On Quantum Superposition

Guang-Liang Li
University of Hong Kong
glli@eee.hku.hk

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Abstract

Lying in the core of the conceptual scheme of current quantum theory, the notion of quantum superposition is a consequence of the quantum-mechanical superposition principle. However, relying on precise but not attainable space or time coordinates, quantum superposition makes little sense physically if we use it to describe or explain experimental results obtained by measurement. Space or time is usually modeled by a metric space endowed with the usual distance function. Irrelevant to instrument precision and measurement accuracy, unattainability of precise coordinates is a well-established mathematical fact following from properties of the corresponding metric topology, and hence rules out completely, in principle, the possibility for us to obtain precise coordinates by measurement in actually performed experiments. In classical mechanics, unattainability of precise coordinates is hardly noticeable, and measurement results obtained in classical physics can still be reasonably explained even though precise coordinates are unattainable. However, when we explain measurement results obtained in quantum physics, unattainability of precise coordinates should not be omitted. In the conceptual scheme of current quantum theory, omission of unattainability of precise coordinates is an unfortunate flaw, which is largely responsible not only for the invalidity of quantum superposition in description of the physical world but also for various ineligible applications of quantum mechanics. Nevertheless, in no sense does the flaw imply quantum mechanics as a successful theory failing to be correct. As shown in this article, the flaw is repairable. By taking into account unattainability of precise coordinates, we can reinterpret the meaning of quantum superposition to repair the flaw without affecting eligible applications of quantum mechanics while avoiding all the ineligible applications.

Keywords: Foundations of quantum theory, Quantum superposition, Quantum randomness, Quantum measurement

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1 Introduction

At the level of quantum objects, such as electrons, atoms, and photons, the physical world is described by quantum mechanics. The quantum-mechanical description is typically based on the notion of quantum superposition. Lying in the core of the conceptual scheme of current quantum theory, quantum superposition is intended to describe physical quantities concerning microscopic objects, which can be observed by measurement. Although quantum-mechanical predictions are always in agreement with experimental results obtained by measurement, there are some unresolved issues relevant to quantum superposition.

For example, according to quantum theory in its current form, if no measurement is performed on a system, the system is in a quantum superposition of orthogonal states corresponding to mutually exclusive properties. Each orthogonal state represents a deterministic outcome obtained by measurement. Orthogonal states are added by vector addition, which requires use of conjunction ("and") as the logical relation between mutually exclusive properties of the system before measurement. Described by quantum mechanics, the system is simultaneously in each orthogonal state, and hence possesses mutually exclusive properties at the same time. However, after a measurement is performed on the system, the quantum superposition collapses immediately onto one of orthogonal states. Beginning initially in the quantum superposition with conjunction as the logical relation between mutually exclusive properties before measurement, the system, as time evolves, ends up inexplicably in one of orthogonal states, which requires use of disjunction ("or") as the logical relation between mutually exclusive properties after measurement. In other words, without any physical meaning, the logical relation between mutually exclusive properties before and after a measurement changes from conjunction to disjunction. A question then appears as John S. Bell put it: How does an "and" get converted into an "or"? This is an important question concerning the conceptual foundations of quantum mechanics, which characterizes the essence of the quantum measurement problem. The question is still open.

By raising the above question, Bell assumed implicitly the validity of quantum superposition as a meaningful description of the physical world at the level of quantum objects. If this assumption holds, we are forced to choose one of the two options regarding description of the physical world:

(a) Quantum superposition is valid for description of both quantum objects and macroscopically distinguishable objects, i.e., objects with macroscopically distinct states.

(b) Quantum superposition can only describe quantum objects, but is invalid for description of macroscopically distinguishable objects. There exists a boundary between the former and the latter.

Option (a) is the choice of most physicists, although they have never seen any effects of quantum superposition at the level of macroscopically distinguishable objects. The reason for them to choose this option is the so-called decoherence.

They believe, without any evidence, decoherence making effects of quantum superposition disappear beyond the level of quantum objects. Option (b) is the choice of the minority [1, 2, 3]. This option has been intensively investigated by experiment. The aim is to find macroscopically distinguishable objects, which cannot be described by quantum superposition. So far, such objects have not been found yet [4]. On the other hand, if the assumption implied by options (a) and (b) is false, then there also exists another option:

(c) The validity of quantum superposition for description of the physical world is questionable even at the level of quantum objects.

Option (c) is the choice of Einstein, but dismissed by Bohr in the Einstein-Bohr debate [5, 6, 7]. According to Einstein, the use of quantum superposition for description of the physical world may not make sense physically, because the quantum-mechanical description involves instantaneous collapse of a quantum superposition onto one of orthogonal states upon measurement, which contradicts relativity. Nowadays, this third option seems to have been excluded by the standard interpretation of the experimental invalidation of Bell inequalities [8, 9, 10, 11, 12, 13]. However, as we shall see in this article, Einstein's choice, i.e., option (c) is correct. Even at the level of quantum objects, description of the physical world based on quantum superposition is physically meaningless. Consequently, we may rule out options (a) and (b), but need reinterpret the meaning of quantum superposition in the conceptual scheme of current quantum theory.

As we all know, physical quantities described by quantum superposition can be observed by measurement in experiments with quantum objects. Only in space and time can physical quantities exist and be measured. To measure physical quantities in space and time, we must model space and time mathematically. Space or time is usually modeled by a metric space endowed with the usual distance function. By using the distance function together with precise space or time coordinates, we can calculate the distance between two points in space or the length of a time interval between two instants. However, precise coordinates are practically unattainable. Irrelevant to instrument precision and measurement accuracy, unattainability of precise coordinates is a well-established mathematical fact following from properties of the corresponding metric topology. When reinterpreting the meaning of quantum superposition, we must take into account unattainability of precise space and time coordinates.

For actually performed experiments, unattainability of precise coordinates rules out completely, in principle, the possibility for us to obtain precise coordinates by measurement. In classical mechanics, unattainability of precise coordinates is hardly noticeable and hence can be safely omitted. This is why we may still reasonably explain measurement results obtained in classical physics. Although the role played by unattainability of precise coordinates in classical mechanics is insignificant, when we explain measurement results obtained in quantum physics, unattainability of precise coordinates must not be omitted. In the conceptual scheme of current quantum theory, omission of unattainability of precise coordinates is an unfortunate flaw, which is largely responsible not only for the invalidity of quantum superposition in description of the physical world, but also for various ineligible applications of quantum mechanics. Nevertheless, in no sense does the flaw imply quantum mechanics as a successful theory failing to be correct. As shown in this article, the flaw is repairable. By taking into account unattainability of precise coordinates, we can reinterpret the meaning of quantum superposition to repair the flaw without affecting eligible applications of quantum mechanics while getting rid of all the ineligible applications.

