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On Quantum Superposition

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Abstract

Lying at the heart of quantum mechanics, the notion of quantum superposition causes enormous difficulty in understanding the problem of quantum measurement, which concerns the Einstein-Bohr debate on the conceptual foundations of the quantum theory. According to Bell's theorem, Einstein's viewpoint in his debate with Bohr seems to be wrong. In the present paper, a new principle (referred to as the general principle of measurements) is introduced and proved as a mathematical theorem. Based on this principle, various forms of quantum superposition (including quantum entanglement) and the corresponding experiments are scrutinized. The main findings are as follows. (a) Einstein's viewpoint is correct and Bell's theorem is problematic. (b) Measurement outcomes of any actual experiment with individual microscopic objects described in a form of quantum superposition, in particular, the experimental results of testing Bell inequalities, are all erroneously explained. (c) Bell inequalities failed to capture the essence of the Einstein-Bohr debate, i.e., whether quantum superposition is legitimate for describing individual microscopic objects. (d) All kinds of quantum superposition violate the general principle of measurements and hence are illegitimate. (e) Quantum mechanics can be completed by using disjunction ("or") as the logical relation between the orthonormal vectors that span an arbitrarily given Hilbert space for describing a single microscopic object, and the mathematical setting will remain essentially unchanged; hidden-variable theories are irrelevant to the real world. (f) After completing quantum mechanics as above, the difficulty in understanding the problem of quantum measurement disappears naturally. Implications of the findings are also discussed.

Keywords: Conceptual foundations of quantum mechanics, Einstein-Bohr debate, Quantum superposition, Quantum measurement, Bell's theorem, Bell inequalities

1 Introduction

The basic mathematical theory for describing microscopic objects studied by quantum physics, such as electrons, atoms, and photons, is quantum mechanics.

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The quantum-mechanical description is typically based on the notion of quantum superposition. Lying at the heart of current quantum theory, this notion is the cause of enormous difficulty in understanding the problem of quantum measurement, and its legitimacy for describing individual microscopic objects is the essence of the Einstein-Bohr debate on the conceptual foundations of the quantum theory [1, 2, 3, 4]. The difficulty still remains today, although most physicists seem to believe that Bell and his followers already solved the problem by experimentally testing Bell's inequality and its various variants, collectively called Bell inequalities derived by resorting to a hidden-variable theory [5, 6, 7], with the experimental results explained by Bell's theorem proved in theoretical physics [8, 9, 10]. However, although quantum-mechanically calculated probabilities always agree with the corresponding experimental results obtained by measurements, in no sense can such probabilities diminish the difficulty as illustrated below.

According to the quantum theory in its current form, before a measurement is performed on a single microscopic object described by a wave function in a form of quantum superposition, the object is claimed to be simultaneously in the superposed states representing mutually exclusive properties, and no definite values can be assigned to the corresponding physical quantities. Being simultaneously in the superposed states, the object is actually claimed to possess mutually exclusive properties at the same time before measurement, which amounts to using conjunction ("and") as the logical relation between the states. Because each of the states corresponds to a definite outcome obtained by measurement, once a measurement is performed on the object, the measurement triggers an abrupt collapse of the wave function onto one of the states. Initially, the object is described by the wave function in the form of quantum superposition with conjunction as the logical relation between the superposed states before measurement; however, as time evolves, the object ends up inexplicably in one of the states after measurement, which amounts to using disjunction ("or") as the logical relation between the states. In other words, without any reasonable explanation, the logical relation between the states before and after a measurement changes from conjunction to disjunction. A question then appears as John S. Bell put it: How does an "and" get converted into an "or"? This is an important question concerning the conceptual foundations of quantum mechanics, which characterizes pithily the problem of quantum measurement.

Bell and his followers attempted to provide an answer to the above question based on experimentally testing Bell inequalities. But the supreme success of the quantum theory has prevented anyone from considering the theory entirely wrong. This is why Bell and his followers merely tried to reinterpret quantum mechanics and kept the theory in its current form intact [11]. Keeping current quantum theory intact presumes the legitimacy of quantum superposition. Consequently, Bell inequalities cannot capture the essence of the Einstein-Bohr debate and failed when tested by actual experiments [12, 13, 14].

