

It is standard in economics to attribute the compensation of individuals to their marginal productivities. Our model provides for a variant of this approach if each member of the firm (i.e., each vertex) is paid an amount equal to the “work” it performs, where work is measured by the cost incurred by the vertex in carrying out the tasks. However, suppose some other indication of the individuals’ importance to the firm were the basis of compensation. Network theory offers multiple measures of the “importance” or “centrality” of individual nodes. We consider two possible candidates for centrality-based compensation: BetweennessCentrality (BC) and PageRankCentrality (PRC). Formal definitions of these two quantities are given in equations (1) and (2):

$$BC = \sum_{s,t \in v \wedge s \neq i \wedge t \neq i} n_{s,t}^i / n_{s,t} \quad (1)$$

where $n_{s,t}$ is the number of shortest paths from s to t and $n_{s,t}^i$ is the number of shortest paths from s to t passing through i , and

$$PRC = \text{a list of centralities that are solutions to } c = \alpha a^T \cdot d \cdot c + \beta \quad (2)$$

where a is the adjacency matrix of graph g , a^T is its transpose, and d is the diagonal matrix consisting of $1/\max(1, d_i^{out})$, where d_i^{out} is the out-degree of the i^{th} vertex. In our model, β is the unit vector (Mathematica Documentation, Wolfram Research, Inc., 2023).

BetweennessCentrality is “a widely used measure that captures a person’s role in allowing information to pass from one part of the network to the other”^[15]. PageRankCentrality is the recursive measure of a node’s influence pioneered by Sergei Brin and Lawrence Page, the founders of Google. It reflects the value of all the nodes that influence each particular node. Either of these measures could be the basis for compensation, particularly if compensation is determined in part by political or bureaucratic power. Other centrality measures are less suitable for use in our model. The Eigenvector Centrality of a node can be zero in directed networks, so it could not be the basis for employee compensation in our model; Katz-Bonacich Centrality has the drawback that any vertex linked to an “important” vertex would also have a high value^[16].

5. Results

Whether compensation is based on individuals’ work as measured by vertex cost or their centralities in the firm’s network, our simple network models generate considerable economic inequality. Studies of within-firm pay differences show a considerable range, for example, “the median firm-level total pay Gini coefficient is 0.27, on par with the country Gini for Sweden, whereas the 90th percentile is 0.59, similar to the level for Namibia” (^[17]; see also the range of Ginis reported by^[18]). Our results fall roughly within this wide range, as illustrated by Table 1. The Gini index for vertex-cost-based compensation is greater than 0.6 for all parameter combinations shown. Although within-firm Ginis do not aggregate into country-wide Ginis because of pay differences across firms (such differences as are recognized in Wallskog et al.’s^[19] examination of the relationship between within-firm inequality and productivity, and elsewhere), we note that this Gini of 0.6 is comparable to the highest level of income inequality found in country-wide data – the Gini for household income in South Africa is 0.63. As shown in Table 1, the Ginis we calculate for compensation based on the two centrality measures are considerably lower, with the

BetweennessCentrality Gini somewhat greater than the Gini for compensation based on PageRankCentrality. For comparison, the income Ginis of the five largest world economies are: the United States, 0.415; China, 0.382; Japan, 0.329; Germany, 0.317; and India, 0.357^[20]. Considering wage income only, a recent NBER study^[21] found that the average earnings Gini for the four Nordic countries is 0.23, while for the United States this Gini is 0.38 and for the United Kingdom it is 0.37.

Associative Task Cost Function	x^2	x^2	x^2	x^2
Adoption Task Cost Function	$-(x^2)/50$	$-(x^2)/60$	$-(x^2)/70$	$-(x^2)/80$
Average Total Cost	1613	1567	1484	1567
Average Ginis:				
VertexCost	0.6563	0.6030	0.6240	0.6304
BetweennessCentrality	0.4626	0.4260	0.5337	0.5298
PageRankCentrality	0.3664	0.3193	0.3535	0.3109
Associative Task Cost Function	x^2	x^3	x^4	x^5
Adoption Task Cost Function	$-(x^2)/50$	$-(x^3)/50$	$-(x^4)/50$	$-(x^5)/50$
Average Total Cost	1613	4539	11272	33473
Average Ginis:				
VertexCost	0.6563	0.6011	0.7053	0.6841
BetweennessCentrality	0.4626	0.5116	0.4542	0.3718
PageRankCentrality	0.3664	0.3453	0.3354	0.2779

Table 1. Characteristics of Evolved *Unstructured* Firms after 500 Generations

Parameters: Vertices = 43, Initial Edges = 129, Population = 100,

PageRankDecay = 0.85, Vertex InDegree = x , Seed = 32.