After briefly reviewing a few definitions and mathematical facts about metric topology concerning unattainability of precise space or time coordinates (Section 2), we scrutinize the quantum-mechanical superposition principle and the notion of quantum superposition (Section 3). Based on the scrutiny, we propose a feasible way to repair the flaw in the conceptual scheme of current quantum theory by reinterpreting the meaning of quantum superposition (Section 4), and then discuss briefly implications of unattainability of precise coordinates concerning the foundations of quantum theory (Section 5). The topics discussed include Bell inequalities [14] and the EPR experiment [5], the so-called entangled state in experimental tests of Bell theorem, statistical regularities and probabilities in classical and quantum mechanics, quantum information [15], Heisenberg's uncertainty relation, and Kochen-Specker theorem [16]. Finally, we conclude with a summary of the main findings reported in this article (Section 6).

2 Metric Topology and Unattainability of Precise Coordinates

As a mathematical model of space in which we live, Euclidean-3 space (denoted by $\mathbb{R}^3$) consists of ordered triples of real numbers $\mathbf{r} = (r_1, r_2, r_3)$. The triples are also referred to as points in $\mathbb{R}^3$. Write

$$x(\mathbf{r}) = r_1,\ y(\mathbf{r}) = r_2,\ z(\mathbf{r}) = r_3,$$

where x, y, z are the natural coordinate functions, and their values are precise coordinates of the corresponding points in $\mathbb{R}^3$. To measure physical quantities in space modeled by Euclidean-3 space, we need a metric, also referred to as distance function, defined on $\mathbb{R}^3$. The metric or distance is a real-valued function, which satisfies the well-known properties characterizing the notion of distance. The distance between any two points $\mathbf{r}$ and $\mathbf{r}'$ is usually given by

$$d_3(\mathbf{r}, \mathbf{r}') = \sqrt{(r_1 - r_1')^2 + (r_2 - r_2')^2 + (r_3 - r_3')^2}. \tag{1}$$

By definition, $d_3(\mathbf{r}, \mathbf{r}') = 0$ if and only if $\mathbf{r} = \mathbf{r}'$.

Equipped with the distance function (1), $\mathbb{R}^3$ is a metric space $(\mathbb{R}^3, d_3)$. Consequently, the open subsets of $\mathbb{R}^3$ form a metric topology $\mathscr{T}_3$, and $(\mathbb{R}^3, \mathscr{T}_3)$ is a topological space. For a point $\mathbf{r} \in \mathbb{R}^3$, its neighborhood is a set $V(\mathbf{r}) \subset \mathbb{R}^3$, such that $\mathbf{r} \in U \subset V(\mathbf{r})$, where $U \in \mathscr{T}_3$. Any point $\mathbf{r} \in \mathbb{R}^3$ has uncountably many neighborhoods, and cannot be isolated from any of its neighborhoods. In other words, $\mathbf{r}$ is not an isolated point of any $V(\mathbf{r})$, i.e., for any sufficiently small real number $\gamma > 0$,

$$V(\mathbf{r}) \cap B(\mathbf{r}, \gamma) = B(\mathbf{r}, \gamma) \neq \{\mathbf{r}\}, \tag{2}$$

where

$$B(\mathbf{r}, \gamma) = \{\mathbf{r}' \in \mathbb{R}^3 : d_3(\mathbf{r}, \mathbf{r}') < \gamma\}$$

is an open ball with center $\mathbf{r}$ and radius γ. Consequently, so long as $\gamma > 0$, there always exist uncountably many points different from and arbitrarily close to $\mathbf{r}$ in $B(\mathbf{r}, \gamma)$, and hence the distance between any of such points and $\mathbf{r}$, given by (1), is strictly greater than zero. The point $\mathbf{r}$ considered above is arbitrary.

As shown above, no point in $\mathbb{R}^3$ can be isolated from any of its neighborhoods. Consequently, by using the distance function (1), we cannot obtain any desired point $\mathbf{r}$. Instead of the desired point $\mathbf{r}$, we can only obtain a neighborhood $V(\mathbf{r})$ as an approximation. The approximation is at best an infinitesimal volume. In this sense, precise coordinates are unattainable.

Implied by the properties of the topological space $(\mathbb{R}^3, \mathscr{T}_3)$, unattainability of precise coordinates of any point in $\mathbb{R}^3$ is a well-established mathematical fact based on the distance function (1), and essentially irrelevant to anything about measurement in practice, such as instrument precision or accuracy of measurement results. On the other hand, the distance function (1) is a necessary tool devised for practical measurement in general, and we need it when measuring physical quantities in space. However, because we cannot isolate any desired point in $\mathbb{R}^3$ from any of its neighborhoods by using (1), it is then prohibited, in principle, for us to attain precise coordinates by measurement. In other words, the property (2) rules out completely, in principle, the possibility for us to obtain precise coordinates of any point in space by using (1). Actually, because of the well-established mathematical fact characterized by (1) and (2), no measurement performed in space can be perfectly accurate. A measurement can only determine some neighborhood $V(\mathbf{r}) \supset B(\mathbf{r}, \gamma)$. The open ball $B(\mathbf{r}, \gamma)$ contains $\mathbf{r}$ together with uncountably many other points, such that the distance between any of such points and $\mathbf{r}$ is strictly greater than zero, no matter how small γ is.

Euclidean-2 space (denoted by $\mathbb{R}^2$) is a subset of $\mathbb{R}^3$, and so is $\mathbb{R}^+$, the set of nonnegative real numbers. Points in a plane are typically given by their coordinates in Euclidean-2 space. Instants of time are usually modeled by nonnegative real numbers. By restricting the distance function (1) respectively to Euclidean-2 space (for example, the xy-plane) and $\mathbb{R}^+$, we can make $\mathbb{R}^2$ and $\mathbb{R}^+$ into metric spaces. The restrictions of (1) to $\mathbb{R}^2$ and $\mathbb{R}^+$ are

$$d_2(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

and

$$d_+(s, t) = |s - t|,$$

where $x = (x_1, x_2), y = (y_1, y_2), x, y \in \mathbb{R}^2$, and $s, t \in \mathbb{R}^+$. Define

$$\mathscr{T}_2 = \{U \cap \mathbb{R}^2 : U \in \mathscr{T}_3\}$$

and

$$\mathscr{T}_+ = \{U \cap \mathbb{R}^+ : U \in \mathscr{T}_3\},$$

which are relative topologies inherited from $(\mathbb{R}^3, \mathscr{T}_3)$. The following statements are evident mathematical facts.