In fact, the failure of Bell inequalities is unavoidable even before actual experiments are performed to test them against quantum mechanics, no matter how they are derived based on whatever hypotheses. In current quantum theory, the notion of quantum superposition plays two closely related roles in various experiments with individual microscopic objects. Consider an experiment with individual microscopic objects of a given kind. Quantum-mechanically, a wave function ψ in a form of quantum superposition describes each of the objects. On the other hand, ψ is also used to calculate probabilities of the outcomes corresponding to the objects. In other words, ψ not only describes an arbitrarily given single object to be measured in the experiment but also serves to calculate the probability of the outcome obtained by measuring the object. Because ψ presumes the legitimacy of quantum superposition, the fate of Bell inequalities is already predetermined by the presumed legitimacy of quantum superposition; the failure of Bell inequalities is not surprising at all.

For instance, consider the optical experiment with individual pairs of correlated photons for testing the CHSH inequality derived by Clauser, Horne, Shimony, and Holt [7]. The CHSH inequality is a generalization of Bell's inequality [5]. In this optical experiment [10], one of the roles is to describe each of the pairs by the so-called "entangled state" given in a specific form of quantum superposition; the other is to calculate, based on the same "entangled state", the probability of the outcome obtained by measuring the pair. Although this twofold role can guarantee that the quantum-mechanically calculated probabilities always agree with the corresponding experimental results obtained by measurements, unfortunately, as a consequence of presuming the legitimacy of quantum superposition, the agreement between the quantum-mechanically calculated probabilities and the experimental results conceals the real scientific truth. In general, measurement outcomes of various experiments that involve quantum superposition, including the experimental results obtained by testing Bell inequalities, are all erroneously explained.

In this study, a new principle (referred to as the general principle of measurements) is introduced and proved as a mathematical theorem. Based on this principle, various forms of quantum superposition (including quantum entanglement) and the corresponding experiments are scrutinized. The results obtained from the scrutiny support Einstein's viewpoint and indicates that Bell's theorem is problematic. As shall be demonstrated in the present paper, all kinds of quantum superposition violate the general principle of measurements and are not legitimate for describing individual microscopic objects. In addition, based on the general principle of measurements, quantum mechanics can be completed by using disjunction as the logical relation between the orthonormal vectors that span an arbitrarily given Hilbert space for describing a single microscopic object, and the mathematical setting will remain essentially unchanged; hidden-variable theories are irrelevant to the real world. After completing quantum mechanics as above, the difficulty in understanding the problem of quantum measurement disappears naturally.

In Section 2, the general principle of measurements is introduced and proved as a mathematical theorem; its application is illustrated with examples. In Section 3, completing quantum mechanics based on this principle is demonstrated. In Section 4, implications of the results obtained in this study are discussed briefly. In Section 5, the paper is concluded with a summary of the reported findings.

2 General Principle of Measurements

Physical quantities can only exist in the real world. Consequently, all physical quantities can only be measured based on mathematical models of space and time of the real world, not anywhere else. The mathematical model of space in the real world is the three-dimensional Euclidean space $\mathbb{R}^3$ endowed with the metric given by the usual distance function between two points in the space. The mathematical model of time elapsed in the real world is the set of non-negative real numbers R_0 equipped with the metric given by the usual distance function between two nonnegative real numbers. Points in $\mathbb{R}^3$ represent precise space coordinates, and elements in R_0 are precise time coordinates.

Several basic definitions in topology are needed to prove the general principle of measurements. A metric space is denoted by (X, d), where X is a set, and d is a metric on X. Let r be a positive real number. For $x \in X$, the open ball with center x and radius r is

$$B(x; r) = \{y \in X; d(x, y) < r\}.$$

Any open subset of X is a union of open balls. All open subsets of X constitute a metric topology for X. The set X and the metric topology form a metric topological space. Consider $x \in S$ where S is a subset of X. If there exists $r > 0$ such that

$$B(x; r) \cap S = \{x\},$$

then x is an isolated point of S.

Theorem 2.1. *(***The General Principle of Measurements***) Precise space and time coordinates are practically unattainable by measurements.*

Proof. Measuring a point x in the space perfectly precisely requires x to be an isolated point of $\mathbb{R}^3$. Similarly, unless time t is an isolate point of R_0, it is impossible to measure t perfectly precisely. However, neither $\mathbb{R}^3$ nor R_0 has any isolated point. To see this, consider $t \in S$ where S is an arbitrary subset of R_0. An open "ball" now is an open interval

$$B(t; r) = (t - r, t + r).$$

There are two cases: $t = 0$, and $t > 0$. If $t = 0$, no open interval centered at t is a subset of R_0. If $t > 0$, there is no $r > 0$ such that

$$S \cap B(t; r) = B(t; r) = \{t\}.$$

The condition for t to be an isolated point is not satisfied in either case. Consequently, R_0 has no isolated point. It can be shown similarly that $\mathbb{R}^3$ has no isolated point either. This completes the proof of the general principle of measurements. □