Similar patterns are found if a structure is first imposed and then maintained for the firms as they undergo evolution. This starting point for this type of structure is shown in Figure 1.

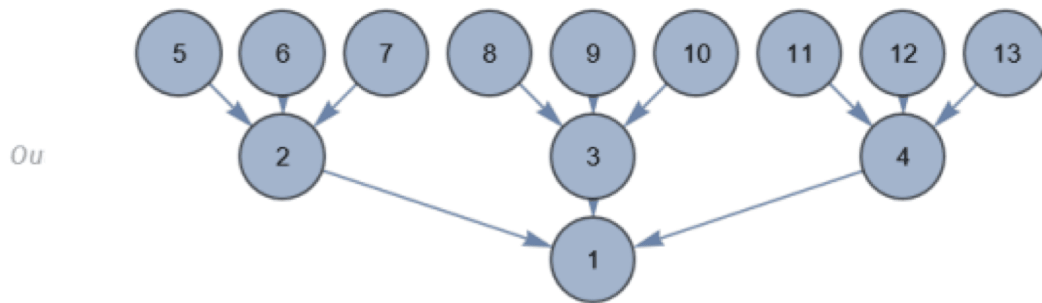


Figure 1. Tree structure with 3 levels, 3 inputs to each superior level

As can be seen here, there are a total of 13 vertices in this graph. The vertex to which all the information flows is Vertex #1; the “middle management” level is made up of Vertices #2, #3, and #4. The model we use for the fixed underlying structure version of our model has three levels but 6 “subordinate” vertices feeding into each superior. This makes a total of 43 vertices ($1 + 6 + (6 \times 6)$). This number is sufficient to display the main results of the evolutionary process and is consistent with the conventional wisdom that a manager can most efficiently supervise 7 ± 2 subordinates^[22].

Associative Task Cost Function	x^2	x^2	x^2	x^2
Adoption Task Cost Function	$-(x^2)/50$	$-(x^2)/60$	$-(x^2)/70$	$-(x^2)/80$
Average Total Cost	4031	4159	4051	4163
Average Ginis:				
VertexCost	0.8205	0.8191	0.8187	0.8261
BetweennessCentrality	0.6597	0.6241	0.6343	0.6706
PageRankCentrality	0.4362	0.4361	0.4309	0.4621
Associative Task Cost Function	x^2	x^3	x^4	x^5
Adoption Task Cost Function	$-(x^2)/50$	$-(x^3)/50$	$-(x^4)/50$	$-(x^5)/50$
Average Total Cost	4031	27373	185877	1.30×10^6
Average Ginis:				
VertexCost	0.8205	0.8516	0.8617	0.8653
BetweennessCentrality	0.6597	0.7094	0.6889	0.6691
PageRankCentrality	0.4362	0.4548	0.4530	0.4457

Table 2. Characteristics of Evolved *Structured* Firms after 500 Generations

Parameters: Vertices = 43, Initial Edges =129, Population = 100,

PageRankDecay = 0.85, Vertex InDegree = x , Initial innovation in Vertex #43, Seed = 32.

Structured firms' average total cost figures are higher than those for the unstructured firms, reflecting the additional "overhead" imposed by maintaining an underlying fixed structure. Interestingly, the Gini coefficients for each comparable set of cost parameters are also higher for the structured than for the unstructured firms. Only the PageRankCentrality Ginis in Table 2 are comparable to the degree of inequality seen in the real world. This finding is consistent with the possibility that power and position in structured organizations are the primary determinants of employee compensation in such firms.