- The topological spaces $(\mathbb{R}^2, \mathscr{T}_2)$ and $(\mathbb{R}^+, \mathscr{T}_+)$ are subspaces of $(\mathbb{R}^3, \mathscr{T}_3)$.
- The collection $\mathscr{T}_2$ (respectively, $\mathscr{T}_+$) consists of relatively open sets of $\mathbb{R}^2$ (respectively, $\mathbb{R}+$).
- An open ball in the metric space $(\mathbb{R}^2, d_2)$ with center x and radius γ is a disc $\{y \in \mathbb{R}^2 : d_2(x, y) < \gamma\}$; an open ball in the metric space $(\mathbb{R}^+, d_+)$ with center $t > 0$ and radius γ is simply an open interval $(t - \gamma, t + \gamma)$, where $\gamma < t$.
- Unattainability of precise coordinates of points in $\mathbb{R}^2$ (respectively, on $\mathbb{R}^+$) is implied by properties of the corresponding metric topology $\mathscr{T}_2$ (respectively, $\mathscr{T}_+$).

With the mathematical facts we have just reviewed, now we are ready to scrutinize the quantum-mechanical superposition principle and the notion of quantum superposition.

3 Quantum Superposition Revisited

The notion of superposition given by its original form in quantum physics concerns waves, and is a consequence of the superposition principle proved in mathematics regarding solutions of a linear differential equation. In quantum mechanics, which is a mathematical theory of quantum physics, abstract quantum states and their superposition may not necessarily be solutions of a linear differential equation. As a generalization of the superposition principle proved for linear differential equations, the quantum-mechanical superposition principle is simply taken to be a basic postulate, which is not reducible to anything more elementary and hence needs no proof. According to the quantum-mechanical superposition principle, if a quantum system can be in either of two states, the system can also be in a linear combination of the two states. This linear combination is also referred to as a quantum superposition.

Thus, roughly speaking, quantum superpositions may be divided into two categories: quantum superposition of waves, resulting from the superposition principle proved in mathematics regarding solutions of a linear differential equation, and quantum superposition of abstract quantum states, where quantum states may be associated either with a single quantum object, or with a system consisting of multiple quantum objects. In this article except Section 5, where we discuss the EPR experiment and the so-called entangled state in experimental tests of Bell inequalities, our focus is mainly on quantum superposition of waves associated with a single particle, and superposition of quantum states associated with a single quantum object.

3.1 Quantum Superposition of Waves

Associated with a single particle, a wave function (also referred to as a quantum state) is a solution of a wave equation. The wave equation can have different solutions, which are wave functions associated with the *same single* particle. Let $\psi_i(\mathbf{r}, t), i = 1, 2, \cdots, n$ be solutions of the wave equation, such that none of them is identically zero. In current quantum theory, the wave functions represent orthogonal states, corresponding to mutually exclusive properties of the particle. The wave equation is a linear differential equation, and hence satisfies the superposition principle, according to which

$$\Psi(\mathbf{r}, t) = \sum_{i=1}^{n} c_i \psi_i(\mathbf{r}, t) \tag{3}$$

is also a solution. The linear combination given by (3) is a quantum superposition, where $c_i, i = 1, 2, \cdots, n$ are complex numbers. By *assumption*, $\Psi(\mathbf{r}, t)$ and $\psi_i(\mathbf{r}, t), i = 1, 2, \cdots, n$ are all associated with the *same single* particle. Applying Born's probabilistic interpretation of wave functions and the normalization condition to (3), we have

$$\sum_{i=1}^{n} |c_i|^2 = 1,$$

where $|c_i|^2$ is the probability of finding the particle in state ψ_i. According to Born's probabilistic interpretation, for a wave function $\psi(\mathbf{r}, t)$,

$$f(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2 \text{ with } \int_{\mathbb{R}^3} f(\mathbf{r}, t) dv = 1$$

is a time-dependent probability density, and hence $f(\mathbf{r}, t)dv$ is the probability of finding the particle in an infinitesimal volume dv containing point $\mathbf{r} \in \mathbb{R}^3$ at time t. Clearly, by treating $f(\mathbf{r}, t)dv$ as a probability, Born's probabilistic interpretation of wave functions has already taken into account unattainability of precise position coordinates of a particle in space.

However, unattainability of precise time coordinates is omitted in Born's probabilistic interpretation. Moreover, in (3), an important physical constraint imposed on measuring individual quantum objects is also omitted. According to this constraint, *the same single quantum object can at most be measured only once*. This is why we need a sequence of identically prepared quantum objects of the same kind for experiments in quantum physics. Because the constraint above and unattainability of precise time coordinates are omitted, (3) is not an experimental fact; it is only mathematically meaningful as a consequence of the superposition principle proved in mathematics regarding solutions of a linear differential equation, but meaningless when we use it to describe or explain phenomena observed in quantum physics. Therefore, although $\psi_i, i = 1, 2, \cdots, n$ and Ψ satisfy the same linear differential equation, Ψ is physically meaningless, and so is the assertion that the same single particle can possess mutually exclusive properties simultaneously before measurement. Nevertheless, the same particle may still have mutually exclusive properties at *different* times.

In fact, what can be observed in an actually performed experiment are *different* particles of the same kind, each of which is associated with *one and only one* of the wave functions $\psi_i, i = 1, 2, \cdots, n$, and can *at most be detected only once* at unknown, almost surely *different* instants. Consider, for example, a sequence $(\rho_k)_{k\geq 1}$ of identically prepared particles of a given kind. Suppose we want to detect $\rho_k, k = 1, 2, \cdots$ at an arbitrarily given instant τ represented by a precise coordinate after a time interval $[0, \tau]$ elapses in different runs of the experiment. At the beginning of the interval, i.e., at $t = 0$, each prepared particle is ready for detection under the purported same condition. In general, once a particle has been detected, it cannot be detected anymore. One at a time, the particles are detected. For $k = 1, 2, \cdots$, we may find the k-th particle ρ_k, which is associated with one of the wave functions, say, ψ_j, with a positive probability $f_j(\mathbf{r}_k, t_k)dv_k$, in an infinitesimal volume dv_k containing $\mathbf{r}_k$ represented by precise space coordinates during an infinitesimal time interval containing both t_k and τ, where t_k, represented by a precise time coordinate, is an unknown instant, and

$$f_j(\mathbf{r}_k, t_k) = |\psi_j(\mathbf{r}_k, t_k)|^2$$

is the corresponding probability density for ρ_k to be detected at point $\mathbf{r}_k$ at instant t_k. All instants $t_k, k = 1, 2, \cdots$ are unknown to us, and cannot be predicted in any way. The instants are random points in an infinitesimal interval.