The general principle of measurements is irrelevant to any issue about measurement instruments and has nothing to do with accuracy of measurement outcomes in practice. But it can reasonably explain the random phenomena observed in outcomes measured in experiments with individual microscopic objects described by quantum superposition. Needless to say, probability can describe any observed random phenomenon. But the observed random phenomenon needs a reasonable explanation. In current quantum theory, it is claimed that quantum mechanics is intrinsically probabilistic and the observed random phenomena require no further explanation. This is why Einstein questioned the theory by calling it "the fundamental dice-game" [4].

A microscopic object can at most be measured only once. But the outcome obtained by measuring a single microscopic object, such as a photon, makes little sense statistically and cannot explain the random phenomenon observed in the corresponding experiment. The random phenomenon can only manifest itself in a large number of measurement outcomes obtained in different repetitions of the experiment under purported the same experimental condition that depends on precisely specified space and time coordinates. Because precisely specified space and time coordinates are unattainable by measurements, "the same experimental condition" violates the general principle of measurements and hence does not exist in the real world. When a microscopic object is measured, it corresponds to only one outcome in one repetition. As illustrated with examples below, the observed random phenomenon can be explained by the general principle of measurements, and quantum mechanics is not intrinsically probabilistic.

Example 2.2. Usually, microscopic objects are not at rest. For instance, photons propagate in space. Consequently, it is necessary to consider their propagating directions and orientations of polarizers for measuring the photons. Any direction or orientation in space corresponds to a unique point on a unit sphere. The sphere is a subset of $\mathbb{R}^3$. It is worth noting the following fact:

Remark 2.3. The points on the unit sphere are irrelevant to spatial positions of microscopic objects.

Consider the individual pairs of perfectly correlated photons for testing the CHSH inequality in the optical experiment [10]. The pairs are described by the "entangled state" in a form of quantum superposition. Let the orientations of two spatially separated polarizers be parallel. There are only two different outcomes, i.e., (+, +) and (-, -), obtained with equal probabilities [10]

$$\mathbb{P}\{(+,+)\} = \mathbb{P}\{(-,-)\} = \frac{1}{2}.$$

At either polarizer, the detected photons have purportedly the same (desired) polarization direction and follow purportedly the same (desired) propagating direction. Consider the precise space coordinates corresponding to the following directions and orientations:

(a) the same (desired) propagating direction and the same (desired) polarization direction specified purportedly for each photon,

(b) the actual propagating direction and the actual polarization direction of each photon, and

(c) the same (desired) orientation of the polarizer specified purportedly for measuring each photon and the actual orientation for measuring each photon.

According to the general principle of measurements, the space coordinates corresponding to the directions and orientations listed above are all unattainable by measurements and hence unknown. The actual propagating directions of different photons are almost surely different; the actual polarization directions of different photons are almost surely different; the desired orientation and the actual orientations for measuring different photons are also almost surely different. Three tiny volumes serve as the approximations to the precise space coordinates. The first volume contains the coordinates of desired and actual propagating directions; the second contains those of desired and actual polarization directions; the third contains those of desired and actual orientations of the polarizer. The "entangled state" takes precise space coordinates for granted and hence violates the general principle of measurements; it is invalid and illegitimate to use the "entangled state" for describing the pairs of correlated photons. As shown above, the random phenomenon exhibited in the outcomes of measuring the polarizations of perfectly correlated photons can be explained by the general principle of measurements, and quantum mechanics is not intrinsically probabilistic.

Example 2.4. Consider a particle described by a wave function $\psi(x,t)$ given by a coherent superposition of energy eigenstates. Clearly, ψ depends on time and spatial position of the particle explicitly. Each of the energy states is assigned a quantum-mechanically calculated, nonzero probability. According to the quantum-mechanical description, before an experiment is performed to measure the energy, the particle is claimed to have more than one energy states at the same time t. Now it is not difficult to see why the quantum-mechanical description makes no sense physically.

Needless to say, the outcome obtained by measuring a single particle is not statistically meaningful. To observe the random phenomenon exhibited in the experimental results, a large number of repetitions of the corresponding experiment are necessary. If a particle is measured, it corresponds to only one outcome obtained in one repetition of the experiment at purported the same time t. According to the general principle of measurements, the value of t is unattainable by measurements and hence unknown. As an approximation to the precise coordinate t, a tiny time interval contains both t and the actual, almost surely different, and unknown time coordinates at which different particles of the same kind in different repetitions are measured. No particle in the real world can be found to have more than one energy states at the same time. Again, because the observed random phenomenon can be explained by the general principle of measurements, quantum mechanics is not intrinsically probabilistic.