We can examine typical evolved firms in greater detail. Figure 2 shows the structure of one of the evolved unstructured firms with a cost structure given by the second numerical column of the first half of Table 1.



Figure 2. Typical Evolved Unstructured Firm

First, notice Vertices #1 and #43. Vertex #1 is where the summation from the associative task is received. It is not particularly prominent visually in the network, but it does have three other vertices feeding into it. Vertex #43 is where the innovation originates. It feeds its information to one vertex, #6, and no other vertex is connected to it. (This is not a general feature of all the evolved firms, however.) More interestingly, consider the top four vertices as measured by their vertex costs, betweenness centralities, and PageRank centralities. These are: Vertex Cost: {1, 26, 41, 5 and 24 (tied)}; BetweennessCentrality: {4, 2, 13, 28}; PageRankCentrality: {1, 26, 41, 9}. The two measures showing overlap are Vertex Cost and PageRankCentrality, but it is not complete. Table 3 shows the average correlations between the three compensation measures, taken across the population of the 100 evolved firms after 500 generations. By way of comparison, the critical value for two-tailed statistical significance at the 1% level for a population

of 100 is 0.254^[23], although we make no claim of a formal hypothesis test because the evolved graphs do not constitute a random sample.

Associative Task Cost Function	x^2	x^2	x^2	x^2
Adoption Task Cost Function	$-(x^2)/50$	$-(x^2)/60$	$-(x^2)/70$	$-(x^2)/80$
Correlations:				
(Vertex Cost, PRC)	0.943	0.942	0.975	0.968
(Vertex Cost, BC)	- 0.060	- 0.213	- 0.222	- 0.236
(BC, PRC)	- 0.020	- 0.232	- 0.209	- 0.194
Associative Task Cost Function	x^2	x^3	x^4	x^5
Adoption Task Cost Function	$-(x^2)/50$	$-(x^3)/50$	$-(x^4)/50$	$-(x^5)/50$
Correlations::				
(Vertex Cost, PRC)	0.943	0.826	0.909	0.513
(Vertex Cost, BC)	- 0.060	0.216	- 0.163	0.034
(BC, PRC)	- 0.020	0.372	- 0.132	- 0.186

Table 3. Average correlations between compensation measures, population of 100 unstructured firms after 500 generations

Parameters: Vertices = 43, Initial Edges = 129, Population = 100,

PageRankDecay = 0.85, Vertex InDegree = x , Seed = 32.

Figure 2 (as well as other evolved firms not shown here) shows looping structures that can be characterized as “teams.” In our model, teams emerge naturally from the evolution of the otherwise unstructured firms. Figure 3 is analogous to Figure 2, except for an evolved firm with a fixed underlying structure. Obviously, because of maintaining the 3-layer structure, the evolved structured firm has more edges than the evolved unstructured firm. The firm in Figure 3 displays the hierarchy: Vertices #1, #2, #3, #4, #5, #6, and #7 all have six edges feeding information to them. Vertex #43, the first innovator, has only

one edge feeding into Vertex #7. Information flows across the bottom tier – Vertex #27 feeds into Vertex #29 as well as into its “manager” Vertex #5, for example, and Vertex #24 feeds into Vertex #40, for example. The top Vertex Costs are #1, #6, and #7 (tied), followed by #2, #3, #4, and #5 (tied). The top four PageRankCentralities are #1, #5, #6, and #7. It is perhaps no surprise that under these two compensation schemes, the highest payments go to the top management layers. Also, the self-loop edge from Vertex #18 is an artifact from the original random population. Different random seeds give rise to evolved firms without such loops. The survival of this loop is confirming evidence that the evolutionary process does not lead to global cost minima.