As shown above, omission of unattainability of precise time coordinates is responsible for confusion of infinitesimal time intervals and instants of time. The confusion eventually leads to various incorrect explanations of experiments involving quantum superpositions. However, in a way similar in spirit to Born's probabilistic interpretation, we can take into account unattainability of precise time coordinates when explaining measurement results obtained by experiments with quantum objects, although we may not be able to find a probability distribution for the involved instants, because they are all in an infinitesimal interval.

3.2 Superposition of Abstract Quantum States

As a basic postulate of current quantum theory, the quantum-mechanical superposition principle is misleading. Firstly, this generalized superposition principle obscures the connection to its original form proved in mathematics regarding solutions of a linear differential equation. Secondly, unattainability of precise coordinates for representing directions in space is omitted. Finally, because the original superposition principle is merely proved for linear differential equations without considering physical constraint imposed on measuring individual quantum objects, its generalized form is not eligible to serve as a basic postulate of quantum theory.

To explain further why the quantum-mechanical superposition principle is not eligible, consider, for example, an ensemble of identically prepared spin-1/2 particles. Let $(q_k)_{k\geq 1}$ be a sequence of particles in the ensemble. Mathematically, a spin-1/2 particle lives in a Hilbert space spanned by two eigenvectors $|\uparrow\rangle$ and $|\downarrow\rangle$. Thus neither time dependence of the states nor spatial motion of the particle needs to be considered here. This system is described by a linear combination of the eigenvectors.

$$|\psi\rangle = c_1 |\uparrow\rangle + c_2 |\downarrow\rangle \,, \tag{4}$$

where c_1 and c_2 are complex numbers. The linear combination given by (4) is a quantum superposition. None of $|\uparrow\rangle$, $|\downarrow\rangle$, and $|\psi\rangle$ is a solution of a linear differential equation. According to (4), when a spin-1/2 particle is in the state $|\psi\rangle$, it has no definite spin in any direction; the particle has two states $|\uparrow\rangle$ and $|\downarrow\rangle$ in every direction before measurement. As a consequence of the quantum-mechanical superposition principle, (4) simply represents a way to account for the weird behavior of the particles observed in a Stern-Gerlach experiment.

Let q_i and q_j, where $i \neq j$, be two arbitrarily given particles in $(q_k)_{k\geq 1}$. Measuring the spin of q_i along a direction yields a deterministic outcome. Measuring the spin of q_j along the purported same direction for measuring the spin of q_i also yields a deterministic outcome, but it may or may not be identical to the outcome obtained by measuring the spin of q_i. The outcomes may correspond to either $|\uparrow\rangle$ or $|\downarrow\rangle$ with a probability equal to $|c_1|^2$ or $|c_2|^2$, where

$$|c_1|^2 + |c_2|^2 = 1.$$

There seems to be no way for us to distinguish any one particle in $(q_k)_{k\geq 1}$ from any other. In other words, the weird behavior of the particles looks inexplicable; this is exactly why (4) is needed here, which seems to be the only way to account for such behavior. In fact, however, (4) is not only unnecessary but also misleading.

Spin measurements are performed in space modeled by $\mathbb{R}^3$. As we can readily see, each direction in $\mathbb{R}^3$ corresponds to one and only one point in the set D given below.

$$D = \{\mathbf{r} \in \mathbb{R}^3 : d(\mathbf{r}, 0) = 1\}.$$

Consequently, directions or orientations in space can be represented by coordinates of the points in D. Because the unattainability applies to points in D, precise coordinates of any point in D are unattainable.

Now consider the Stern-Gerlach experiment again. Suppose we want to measure spins of particles in $(q_k)_{k\geq 1}$ along an arbitrarily given direction in $\mathbb{R}^3$ represented by precise coordinates of the corresponding point $\mathbf{a}$ in D for spin measurements. However, because precise coordinates are unattainable, an infinitesimal volume V containing $\mathbf{a}$ is what we can use for the measurements at best. For the same reason, we are not aware of the exact directions $\mathbf{a}_i$ and $\mathbf{a}_j$ in D, where $i \neq j$, along which the spins of q_i and q_j are measured, so $\mathbf{a}_i$ and $\mathbf{a}_j$ may not necessarily be identical to $\mathbf{a}$. Nevertheless, $\mathbf{a}$, $\mathbf{a}_i$, and $\mathbf{a}_j$ are all contained in V.

Suppose $\mathbf{a}_j$ is inclined at a tiny angle to $\mathbf{a}_i$. The angle between $\mathbf{a}_i$ and $\mathbf{a}_j$ is unknown, and cannot be predicted in any way, so it is a value taken by a random variable θ. Because V is an infinitesimal volume, θ is a continuous, infinitesimal random variable, although the probability distribution and values of θ are unknown. Evidently, $\{\theta = 0\}$ is an event of probability zero so long as $i \neq j$. In other words, $\mathbf{a}_i$ and $\mathbf{a}_j$ are different almost surely. Consequently, the two outcomes obtained by measuring the spins of q_i and q_j may or may not be identical. This may explain, in an intuitively understandable way without involving (4), why the outcomes obtained by measuring the spins of identically prepared particles in an actually performed experiment are unpredictable, although the measurements seem to be performed along the purported same direction.

In the everyday world, we may treat an infinitesimal quantity as zero; such treatment is a useful and sometimes necessary approximation when we measure or calculate physical quantities. However, treating θ as zero amounts to mistaking V, which is an infinitesimal volume, for the exact directions represented by precise coordinates contained in V, along which the measurements are actually performed. In fact, the seemingly inexplicable behavior of the particles is due to ignorance of knowledge about the exact directions contained in V as implied by unattainability of precise coordinates, and (4) is merely a consequence of the misleading quantum-mechanical superposition principle rather than an experimental fact. By distinguishing strictly infinitesimal volumes from points in D when explaining the measurement results obtained in the Stern-Gerlach experiment, we can take into account unattainability of precise coordinates representing exact directions in a way similar in spirit to Born's probabilistic interpretation of wave functions, although we may not be able to find a probability distribution for the involved directions.