Example 2.5. Consider the particle in the previous example again. Instead of measuring energy, investigating the spatial position of the particle at a given time t requires to repeat the procedure used in the previous example for all possible spatial positions of the particle. According to the quantum-mechanical description, until and unless an experiment is performed to measure the position of the particle in space, the particle is claimed to be at infinitely many different positions at the same time. In other words, with a quantum-mechanically calculated probability $|\psi(x,t)|^2 dx > 0$, the particle may be at each of the possible positions at the same time before measurement. Such claim violates the general principle of measurements, and hence describing the particle by ψ in the form of quantum superposition is illegitimate. Because the observed random phenomenon can be explained by the general principle of measurements, quantum mechanics is not intrinsically probabilistic.

Example 2.6. Quantum-mechanically, a spin-1/2 particle is described by a form of quantum superposition with two eigenvectors spanning a Hilbert space. The eigenvectors correspond to possible outcomes obtained by performing a Stern-Gerlach experiment for measuring the spin of the particle in a specified direction. Neither time dependence of the superposed states nor spatial motion of the particle needs to be considered in this example.

According to the quantum-mechanical description, the particle is claimed to be in two states along every direction simultaneously and hence has no definite spin in any direction. When a measure is performed in an arbitrarily given direction, the outcome is either "spin up" or "spin down" with the corresponding probability, which is considered as an evidence for the claim that quantum mechanics is intrinsically probabilistic.

However, spin measurements are performed in space of the real world modeled by $\mathbb{R}^3$ and depend on precisely specified space coordinates for the measurements. The random phenomenon described by the probabilities can only be observed in a large number of measurement outcomes obtained in different repetitions of the experiment. Because precisely specified space coordinates are practically unattainable, taking such coordinates for granted violates the general principle of measurements. Therefore, the claim that quantum mechanics is intrinsically probabilistic is wrong.

3 Completing Quantum Mechanics

The most controversial issue in the Einstein-Bohr debate on the conceptual foundations of quantum mechanics concerns whether the quantum-mechanical description of physical reality is complete. According to Einstein, for a complete physical theory,

- each element of the physical reality must have a counterpart in the theory;
- for a microscopic object, if it is possible to predict the value of a physical quantity almost surely without disturbing the object in any way, then

an element of the physical reality corresponding to the physical quantity exists.

However, if a single microscopic object is described by quantum superposition, then the object is claimed to have no definite properties characterized by values of the corresponding physical quantities prior to measurements or observations. This contradicts Einstein's vision of the physical world, and hence Einstein considered the quantum-mechanical description of physical reality incomplete in his debate with Bohr. Nevertheless, Einstein never excluded the possibility of completing quantum mechanics.

The general principle of measurements paves the way towards completing quantum mechanics within the framework of the Hilbert space for describing individual microscopic objects by replacing conjunction with disjunction between the orthonormal vectors that span the Hilbert space. Using disjunction as the logical relation between the orthonormal vectors not only can be justified by the principle of measurements, it is also consistent with the definition of a general Hilbert space. In fact, the concepts for defining a Hilbert space in general are all highly abstract and have no practical meanings. Orthogonality specified by an inner product is the most important concept to define a Hilbert space. The orthogonality for defining a Hilbert space in general is a purely mathematical concept without any practical meaning. Assigning practical meanings to the orthogonality is unnecessary.

Moreover, for a Hilbert space in general, it is even unnecessary to specify the logical relation between orthogonal vectors. In fact, the logical relation between orthogonal vectors in a specific Hilbert space can even be neither conjunction nor disjunction. For a given application, practically meaningful concepts are necessary to define a specific Hilbert space for describing practically meaningful objects, and conjunction may serve as the logical relation between the orthogonal vectors in that space. But the orthogonal vectors must not correspond to mutually exclusive properties simultaneously belonging to the same object.

Example 3.1. The classical prototype of a Hilbert space was first studied by D. Hilbert with applications to the theory of integral equations. This Hilbert space consists of infinite sequences of complex numbers. The logical relation between the orthogonal vectors is neither conjunction nor disjunction. It is not necessary to specify the logical relation.