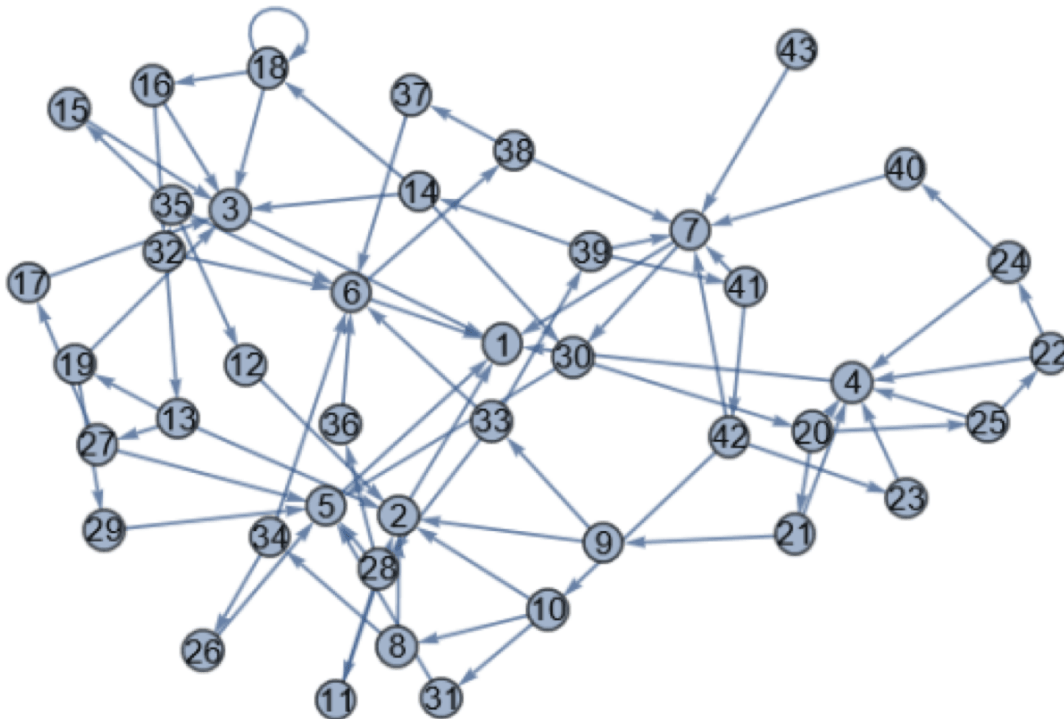


Figure 3. Typical Evolved Structured Firm

6. Discussion and Conclusion

Clearly, inequality in compensation is intrinsic to the network structure of organizations. The average Ginis in the initial population of randomly connected firms are high initially, may decrease and increase over the course of the evolutionary process, and then settle in at values that remain stable, as shown in Figure 4. This figure gives the historical path of the average Ginis for the three kinds of employee

compensation, for one particular set of parameter values (those given in the second column of the top half of Table 2).