4 Reinterpreting Quantum Superposition

According to current quantum theory, orthogonal states in a quantum superposition of a system are orthonormal vectors added by vector addition, which requires use of conjunction as the logical relation between mutually exclusive properties corresponding to the orthogonal states if we do not measure the system. Before measurement, the orthogonal states are supposed to hold for the system simultaneously at any instant represented by a precise time coordinate or along any direction represented by precise space coordinates. As a result, use of conjunction as the logical relation between mutually exclusive properties implies omission of unattainability of precise coordinates, which is an unfortunate flaw in the conceptual scheme of current quantum theory.

Because it is vector addition that requires use of conjunction as the logical relation between mutually exclusive properties, omission of unattainability of precise coordinates is essentially implied by the notion of quantum superposition. On the other hand, because any single quantum object can *at most be measured only once*, what can be actually observed by measurement is a *single* quantum object in one of the orthogonal states, which requires use of disjunction as the logical relation between mutually exclusive properties after measurement. However, as shown in the previous section, independent of our observation, the exclusive properties can only belong to *different quantum objects at different times or in different directions*. The logical relations between these properties before and after measurement remain unchanged. Before measurement, the logical relation is also disjunction, which is just mistaken for conjunction mainly because unattainability of precise coordinates is omitted. Therefore, to repair the flaw in the conceptual scheme of current quantum theory, we may have to reinterpret the meaning of quantum superposition.

In current quantum theory, a quantum superposition ψ is a generalized vector in an abstract multi-dimensional vector space spanned by orthonormal vectors ψ_i with expansion coefficients c_i.

$$\psi = \sum_i c_i \psi_i, \tag{5}$$

where states ψ_i serve as the basis vectors of the vector space. By calculating $\langle \psi_i | \psi \rangle$, we can obtain the expansion coefficients c_i, from which we can further obtain the probabilities $|c_i|^2$. However, if ψ describes the same quantum object as assumed in current quantum theory, this quantum object appears to possess simultaneously the mutually exclusive properties corresponding to the orthonormal states before measurement, because $|c_i|^2 > 0$ for each i. If a measurement is performed on the system, the measurement triggers an abrupt collapse of ψ onto one of the orthonormal states, say, ψ_j, and the probability for ψ to collapse onto ψ_j is $|c_j|^2$. According to the quantum-mechanical interpretation, randomness exhibited in the measurement results is inherent, coming from nowhere. However, as shown in the previous section, quantum superposition is physically meaningless. The quantum-mechanical interpretation just *attaches* some physical meaning to it. Moreover, because unattainability of precise coordinates is omitted in current quantum theory, mutually exclusive properties corresponding to different orthonormal states associated with *different* quantum objects are *attached* to the *same* quantum object.

The above analysis suggests a feasible way to repair the flaw in the conceptual scheme of current quantum theory without affecting eligible applications of quantum mechanics: We may simply remove any physical meaning attached to quantum superposition by taking vector addition merely as a means of calculation to obtain the probabilities $|c_i|^2$. This reinterpretation of quantum superposition differs from the so-called "statistical" interpretation of quantum mechanics [17]. The former is an effort to repair the flaw in the conceptual scheme of current quantum theory, but the latter is not.

Needless to say, the calculated probabilities $|c_i|^2$ can be verified by measurement in actually performed experiments, and of course, quantum-mechanical predictions are always in agreement with the experimental results. However, because of 1) unattainability of precise coordinates, and 2) the physical constraint imposed on measuring individual quantum objects, i.e., any single quantum object can at most be measured only once, we cannot find correct answers to the following two questions by experiment.

1. Why are the measurement outcomes random rather than deterministic?
2. Are the measurement outcomes inherently random or merely due to our ignorance of some relevant knowledge?

The two questions are crucial, as they reflect, in a sense, the most important issue debated by Einstein and Bohr. However, as a version of Bohr's point of view, the "statistical" interpretation [17] is not intended to answer the above questions at all. Actually, the answers to the questions have already been given in the previous section. It may be worth providing the answers again concerning a general experimental setting. By doing so, we can establish a connection between quantum-mechanical probabilities and classical probability theory formulated based on generally accepted Kolmogov's axioms, where quantum-mechanical probabilities are simply probabilities in quantum mechanics and have nothing to do with the so-called quantum probability, as we shall see in the next section. The connection is helpful for us to understand that there is nothing mysterious about quantum-mechanical probabilities; however, because of unattainability of precise coordinates, measurement results obtained by experiment are not sufficient for us to answer the two crucial questions.

Because a single quantum object can at most be measured only once in an actually performed experiment, we must consider a sequence $(q_k)_{k\geq 1}$ of identically prepared quantum objects of the same kind. For each k, let $H(q_k)$ be the result obtained by measuring q_k. Because precise coordinates are unattainable, in different runs of the experiment, $H(q_k)$ cannot remain unchanged. Suppose each q_k has n possible measurement outcomes, i.e.,

$$H(q_k) \in \{h_1, h_2, \cdots, h_n\}.$$

Denote by $\mathbb{N}$ the set of positive integers. The outcomes obtained by measuring $q_k, k = 1, 2, \cdots$ then constitute a set Ω given below.

$$\Omega = \{H(q_k) : k \in \mathbb{N}\}.$$

To connect classical probability theory with probabilities in quantum mechanics, we show that quantum-mechanical probabilities can be obtained by conventional analytic techniques in classical probability theory. Let $\mathscr{A}$ be the σ-algebra of subsets of Ω, and $\mathbb{P}$ the probability measure on $\mathscr{A}$. Now we have a probability space $\{\Omega, \mathscr{A}, \mathbb{P}\}$ in classical probability theory. For $j = 1, 2, \cdots, n$, write

$$N_j = \{k \in \mathbb{N} : H(q_k) = h_j\},\ \Omega_j = \{H(q_k) \in \Omega : k \in N_j\}.$$

Clearly, $N_j, j = 1, 2, \cdots n$ form a partition of $\mathbb{N}$.

$$\bigcup_{j=1}^{n} \mathbb{N}_j = \mathbb{N},\ \bigcap_{j=1}^{n} \mathbb{N}_j = \emptyset.$$

Accordingly, $\Omega_j, j = 1, 2, \cdots n$ constitute a partition of Ω.