Example 3.2. With the inner product defined for the Euclidean vectors, $\mathbb{R}^3$ is a Hilbert space. For this Hilbert space, the orthogonal Euclidean vectors do not represent mutually exclusive properties simultaneously belonging to any geometric object, and the logical relation between the orthogonal vectors is conjunction.

Needless to say, the logical relation between the orthogonal vectors that span a specific Hilbert space can also be disjunction. For the Hilbert space in quantum mechanics, the logical relation between the orthonormal vectors must be disjunction as required by the general principle of measurements. Different outcomes corresponding to mutually exclusive properties are obtained by measuring different microscopic objects of the same kind in different repetitions of the experiment in question. Each outcome yields a definite property of the physical reality belonging to the corresponding microscopic object. The definite property exists independently of human consciousness.

Consequently, a definite value corresponding to the outcome can be assigned to the object, even though the precise space and time coordinates for measuring it are unknown; the value can even be taken from a continuum and hence cannot be obtained by measurements, such as position and momentum of a particle moving in space. Therefore, by using disjunction as the logical relation between the orthonormal vectors, quantum mechanics can indeed be completed without changing the mathematical setting essentially! Hidden-variable theories are irrelevant to the real world.

On the other hand, violating the general principle of measurements can result in using an imaginary microscopic object to characterize different microscopic objects measured in different repetitions. No outcome is obtained by measuring the imaginary object described by quantum superposition. The imaginary object does not exist in the real world.

After completing quantum mechanics within the framework of the Hilbert space without resorting to any hidden-variable theory, there will be two entirely different notions of quantum superposition: one lies at the heart of current quantum theory, which will be referred to as "superposition (conjunction)", and the other uses disjunction to serve as the logical relation between the superposed orthonormal vectors, which will be denoted by "superposition (disjunction)" to avoid confusion.

Example 3.3. In current quantum theory, the notion of "commutator" used to prove uncertainty relations precludes simultaneous assignment of values to some physical quantities for a particle described by superposition (conjunction). The commutators and uncertainty relations serve to argue against Einstein's vision of the physical world and are hindrances of completing quantum mechanics. For instance, the commutator used to prove Heisenberg's uncertainty relation precludes simultaneous assignment of values to position and momentum of the same particle. Because superposition (conjunction) merely describes imaginary particles that do not exist in the real world, the arguments based on the commutators and uncertainty relations are not physically meaningful.

Example 3.4. Consider, again, the outcomes of jointly measuring the individual pairs of perfectly correlated photons in the optical experiment for testing the CHSH inequality [7, 10]. The pairs are described by the "entangled state" in a form of superposition (conjunction). As elucidated in the previous sections, the twofold role played by the "entangled state" in the experiment leads to the erroneous explanation of the experimental results, which claims that Einstein's vision of the physical world is wrong.

The "entangled state" violates the general principle of measurements and hence is illegitimate for describing the individual pairs of the correlated photons in the real world. Violating the general principle of measurements brings about using an imaginary pair to characterize different pairs of the same kind measured in different repetitions of the experiment. No outcome is obtained by measuring the imaginary pair described by the "entangled state". The imaginary pair is claimed to have no definite polarizations before measurements [10]. By no means can such an imaginary pair exist in the real world. The measurement outcome corresponding to each single pair in the real world yields an element of the physical reality independent of human consciousness, even though the actual orientations of the polarizers for measuring the pair are unattainable by measurements and unknown.

After completing quantum mechanics by replacing superposition (conjunction) with superposition (disjunction), the difficulty caused by the former in understanding the problem of quantum measurement disappears naturally. Superposition (conjunction) is also the cause of various paradoxes related to quantum measurement, such as the paradox of Schrödinger's cat. The paradoxes can be easily resolved by replacing superposition (conjunction) with superposition (disjunction) in the completed quantum theory.

4 Discussion

The quantum measurement problem concerns the Einstein-Bohr debate on the conceptual foundations of current quantum theory. The essence of the debate is the legitimacy of superposition (conjunction) for describing individual microscopic objects. The difficulty in understanding the problem of quantum measurement and various paradoxes related to quantum measurement, such as the paradox of Schrödinger's cat, all stem from superposition (conjunction). Bell and his followers provided a solution to the above problem by experimentally testing Bell inequalities and explaining the experimental results based on Bell's theorem.