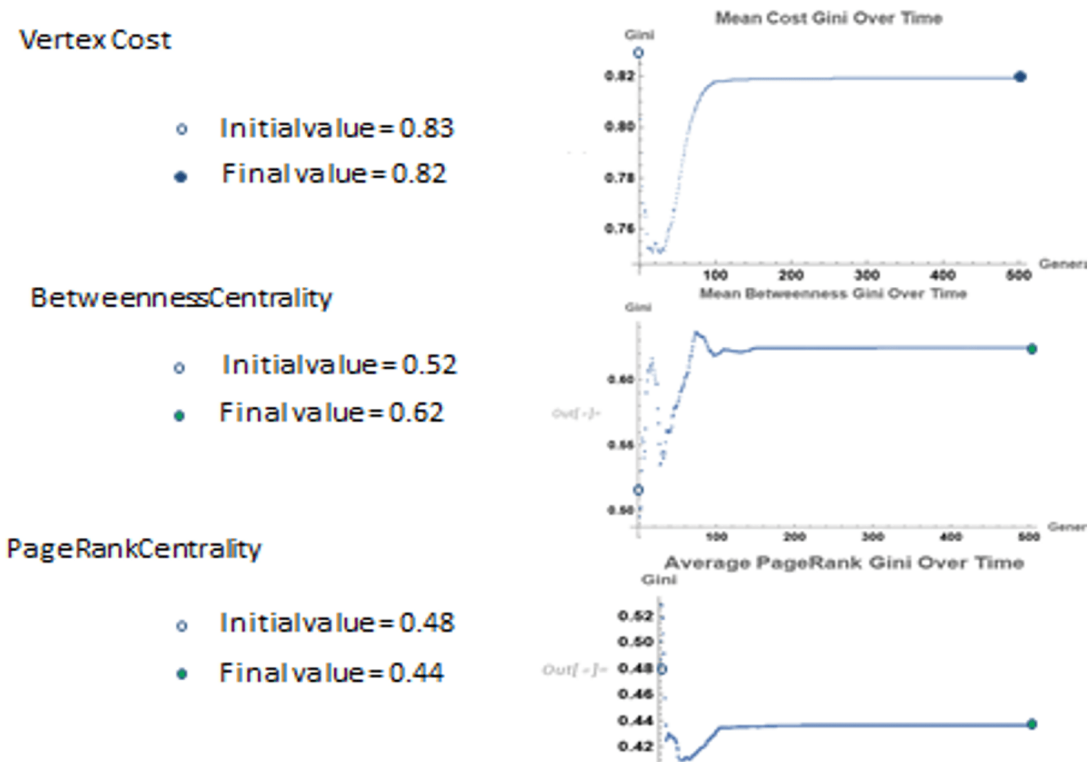


Figure 4. Inequality (Mean Gini) histories, three compensation schemes, structured firm

These results indicate that considerable inequality can emerge solely from the network structure of the firms, independent of the characteristics of the individual agents making them up. All that is required is that market conditions exert selection pressure on the firms. Structured firms give rise to greater inequality than unstructured firms, a finding consistent with economic intuition. Additionally, selection pressure for lower total costs leads to “informal ties” across tiers in order to accomplish the adoption task more efficiently.

Of course, network structure is not the only source of economic inequality. Many other factors contribute, as evidenced by the very large literature on inequality across the social sciences. Classic treatments by Sen [24] (with Annexe by Sen and Foster, 1997) and Atkinson [25][26] have stood the test of time. The “econophysics” literature, in which economic analogues of physical concepts such as entropy and temperature are calculated, also yields substantial inequalities in the economic variables those models

simulate ^{[27][28][29][30]}, are a few examples). The Greenberg and Gao^[31] survey of this literature notes that “a large proportion of observed economic inequality is the result of luck and the inherently diffusive (entropy-increasing) nature of exchange itself, and not the result of interpersonal differences in industriousness, entrepreneurialism, or intelligence” (p. 18).

In our model, it is also the case that inequality emerges even though the individual agents are identical in all their capabilities. It is a topic of further research to see how (and whether) this inequality would increase if the agents’ abilities were drawn from a distribution of skills. Even so, we have shown that under alternative compensation schemes, considerable inequality arises solely from the network structure of the firms and the selection pressure for efficiency resulting from competitive market forces. We propose only to add the ubiquitous presence of organizational networks to the set of causes of this early and perhaps most salient of economic questions – the sources of inequality.

Notes

JEL Codes: A12, D2, D3, J3

References

1. [△]Jackson MO (2016). "The Past and Future of Network Analysis in Economics." *The Oxford Handbook of the Economics of Networks*. Oxford University Press.
2. [△]Goyal S (2023). *Networks: An Economic Approach*. Cambridge: The MIT Press.
3. [△]Graicunas VA (1933). "Relationship in Organization." *Bulletin of the International Management Institute*. 7:3 9–42.
4. [△]Watts D, Strogatz S (1998). "Collective dynamics of ‘small-world’ networks." *Nature*. 393:440–442. doi:10.1038/38/30918.
5. [△]White H (2004). *Markets from Networks: Socioeconomic Models of Production*. Princeton, NJ: Princeton University Press.
6. [△]Samoylenko DA, Primo E, Alfaro-Bittner K, Vasilyeva E, Kovalenko K, Musatov D, Raigorodskii AM, Criado R, Romance M, Papo D, Perc M, Barzel B, Boccaletti S (2023). "Why Are There Six Degrees of Separation in a Social Network?" *Physical Review X*. 13:021032.
7. [△]Radner R (1992). "Hierarchy: The Economics of Managing." *Journal of Economic Literature*. 30(3):1382–1415.