$$\bigcup_{j=1}^{n} \Omega_j = \Omega,\ \bigcap_{j=1}^{n} \Omega_j = \emptyset.$$

The partitions of $\mathbb{N}$ and Ω formed respectively by $\{N_j : j = 1, 2, \cdots, n\}$ and $\{\Omega_j, j = 1, 2, \cdots, n\}$ are random, because both $\{k \in N_j\}$ and $\{H(q_k) \in \Omega_j\}$ are random events for arbitrarily given k and j. Needless to say, we can measure the frequencies of $\{H(q_k) \in \Omega_j\}$ in the experiment. The strong law of large numbers in classical probability theory will guarantee the frequencies converging to the corresponding probabilities calculated in quantum mechanics. To see this, define

$$X_{j,k} = \begin{cases} 1, & k \in N_j \\ 0, & k \notin N_j. \end{cases}$$

Clearly, for any fixed j, $X_{j,k}, k = 1, 2, \cdots$ are independent and identically distributed random variables. Because $k \in N_j$ if and only if $H(q_k) \in \Omega_j$, we have

$$\mathbb{E}(X_{j,k}) = \mathbb{P}(k \in N_j) = \mathbb{P}[H(q_k) \in \Omega_j].$$

According to quantum mechanics, $|c_j|^2$ is the probability for us to find the outcome $H(q_k)$ in Ω_j, i.e.,

$$\mathbb{P}[H(q_k) \in \Omega_j] = |c_j|^2.$$

Write

$$M_{j,\ell} = \sum_{k=1}^{\ell} X_{j,k}.$$

By the strong law of large numbers,

$$\mathbb{P}\left\{\lim_{\ell\to\infty} \sum_{k=1}^{\ell} \frac{M_{j,\ell}}{\ell} = \mathbb{E}(X_{j,k}) = |c_j|^2\right\} = 1.$$

As demonstrated above, quantum-mechanical probabilities can indeed be obtained by conventional analytic techniques in classical probability theory, and hence are not mysterious at all. However, without taking into account unattainability of precise coordinates, we cannot answer the two crucial questions by means of measurement and experiment.

Answer to question (i): The outcomes $h_j, j = 1, 2, \cdots, n$ are random rather than deterministic, because they are results obtained by measurement at *almost surely different*, unknown instants contained in an infinitesimal time interval, or along *almost surely different*, unknown directions represented by precise coordinates contained in an infinitesimal volume. Because unattainability of precise coordinates is omitted, the infinitesimal time interval is mistaken for a desired instant represented by a precise time coordinate (Subsection 3.1), and similarly, the infinitesimal volume is mistaken for a desired direction represented by precise space coordinates (Subsection 3.2).

Answer to question (ii): Randomness exhibited in the measurement outcomes is not inherent. As implied by unattainability of precise coordinates, quantum randomness is due to our ignorance of knowledge concerning space and time, but we may not be able to find a probability distribution to characterize the desired time or space coordinates involved in the experiment, because such coordinates are contained in infinitesimal intervals or volumes as shown in Section 3. From the measurement outcomes we cannot identify the origin of quantum randomness. Failing to capture the origin of quantum randomness can lead to serious consequences.

5 Discussion

One of the most controversial issues concerning the foundations of current quantum theory is the standard interpretation of randomness exhibited in quantum superposition for description of an individual microscopic object. According to the standard interpretation, quantum randomness is inherent, irrelevant to our subjective ignorance of relevant knowledge. Although numerous experiments have confirmed quantum-mechanical predictions, scientists and researchers in the minority of the scientific community have never ceased to question the standard interpretation since the inception of quantum mechanics.

5.1 Bell Inequalities and the EPR Experiment

As we all known, in his debate with Bohr, Einstein argued against the standard interpretation, and never changed his objection. Together with Podolsky and Rosen, Einstein proposed the famous EPR experiment [5], in which two correlated and spatially separated systems, particle I and particle II, are considered. The two systems had previously interacted, then separated, and there is no longer any interaction between them. After the two particles are separated, by assuming freedom of choice, one can choose to measure either of two complementary observables, such as momentum and position of a system, say, particle I. From the measured outcome of particle I, the value of the same observable of particle II can be obtained by prediction without measurement, and measuring particle I will not in any way disturb particle II.

In the following, we consider a slightly different version of the EPR experiment to simplify the argument. With the simplified argument, we no longer assume freedom of choice but impose explicitly the constraint on measuring individual quantum objects, according to which *the same particle can at most be measured only once.* Because the two systems are correlated, by measuring momentum of particle I and position of particle II, position of particle I and momentum of particle II can be obtained by prediction from the measured outcomes corresponding to position of particle II and momentum of particle I, respectively. Thus, each system can be measured without disturbing in any way the other system while values of position and momentum can be obtained for both systems. The EPR experiment actually reveals a contradiction implied by Heisenberg's uncertainty relation and the quantum-mechanical description of the combined system consisting of particle I and particle II. According to the uncertainty relation, momentum and position of a particle cannot be simultaneously measured to arbitrary precision because of disturbance caused by simultaneous measurements of position and momentum of the same particle.

Motivated by the Einstein-Bohr debate, and inspired by David Bohm's work [18], John S. Bell derived the first of Bell inequalities [14, 19], attempting to express the EPR argument mathematically by introducing some hidden variable to account for quantum randomness, purportedly under the assumptions of locality, realism, and freedom of choice. Unlike the EPR experiment, real experiments can be performed to test Bell inequalities [20, 21, 22]. After three Bell experiments attempting to close all the relevant loopholes at once [23, 24, 25], it is claimed that the Einstein-Bohr debate is purportedly settled ultimately [12].

Unfortunately, Bell inequalities fail to capture the origin of quantum randomness. Irrelevant to the hidden variable and the corresponding assumptions introduced by Bell, quantum randomness is due to our subjective ignorance of knowledge concerning space and time. The ignorance is implied by unattainability of precise space or time coordinates. As a well-established mathematical fact irrelevant to anything about measurement in practice, unattainability of precise coordinates does not allow us to obtain precise space or time coordinates by measurement in principle. Therefore, it is impossible to resolve the Einstein-Bohr debate by experimental tests of Bell inequalities, and hence the Einstein-Bohr debate remains unsettled.

In fact, quantum randomness is not necessarily in conflict with the existence of reality corresponding to physical quantities independent of our observation. As correctly pointed out by EPR [5], the question whether elements of physical reality exist can only be answered "by an appeal to results of experiments and measurements" rather than by a priori philosophical considerations, yet measurement results obtained by experiment are not sufficient for us to capture the origin of quantum randomness caused by unattainability of precise coordinates.