Unfortunately, the approach adopted by Bell and his followers presumes the legitimacy of superposition (conjunction), which leads to the failure of Bell inequalities. More unfortunately, problematic Bell's theorem and the experimental invalidation of Bell inequalities opened the door to so-called quantum information technologies, such as quantum computation and quantum communication [16]. All such techniques are based on "quantum bit" in the form of superposition (conjunction). Because physically meaningless superposition (conjunction) can only describe imaginary objects that do not exist in the real world, quantum information has no physical carriers, and hence none of such technologies can be realized physically.

The difficulty in understanding the quantum measurement problem, various paradoxes related to quantum measurement, and physically unrealizable "quantum bit" will all disappear naturally by replacing superposition (conjunction) with superposition (disjunction) in the completed quantum theory. Furthermore, various far-fetched interpretations of the physical world based on superposition (conjunction) and problematic Bell's theorem, such as the many-worlds interpretation, will also disappear naturally. In addition, quantum mechanics, after it is completed by replacing superposition (conjunction) with superposition (disjunction), will become relatively comprehensible compared to current quantum theory. It is also reasonable to believe that the completed quantum theory can explain more physical phenomena and provide us with more practical applications.

5 Conclusion

In this study, various forms of superposition (conjunction) and the corresponding experiments are scrutinized. The scrutiny is based on the general principle of measurements proved as a mathematical theorem. The results obtained from the scrutiny indicate that the measurement outcomes of the experiments involving superposition (conjunction), such as those obtained by testing Bell inequalities, are all erroneously explained. Superposition (conjunction) is illegitimate because it violates the general principle of measurements. Quantum mechanics can be completed by using superposition (disjunction) as the logical relation between the superposed orthonormal vectors that span an arbitrarily given Hilbert space for describing a single microscopic object, and the mathematical setting will remain unchanged essentially. Hidden-variable theories are irrelevant to the real world. In the completed quantum theory, there will be no difficulty in understanding the problem of quantum measurement.

References

[1] A. Einstein, B. Podolsky and N. Rosen, Can quantum-mechanical description of physical reality be considered completed? *Physical Review*, 47(1935), 777–80, DOI 10.1103/PhysRev.47.777.

[2] N. Bohr, Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 48(1935), 696–702, DOI 10.1103/PhysRev.48.696.

[3] N. Bohr, Discussion with Einstein on epistemological problems in atomic physics, in **Albert Einstein: Philosopher-Scientist**, 1949, ed. P. A. Schilpp, The Library of Living Philosophers, Evanston, Illinois.

[4] **The Born-Einstein Letters**, Translated by Irene Born, MACMILLAN, 1971.

[5] J. S. Bell, On Einstein Podolsky Rosen paradox, *Physics*, 1(1964), 195-200, DOI 10.1103/PhysicsPhysiqueFizika.1.195.

[6] J. S. Bell, Introduction to the hidden-variable question, in **Foundations of Quantum Mechanics**, 171-81, 1971, ed. B. d'Espagnat, Academic Press, New York.

[7] J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Proposed experiment to test local hidden variable theories, *Physical Review Letters*, 23(1969), 880-84, DOI: 10.1103/PhysRevLett.23.880.

[8] J. F. Clauser and M. A. Horne, Experimental consequences of objective local theories, *Physical Review D*, 10(1974), 526-35, DOI 10.1103/PhysRevD.10.526.

[9] J. F. Clauser and A. Shimony, Bell's theorem: experimental tests and implications, *Reporting Progress Physics*, 41(1978), 1881–927, DOI 10.1088/0034-4885/41/12/002.

[10] A. Aspect, Bell's theorem: the naive view of an experimentalist, in **Quantum [Un]speakables: From Bell to Quantum Information**, 119-53, 2002, Springer, Berlin, Heidelberg, DOI 10.1007/978-3-662-05032-3-9.

[11] M. Jammer, **The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective**, John Wiley & Sons, 1974.

[12] B. Hensen, et al., Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, *Nature*, 526(2015), 682-686, DOI 10.1038/nature15759.

[13] M. Giustina, et. al., Significant-loophole-free test of Bell's theorem with entangled Photons, *Physical Review Letters*, 115(2015), DOI 10.1103/PhysRevLett.115.250401.

[14] L. K. Shalm, et. al., Strong loophole-free test of local realism, *Physical Review Letters*, 115(2015), DOI 10.1103/PhysRevLett.115.250402.

[15] A. Aspect, Closing the door on Einstein and Bohr's quantum debate, *Physics*, 8(2015), DOI 10.1103/Physics.8.123.

[16] M. A. Nielsen and I. L. Chuang, **Quantum Computation and Quantum Information**, 2000, Cambridge University Press, Cambridge.