8. [△]DeCanio SJ, Watkins WE (1998). "Information processing and organizational structure." *Journal of Economic Behavior and Organization*. 36:275–294.
9. [△]DeCanio SJ, Dibble C, Amir-Atefi K (2000). "The Importance of Organizational Structure for the Adoption of Innovations." *Management Science*. 46(10):1285–1299.
10. [△]DeCanio SJ (1993). "Barriers within firms to energy-efficient investments." *Energy Policy*. 21(9):906–914.
11. [△]Alchian AA (1950). "Uncertainty, Evolution, and Economic Theory." *Journal of Political Economy*. 58(3):211–221.
12. [△]Alchian AA (1953). "Biological Analogies in the Theory of the Firm: Comment." *American Economic Review*. 43(4):600–603.
13. [△]Wolfram Research, Inc. (2024). *Mathematica*, Version 14.0. Champaign, IL.
14. [△]Stark S, Peter D, Tutić A (2024). "Homophily and the evolution of cooperation in the Volunteer's Dilemma: A computational study on dynamic graphs." *Social Networks*. 78:25–39.
15. [△]Goldbeck J (2015). *Introduction to Social Media Investigation: A Hands-On Approach*. 1st ed. Syngress.
16. [△]Wolitzky A (2022). "Lecture 3: Eigenvector Centrality Measures (PDF)." MIT OpenCourseWare. https://ocw.mit.edu/courses/14-15-networks-spring-2022/resources/mit14_15s22_lec3/.
17. [△]Green TC, Zhou D (2019). "Pay Inequality, Job Satisfaction, and Firm Performance." Baruch College Zicklin School of Business Research Paper.
18. [△]Morais F, Kakabadse NK (2014). "The Corporate Gini Index (CGI) determinants and advantages: Lessons from a multinational retail company case study." *International Journal of Disclosures and Governance*. 11(4):380–397.
19. [△]Wallskog M, Bloom N, Ohlmacher SW, Tello-Trillo C (2024). "Within-Firm Pay Inequality and Productivity." Working Paper 32240. National Bureau of Economic Research. Cambridge, MA. <http://www.nber.org/papers/w32240>.
20. [△]U.S. Government, Central Intelligence Agency (2024). *The World Factbook: Country Comparisons – Gini index coefficient – distribution of family income*. <https://www.cia.gov/the-world-factbook/field/gini-index-coefficient-distribution-of-family-income/country-comparison/>.
21. [△]Morgstad M, Salvanes KG, Torsvik G (2025). "Income Equality in the Nordic Countries: Myths, Facts, and Lessons." NBER Working Paper 33444. Cambridge, MA: National Bureau of Economic Research.
22. [△]Chin C (2024). "How Many People Can One Person Effectively Manage?" *Management for Startups*. <https://managementforstartups.com/articles/whats-the-ideal-number-of-direct-reports/#:~:text=In%20management%20circles,%20it%20is,time%20on%20individual%20contributor%20work>.

23. [△]Aspelmeier J (2005). *Table of critical values for Pearson's r*. real-statistics.com. <https://real-statistics.com/statistics-tables/pearsons-correlation-table/>.
24. [△]Sen A (1973). *On Economic Inequality*. Oxford: Oxford University Press.
25. [△]Atkinson AB (1970). "On the Measurement of Inequality." *Journal of Economic Theory*. 2:244–263.
26. [△]Brandolini A (2017). "Inequality and economics: Tony Atkinson's enduring lessons." *VoxEU*. <https://cepr.org/voxeu/columns/inequality-and-economics-tony-atkinsons-enduring-lessons>.
27. [△]Wright I (2005). "The social architecture of capitalism." *Physica A*. 346:589–620.
28. [△]Manna SS, Biswas S, Chakrabarti BK (2022). "Near universal values of social inequality indices in self-organized critical models." *Physica A*. 596:127121.
29. [△]Banerjee A, Yakovenko V (2010). "Universal Patterns of Inequality." *New Journal of Physics*. 12.
30. [△]Banerjee S, Biswas S, Chakrabarti BK, Ghosh A, Mitra M (2023). "Sandpile Universality in Social Inequality: Gini and Kolkata Measures." *Entropy*. 25:735.
31. [△]Greenberg M, Gao HO (2024). "Twenty-five years of random asset exchange modeling." *The European Physical Journal B*. 97:69.

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