5.2 Quantum Entanglement

In the derivation of the original Bell inequality [14], Bell considered a sequence of ordered pairs (p_1, p_2) of spin 1/2 particles. Each pair is quantum-mechanically described by a singlet state.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle), \tag{6}$$

which corresponds to the spins (σ_1, σ_2) of the particles in the same pair (p_1, p_2), where σ_1 and σ_2 are aligned anti-parallel to each other in any direction before measurement. This $|\psi\rangle$ is a so-called entangled state, which is a linear combination of spin states of the two particles in the same pair. In other words, $|\psi\rangle$ is a quantum superposition of $|\uparrow_1\rangle|\downarrow_2\rangle$ and $|\downarrow_1\rangle|\uparrow_2\rangle$.

Taking the entangled state given by (6) as an example, now we can apply the results presented in previous sections to scrutinize the notion of quantum entanglement. Suppose we want to measure (σ_1, σ_2) for each pair (p_1, p_2) along a given direction $\mathbf{a}$. For $k = 1, 2, \cdots$, denote by $H_k(p_1, p_2)$ the outcomes obtained by measuring (σ_1, σ_2) for the k-th pair.

$$H_k(p_1, p_2) \in \{h_1, h_2\},$$

where h_1 and h_2 correspond to $|\uparrow_1\rangle|\downarrow_2\rangle$ and $|\downarrow_1\rangle|\uparrow_2\rangle$, respectively. The outcomes $h_j, j = 1, 2$ are random rather than deterministic, because they are results of measuring the spins for *different* pairs in *almost surely different, unknown directions* $\mathbf{a}_k, k = 1, 2, \cdots$. The precise coordinates of $\mathbf{a}_k$ and $\mathbf{a}$ are all contained in an infinitesimal volume V. The outcomes $H_k(p_1, p_2)$ are elements of the following set.

$$\Omega = \{H_k(p_1, p_2) : k \in \mathbb{N}\}.$$

For $j = 1, 2$, write

$$N_j = \{k \in \mathbb{N} : H_k(p_1, p_2) = h_j\}, \ \Omega_j = \{H_k(p_1, p_2) \in \Omega : k \in N_j\}.$$

Clearly, $\{N_1, N_2\}$ form a random partition of $\mathbb{N}$, and $\{\Omega_1, \Omega_2\}$ constitute a random partition of Ω. For each k, the outcome $H_k(p_1, p_2)$ falls into either Ω_1 or Ω_2 with equal probabilities $1/2$.

Because unattainability of precise coordinates is omitted, the infinitesimal volume V is mistaken for the desired direction $\mathbf{a}$. Consequently, randomness exhibited in the measurement outcomes is due to our ignorance of knowledge concerning exact directions represented by precise space coordinates for spin measurements rather than inherent, and has nothing to do with the hidden variable and the corresponding assumptions, i.e., realism, locality, and freedom of choice introduced by Bell. Actually, the notion of quantum entanglement is not physically meaningful; the standard interpretation of the experimental invalidation of Bell inequalities [8, 9, 10, 11, 12, 13] just attaches some physical meaning to it. The above discussion also applies to the entangled state describing ordered pairs of correlated photons in optical tests of Bell inequalities [22].

5.3 Statistical Regularities in Quantum and Classical Physics

As a consequence of unattainability of precise coordinates, quantum randomness is due to lack of knowledge concerning time and space, such as exact instants or directions involved in actual measurements of physical quantities. The involved instants or directions are unknown to us, and cannot be predicted in any way. In this sense, they must be described by random quantities. On the other hand, however, precise coordinates of such random quantities are contained in infinitesimal intervals or volumes, and hence cannot be characterized by statistical regularities usually observed in the everyday world, where "statistical regularities" refer to probability distributions or statistics such as expectation values and mean-square deviations calculated with probability distributions or probability densities. In this sense, quantum randomness is subtle. Because of the subtlety, infinitesimal intervals or volumes are mistaken for precise but practically unattainable coordinates.

Macroscopic objects in classical physics and microscopic objects in quantum physics are all measured in time and space. In classical physics, the same macroscopic object may be measured as many times as we can. For instance, a coin is a macroscopic object. Tossing the coin is a random experiment, as the outcome of each toss may be either a "head" or a "tail", and cannot be predicted with certainty, where randomness is due to subjective ignorance of initial condition and environment for each toss. Nevertheless, the *same* coin can be tossed repeatedly. As the number of tosses increases, the frequency of each outcome gradually approaches a definite probability. After a large number of tosses, a statistical regularity can be observed.

Unlike macroscopic objects, any single quantum object can *at most be measured only once*. For example, pairs of spin 1/2 particles considered by Bell [14] are quantum objects. Needless to say, detection of individual particles are also random events, where randomness is due to lack of relevant knowledge concerning directions along which the spins of the particles are actually measured. After a particle is registered at a detector, the *same single* particle is not available for detection anymore.

Therefore, because no statistical regularity can be observed by measuring the *same single* quantum object, a statistical regularity concerning individual quantum objects of a given kind must be a consequence of measuring a large number of *different* quantum objects of the same kind. Such a statistical regularity is useful for us to understand a *typical* quantum object representative of a given kind of individual quantum objects, but may not be appropriate to describe a *particular* quantum object.

5.4 Quantum-Mechanical Probabilities and Classical Probability

According to current quantum theory, probabilities in quantum mechanics differ from classical probability mainly because the former can be obtained by calculation in quantum mechanics.

$$|\langle\psi_i|\psi\rangle|^2 = |c_i|^2,$$

where c_i are the expansion coefficients of a vector given by (5) in a Hilbert space, purportedly showing that quantum mechanics is intrinsically probabilistic and the so-called quantum probability contradicts Kolmogorov's axioms adopted in classical probability.

However, as shown in Section 3, quantum randomness is due to lack of knowledge concerning time and space, such as exact instants or directions involved in actual measurements of physical quantities. The involved instants or directions must be described by random quantities but cannot be characterized by statistical regularities usually observed in classical physics, because their precise coordinates are contained in infinitesimal intervals or volumes. The seemingly intrinsically probabilistic character of current quantum theory is actually due to omission of unattainability of precise space or time coordinates, which is an unfortunate flaw in the conceptual scheme of current quantum theory. Because of the flaw, the origin of quantum randomness is missing from quantum theory in its current form.

Characterized by classical probability, randomness exhibited in results of measuring macroscopic objects is due to subjective ignorance, which may be reduced by improving our knowledge. Characterized by probabilities in quantum mechanics, quantum randomness is also due to subjective ignorance, but improving our knowledge may not reduce the subjective ignorance concerning quantum objects about exact instants of time or directions in space represented by precise coordinates contained in infinitesimal intervals or volumes. This is the principal difference between probabilities in quantum mechanics and classical probability. So long as we take into account unattainability of precise coordinates when describing or explaining measurement results obtained by experiments in quantum physics, we can still use classical probability as shown in Section 4. The notion of quantum probability is unnecessary and misleading.

5.5 Uncertainty Relation and Kochen-Specker Theorem

As demonstrated by the EPR experiment [5], Heisenberg's uncertainty relation and the quantum-mechanical description for a pair of spatially-separated but correlated particles imply a contradiction. The revealed contradiction indicates at least one thing: the uncertainty relation is irrelevant to the individual particles considered in the EPR experiment. Expressed in terms of statistics concerning position and momentum corresponding to measurement results of a large number of *different* particles of the same kind, Heisenberg's uncertainty relation makes little sense if we use it to describe a *particular* particle as shown in the preceding two subsections.

Furthermore, the proof of the uncertainty relation is based on the commutator for operators corresponding to position and momentum of the *same single* particle, with Planck's constant $\hbar$ serving to determine the scale of the quantum fuzziness responsible for the incompatibility, which results in the uncertainty relation. Consider two observables α and β of a *single* quantum object described by a general state ψ in the form of a quantum superposition. Let $\hat{A}$ and $\hat{B}$ represent two operators associated with the two observables. The commutator for the two operators is defined by

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.$$

For example, α and β may represent position and momentum of a *single* particle, such as particle I or particle II in the EPR experiment.

According to current quantum theory, the only impediment to simultaneous determination of values for α and β by measurement is

$$[\hat{A}, \hat{B}]\psi \neq 0. \tag{7}$$

However, the commutator allows the *same single* quantum object to be measured more than once, which violates the constraint imposed on measuring individual quantum objects, i.e, the *same single* quantum object cannot be measured twice. This constraint is more stringent than (7). Therefore, after measuring either α or β but not both, we cannot measure the quantum object anymore. Nevertheless, the constraint imposed on measuring individual quantum objects is not necessarily an impediment for α and β to have definite values before measurement, as shown correctly by EPR [5].

In quantum theory in its current form, Kochen-Specker theorem [16] serves as an alternative demonstration that quantum mechanics is intrinsically probabilistic. However, just like the proof of Heisenberg's uncertainty relation, in all the proofs of Kochen-Specker theorem, commutators for operators corresponding to observables of the *same single* quantum object are taken for granted, and measuring the *same single* quantum object is treated as tossing the same coin, as if we could measure the *same single* quantum object as many times as we toss the same coin. Clearly, Kochen-Specker theorem also violates the physical constraint imposed on measuring individual quantum objects, and hence appears to make little sense physically.

On the other hand, without taking into account unattainability of precise coordinates, the uncertainty relation, Kochen-Specker theorem, and Bell inequalities all fail to capture the origin of quantum randomness. Although Bell's work [14, 19] is motivated by the Einstein-Bohr debate, introducing the hidden variable under the corresponding assumptions (i.e., locality, realism, and freedom of choice) appears to be an unsuccessful effort to interpret quantum mechanics in terms of a statistical account, and eventually results in the standard interpretation of the experimental invalidation of Bell inequalities [8, 9, 10, 11, 12, 13]. The standard interpretation is misleading, as it is exactly what EPR argued against reasonably.

5.6 Quantum Information

Quantum mechanics has led to a large number of successful applications in practice so far. A successful application of quantum mechanics in practice must be an eligible application, such that its aim is not to realize anything that is not physically meaningful. However, because of the standard interpretation of the experimental invalidation of Bell inequalities [8, 9, 10, 11, 12, 13], physically meaningless quantum superposition and quantum entanglement are considered resources physically real "out there" in the physical world. By trying to close the door on the Einstein-Bohr debate [12], the standard interpretation opens the door to various ineligible applications, such as quantum computation and quantum communication [15]. In these ineligible applications, a quantum superposition of two orthogonal states is treated as a "quantum bit" for quantum information processing, and quantum entanglement is considered necessary for quantum cryptography and teleportation.

As we can readily see, the ineligible applications share two things in common: One is omission of unattainability of precise coordinates, which is an unfortunate flaw in the conceptual scheme of current quantum theory; the other is trying to exploit physically meaningless quantum superposition and entanglement. Such applications are doomed to failure. Instead of doing nothing about the above already identified flaw, we must take into account unattainability of precise coordinates to repair the flaw. As shown in Section 4, by reinterpreting the meaning of quantum superposition, we can indeed repair the flaw without affecting eligible applications of quantum mechanics while avoiding all the ineligible applications.

6 Conclusion

Unattainability of precise coordinates is a well-established mathematical fact implied by properties of the corresponding metric topology. Based on this fact, the main findings reported in this article are as follows.

- Irrelevant to instrument precision and measurement accuracy, unattainability of precise coordinates rules out completely, in principle, the possibility for us to obtain precise time or space coordinates by measurement.
- Instead of precise coordinates, infinitesimal intervals or volumes containing desired coordinates are our best approximations for measuring physical quantities in actually performed experiments.
- Unattainability of precise coordinates is hardly noticeable and hence can be safely omitted in classical mechanics. This is why results obtained by measurement in classical physics can still be reasonably explained even though precise coordinates are unattainable.
- Because a single quantum object can at most be measured only once, unattainability of precise coordinates plays a vital role in quantum mechanics, and should not be omitted when we explain results obtained by measurement in quantum physics.
- As a consequence of unattainability of precise time or space coordinates, quantum randomness is epistemic, due to our ignorance of relevant knowledge concerning time and space, rather than inherent.
- The notion of quantum superposition implies omission of unattainability of precise coordinates, which is an unfortunate flaw in the conceptual scheme of current quantum theory. The flaw makes quantum superposition physically meaningless, and is largely responsible for various ineligible applications of quantum mechanics.
- In no sense does the flaw imply quantum mechanics as a successful theory failing to be correct. By reinterpreting the meaning of quantum superposition, we can repair the flaw without affecting eligible applications of quantum mechanics while getting rid of all the ineligible applications.

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